DIOPTRICAE

PARS PRIMA

CONTINENS

LIBRVM PRIMVM,

DE

EXPLICATIONE PRINCIPIORVM,

EX QVIBVS

CONSTRVCTIO TAM TELESCOPIORVM

QVAM

MICROSCOPIORVM

EST PETENDA.

AVCTORE

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EXPLICATIONEM PRINCIPIORVM,

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EST PETENDA.

CAPUT I.

DE DIFFUSIONE IMAGINIS PER UNICAM LENTEM REPRAESENTATAE.

Definitio I.

1.

Imago principalis vocatur ea, quae a radiis axi lentis proximis per lentem refractis repraesentatur. Tab. I. Fig. 1.

Scilicet si lentis PP axis sit EF in eaque tantum spatium minimum aAa sit apertum, per quod radiis transitus concedatur, radii a puncto lucido E emissi in uno puncto F colligentur, quod punctum imago principalis vocatur.

Coroll. 1.

2. Cum nempe apertura aAa ſit minima, omnes radii, qui a puncto quocunque in eam incidunt, ita aequabilem patiuntur refractionem, vt omnes iterum in vnum punctum colligantur, id quod non ſolum de puncto lucido E in axe lentis ſito, ſed etiam de quibusuis aliis extra axem poſitis eſt intelligendum.

Coroll. 2.

3. Quodſi ergo lentis apertura fuerit minima, ſingula cuiusuis obiecti puncta poſt refractionem iterum ſingulis punctis referentur, ſicque imago principalis erit diſtincta et confuſione carebit: ſiquidem confuſio tum demum oritur, quando radii ex vno puncto emiſſi non iterum in vno puncto colliguntur.

Coroll. 3.

4. Et quamdiu apertura lentis aAa eſt minima, nihil intereſt, ſecundum quamnam figuram facies lentis ſint elaboratae; quaecumque enim earum figura fuerit, quoniam tantum portiuncula minima in computum venit, ea ſemper vt ſphaerica ſpectari poterit.

Coroll. 4.

5. Pendet ergo locus imaginis principalis primum a loco puncti lucidi E, ſiue id in axe ſiue extra axem fuerit ſitum; deinde a ſphaericitate vtrius-

vtriusque faciei aAa et bBb refringentis ; tertio ab earum distantia AB seu lentis crassitie, et quarto a ratione refractionis, quam radii in transitu per lentem patiuntur.

Scholion 1.

6. Refractio radiorum per huiusmodi lentium aperturas minimas transmissorum, ideoque determinatio imaginum principalium satis accurate in elementis Dioptricis tradi solet, quod igitur negotium hic fusius non prosequar; sed potius in eam refractionis rationem hic inquirere constitui, quando lentium apertura est modicae quantitatis, in quo imprimis ad vtriusque faciei figuram est spectandum. Hic autem perpetuo lentium figuras sphaericas assumo, propterea quod haec figura vulgo lentibus induci, vel saltem, nisi forte accurate successerit, intendi solet. In praxi certe nulla adhuc alia figura lentibus commode et accurate tribui poterit, atque adeo a sphaerica figura, etsi ad praxin maxime est accommodata, ab artificibus frequenter aberrari solet. A sollertioribus autem talia vitia iam plerumque satis feliciter euitantur, vnde non adeo erit verendum, ne ea, quae per calculum ex hypothesi sphaericae figurae eliciantur, experientiae non consentanea sint futura. Hanc ob rem hic perpetuo postulo, vt ambae facies lentium exactissime secundum sphaericam figuram sint elaboratae.

Scholion 2.

7. Sphaerica autem figura hoc laborat incommodo, quod statim ac lenti maior apertura tribuatur, non amplius omnes radii, qui quidem ab vno obiecti puncto sunt profecti, post refractionem ad vnum punctum dirigantur radiique EM longius ab axe lentis transmissi non amplius in puncto F concurrant. Vnde confusionen eo maiorem nasci necesse est, quo magis hi radii remotiores ab iis, qui prope axem transeunt, declinauerint; et quoniam talis declinatio a figura sphaerica originem ducit, eo maior euadet, quo maior lenti apertura tribuatur. Quanta igitur quouis casu futura sit haec confusio, hoc capite definire constitui; quae cum ceteris paribus a quantitate aperturae pendeat, hic in perpetuum moneo, me cuiusuis lentis aperturam circularem assumere, per cuius centrum axis lentis transeat: ita vt semidiameter huius circuli simul mensuram aperturae exhibeat. Ita si in superficie lentis PP spatium MAM apertum relinquatur, reliqua parte MP velamine opaco obducta, punctorum extremorum MM distantia diametrum aperturae eiusque semissis semidiametrum aperturae praebebit.

Definitio 2.

8. *Imago extrema est ea, quam radii per extremitatem aperturae transmissi exhibent.*

Ita

Ita si MM sit apertura lentis, radiique EM, EM a puncto lucido E circa oram aperturae transmissi conueniant in puncto f, in hoc ipso puncto f erit imago extrema.

Coroll. 1.

9. Si punctum lucidum E est in ipso axe lentis, nullum est dubium, quin radii inde per marginem circularem MM transeuntes iterum in vno axis puncto f concurrant, imaginemque distinctam repraesentent, quae imago extrema vocatur.

Coroll. 2.

10. Verum si punctum lucidum non esset in axe lentis, hoc neutiquam eueniet, radiique per marginem illum circularem transmissi non amplius in vno puncto colligentur; vnde hoc casu imago extrema eo magis erit confusa, quo magis punctum lucidum ab axe fuerit remotum.

Scholion.

11. Quomodo se habeat refractio radiorum, quando punctum lucidum extra axem lentis fuerit constitutum, quaestio est non solum difficillima, sed etiam ita prolixis calculis inuoluitur, vt inde vix quicquam concludi possit. Ceterum in vsu, ad quem lentes accommodantur, nunquam obiecta ab axe remotiora spectari solent, atque contentos esse nos oportet, dummodo obiecta in ipso axe lentis sita distincte

cte repraesententur, neque etiam confusio, qua obiecta axi proxima afficiuntur, sensibilis esse potest; nam cum imago extrema puncti E in ipso lentis axe siti sit punctum f, nulla confusione inquinatum; etiamsi id parumper esset remotum ab axe, vix sensibilis confusio se immiscere poterit. Quam ob causam inuestigationes sequentes tantum ad obiecta in ipso lentis axe sita adstringam.

Definitio 3.

12. *Spatium diffusionis vocatur interuallum, inter imaginem principalem et extremam interceptum.*

Ita si imago principalis sit in F, extrema vero in f, interuallum Ff appellatur spatium diffusionis.

Coroll. 1.

13. Si ergo apertura lentis MM euanescat, spatium simul diffusionis euanescit, tum enim tantum radii axi proximi transmittuntur, quibus imago distincta in F effingitur. Ex quo intelligi licet, quo maior fuerit apertura lentis, eo maius fore spatium diffusionis Ff.

Coroll. 2.

14. Cum in F imago a radiis axi proximis, in f autem imago a radiis circa marginem circularem MM transmissis formetur; si totam aperturam lentis infinitis circulis concentricis diuisam concipiamus, radii

radii per ſingulos circulos transmiſſi imagines intermedias exhibebunt, quibus interuallum Ff replebitur.

Coroll. 3.

15. Si enim apertura primum nulla, tum vero continuo increſcens ſtatuatur, imago extrema primum cum principali congruet; tum vero continuo magis ab ea diſcedet, ſicque cum vsque ad MM fuerit aucta, omnes illae imagines etiam nunc ſubſiſtent, ſpatiumque Ff implebunt.

Scholion I.

16. Spatium hoc diffuſionis cauſam continet confuſionis, qua repraeſentatio imaginis perturbatur; cum enim eiusdem puncti lucidi E infinitae imagines per interuallum Ff diſpoſitae exhibeantur; earum commiſtio confuſionem pariat neceſſe eſt, quae eo maior erit, quo maius fuerit ſpatium diffuſionis Ff. Quemadmodum enim ad repraeſentationem diſtinctam requiritur, vt omnes radii ex eodem obiecti puncto emiſſi iterum in vnico puncto colligantur; ita ſi hi radii in plura puncta coeant, pluresque eiusdem puncti imagines referant, prout hae magis minusue inter ſe diſcrepant, maior inde minorue confuſio naſcetur. Quemadmodum autem hanc confuſionem aeſtimari oporteat, deinceps demum explicare licebit, cum ante ſpatium diffuſionis accurate definire docuerimus: quo circa in hoc capite, cum propoſita fuerit lens quaecunque vitrea faciebus ſphaericis terminata

pro quauis puncti lucidi E ab ea distantia et quauis apertura spatium diffusionis Ff inuestigare constitui. Quo facto eandem inuestigationem pro duabus pluribusue lentibus inter se coniunctis suscipi conueniet, vt tandem inde confusionem in quibusuis instrumentis dioptricis assignare valeamus.

Scholion 2.

17. Quaestio igitur principalis huius capitis in hoc versatur, vt proposita lente PP in eiusque axe puncto lucido E, radius quicunque incidens EM consideretur, eiusque per lentem refractio definiatur, vnde punctum f, vbi iterum in axem incidat, assignari queat. Cum enim in eodem puncto f omnes radii per totam peripheriam circularem MM transmissi concurrant; ab his imago quaepiam puncti E in f exprimetur, quae erit extrema, si circulus MM in lente eius aperturam determinet; sin autem capiatur minor, imago quaedam intermedia habebitur. Cum autem hic duplex refractio eueniat, altera in ingressu radii EM in vitrum, altera in egressu eiusdem e vitro, quemadmodum eius directio in vtraque inflectatur, seorsim est inuestigandum; vnde duo nascuntur problemata quasi praeliminaria, ex quorum combinatione deinceps negotium conficietur. Verum quo haec problemata commodius calculo expediri queant, quaedam lemmata ex doctrina angulorum petita praemitti oportet.

Lem-

Lemma 1.

18. *Si angulus* Φ *triginta gradus non excedat eius finus fatis accurate erit* fin. $\Phi = \Phi - \frac{1}{6}\Phi^3$, *fi quidem in circulo, cuius radius eft* $= 1$, *arcus* Φ, *qui pro illius anguli menfura habetur, in partibus radii exprimatur.*

Demonftratio.

Quantuscunque fuerit angulus Φ, notum eft, eius finum hac ferie infinita exprimi:

$$\text{fin. } \Phi = \Phi - \tfrac{1}{6}\Phi^3 + \tfrac{1}{120}\Phi^5 - \tfrac{1}{5040}\Phi^7 + \text{etc.}$$

fumtis igitur tantum binis primis terminis, error committitur reliquis neglectis aequalis, fi ergo ftatuamus fin. $\Phi = \Phi - \frac{1}{6}\Phi^3$, hincque pro variis angulis Φ finus colligamus, eorum comparatio cum tabulis finuum errores manifeftabit. Ita fi fumatur $\Phi = 30°$, quia arcus $180°$ valet $3, 14159265$, erit in partibus radii $\Phi = 0, 52359877$, et $\frac{1}{6}\Phi^3 = 0, 0239246$, hincque $\Phi - \frac{1}{6}\Phi^3 = 0, 4996741$. at eft reuera

fin. $\Phi = 0, 5000000$ vnde habetur

error $= 0, 0003259$, qui ergo ne ad $\frac{1}{3000}$ partem radii quidem affurgit. At fi angulus Φ caperetur duplo minor, fcilicet $\Phi = 15°$, reperiretur

$$\Phi - \tfrac{1}{6}\Phi^3 = 0, 2588088$$

cum tamen fit fin. $\Phi = 0, 2588190$

errore exiftente $= 0, 0000102$

qui tricies bis minor eft quam cafu praecedente. Cum ergo in praxi error partem adeo termillefimam radii adaequans facile tolerari poffit, multo magis, fi angulus

gulus Φ fuerit $30°$ minor, expreſſio $\Phi - \frac{1}{6}\Phi^3$ verum eius ſinum exhibere cenſenda erit.

Lemma 2.

19. *Viciſſim ſi anguli triginta gradibus minoris detur ſinus* $= s$, *ex eo ipſe angulus ita proxime definitur, vt ſit in circulo, cuius radius* $= 1$, *arcus eum metiens* $= s + \frac{1}{6}s^3$.

Demonſtratio.

Si enim Φ deſignet iſtum angulum, cuius ſinus proponitur $= s$; modo vidimus, eſſe ſatis exacte $s = \Phi - \frac{1}{6}\Phi^3$, hinc autem per conuerſionem oritur proxime $\Phi = s + \frac{1}{6}s^3$, quae expreſſio quantum peccet in angulo triginta graduum, vt videamus, ſit $s = \frac{1}{2}$ eritque $s + \frac{1}{6}s^3 = 0,5208333$. Cum autem numerus $3,14159265$ reſpondeat angulo $180°$, hic numerus $0,5208333$ praebet angulum $29° \; 50' \; 30''$: verus autem angulus eſt $30°$, ita vt error ſit $9' \; 30''$. Sit autem $s = \frac{1}{4}$ cui ſinui reſpondet angulus $= 14° \; 28' \; 39''$, erit $s + \frac{1}{6}s^3 = 0,2526041 = 14° \; 28' \; 23''$, ita vt hoc caſu error non excedat $16''$. Patet ergo dum angulus minor ſit $30°$, hoc modo ſatis exacte ex dato ſinu reperiri angulum.

Problema 1.

Tab. I. Fig. 2. 20. Si ex puncto lucido E in ſuperficiem ſphaericam reuolutione arcus circularis AM circa axem EC genitam incidat radius EM, definire eius, poſtquam fuerit refractus, concurſum cum axe EC.

Solutio.

Solutio.

Sit C centrum ſuperficiei ſphaericae, eiusque radius $CA = CM = f$, et diſtantia puncti lucidi E a ſuperficie refringente $EA = a$, tum vero ſit ratio refractionis radiorum ex aere in vitrum $= n : 1$. Vocetur pro radio incidente CM angulus $ACM = \Phi$, eritque huius anguli ſinus $= \Phi - \frac{1}{6}\Phi^3$. Ponatur iam breuitatis gratia diſtantia $EC = a + f = c$, et quia in triangulo ECM dantur latera $CM = f$, $EC = c$ cum angulo $CEM = \Phi$, fiet producta CM in c

$$CM\ (f) : \text{ſin. } CEM = CE\ (c) : \text{ſin. } EMc$$

ideoque

$$\text{ſin. } EMc = \frac{c}{f}\ \text{ſin. } \Phi = \frac{c}{f}(\Phi - \tfrac{1}{6}\Phi^3);$$

vnde ipſe angulus EMc elicitur, neglectis tertia altioribus poteſtatibus ipſius Φ:

$$EMc = \frac{c}{f}\Phi - \frac{c}{6f}\Phi^3 + \frac{c^3}{6f^3}\Phi^3 = \frac{c}{f}\Phi + \frac{c(cc-ff)}{6f^3}\Phi^3$$

qui cum ſit $= ECM + CEM$, erit

$$ECM = \frac{c-f}{f}\Phi + \frac{c(cc-ff)}{6f^3}\Phi^3$$

Verum angulus EMc eſt angulus incidentiae, ac ſi MO referat radium refractum, erit CMO angulus refractionis, ſicque per hypotheſin

$$\text{ſin. } EMc : \text{ſin. } CMO = n : 1.$$

vnde colligitur

$$\text{ſin. } CMO = \frac{c}{nf}(\Phi - \tfrac{1}{6}\Phi^3)$$

hincque ipſe angulus

$$CMO = \frac{c}{nf}\Phi + \frac{c(cc-nnff)}{6n^3f^3}\Phi^3$$

 quo

quo ablato ab angulo ECM relinquitur angulus

$$COM = \frac{(n-1)c-nf}{nf}\Phi + \frac{c((n^3-1)cc-nn(n-1)ff)}{6n^3f^3}\Phi^3$$

cuius finus propterea erit

$$\text{fin. } COM = \frac{(n-1)c-nf}{nf}\Phi + \frac{3(n-1)c^3+3(n-1)^2ccf-4n(n-1)cff+nnf^3}{6nnf^3}\Phi^3$$

Iamuero ob

fin. $COM : CM\,(f) =$ fin. $CMO : CO$, obtinebitur

$$CO = \frac{\frac{c}{n} - \frac{c}{6n}\Phi\Phi}{\frac{(n-1)c-nf}{nf} + \frac{3(n-1)c^3+3(n-1)^2ccf-4n(n-1)cff+nnf^3}{6nnf^3}\Phi^2}$$

cuius formulae euolutio praebet:

$$CO = \frac{cf}{(n-1)c-nf} - \frac{cf\Phi\Phi}{6((n-1)c-nf)} - \frac{cf(3(n-1)c^3+3(n-1)^2ccf-4n(n-1)cff+nnf^3)}{6nf((n-1)c-nf)^2}\Phi\Phi$$

quae porro reducitur ad hanc formam:

$$CO = \frac{cf}{(n-1)c-af} - \frac{(n-1)cc(c-f)(c+nf)}{2nf((n-1)c-nf)^2}\Phi\Phi$$

cui fi addamus $CA = f$ prodibit

$$AO = \frac{nf(c-f)}{(n-1)c-nf} - \frac{(n-1)cc(c-f)(c+nf)}{2nf((n-1)c-nf)^2}\Phi\Phi$$

atque hinc pofitio radii refracti MO primo per interuallum AO modo inuentum definitur, tum vero infuper ex angulo AOM, qui erit:

$$AOM = \frac{(n-1)c-nf}{nf}\Phi + \frac{c((n^3-1)cc-nn(n-1)ff)}{6n^3f^3}\Phi^3$$

fiue

$$AOM = \frac{(n-1)c-nf}{nf}\Phi + \frac{(n-1)c((nn+n+1)cc-nnff)}{6n^3f^3}\Phi^3$$

Coroll. I.

21. Cum pofuerimus $c = a+f$ erit
$c-f=a$, et $(n-1)c-nf=(n-1)a-f$,

atque

atque

$$(nn+n+1)cc-nnff=(nn+n+1)aa+2(nn+n+1)af+(n+1)ff$$

quibus valoribus ſubſtitutis habebimus

$$AO=\frac{naf}{(n-1)a-f}-\frac{(n-1)a(a+f)^2(a+(n+1)f)}{2nf((n-1)a-f)^2}\Phi\Phi \text{ et}$$

$$AOM=\frac{(n-1)a-f}{nfa}\Phi+\frac{(n-1)(a+f)((nn+n+1)(a+f)+(n+1)ff)}{6n^3f^3}\Phi^3$$

Coroll. 2.

22. Si M ſit in extremitate aperturae, erit ſemidiameter aperturae $=f$ ſin. $ECM=a\Phi$ ſatis exacte: neque enim neceſſe eſt, aperturam tam exacte noſſe. Vnde ſi ſemidiameter aperturae ponatur $=x$, erit prope $x=a\Phi$, ideoque $\Phi=\frac{x}{a}$.

Coroll. 3.

23. Si accuratius rem definire velimus, quia eſt

$$\text{ſin. } ECM=\frac{c-f}{f}\Phi+\frac{(c-f)(3c-f)}{6ff}\Phi^3=\frac{a}{f}\Phi+\frac{a(3a+2f)}{6ff}\Phi^3$$

erit $x=a\Phi+\frac{a(3a+2f)}{6f}\Phi^3$

ideoque $\Phi=\frac{x}{a}-\frac{(3a+2f)x^3}{6a^3f}$

ſed quia in valore ipſius AO non vltra ſecundam dimenſionem ipſius Φ aſcendimus, hac expreſſione non indigemus.

Coroll. 4.

24. Etſi hae expreſſiones tantum ſunt prope verae; tamen in praxi ſine errore adhiberi poterunt, dummodo anguli, qui in calculum ſunt ingreſſi, infra

30°

30° ſubſiſtant. Non ſolum ergo neceſſe eſt, vt angulus $AEM = \Phi$ ſed etiam angulus EMc ſeu $\frac{c}{f}\Phi = \Phi + \frac{a}{f}\Phi$ minor ſit 30 gradibus.

Scholion.

25. Summo quidem rigore geometrico diſtantiam EO definire potuiſſemus, neque opus fuiſſet ad approximationes confugere: ſcilicet ſi poſuiſſemus angulum $ECM = \omega$, et diſtantiam $CO = u$; deueniſſemus ad hanc determinationem.

$$\frac{cf}{u} = -c\,\mathrm{cof.}\,\omega + \sqrt{(nn(c\,\mathrm{cof.}\,\omega - f)^2 + (nn-1)cc\,\mathrm{fin.}\,\omega^2)}$$

foretque

$$\mathrm{cof.}\,\omega = \frac{c}{f}\,\mathrm{fin.}\,\Phi^2 + \mathrm{cof.}\,\Phi\sqrt{(1 - \tfrac{cc}{ff}\,\mathrm{fin.}\,\Phi^2)} \text{ et}$$

$$\mathrm{fin.}\,\omega = \frac{c}{f}\,\mathrm{fin.}\,\Phi\,\mathrm{cof.}\,\Phi - \mathrm{fin.}\,\Phi\sqrt{(1 - \tfrac{cc}{ff}\,\mathrm{fin.}\,\Phi^2)}$$

hisque valoribus ſubſtitutis

$$\frac{cf}{u} = \frac{-cc\,\mathrm{fin.}\,\Phi^2}{f} - c\,\mathrm{cof.}\,\Phi\sqrt{(1 - \tfrac{cc}{ff}\,\mathrm{fin.}\,\Phi^2)}$$
$$+ (c\,\mathrm{cof.}\,\Phi - \sqrt{(ff - cc\,\mathrm{fin.}\,\Phi^2)})\sqrt{(nn - \tfrac{cc}{ff}\,\mathrm{fin.}\,\Phi^2)}$$

vnde ponendo angulo Φ valde paruo per aproximationem ſuperior expreſſio eliceretur. Verum praecedens analyſis magis ad praeſens inſtitutum videtur accommodata. Interim ſi quis propius ad veritatem accedere voluerit, ex formula vera hic exhibita adipiſcetur.

$$\frac{cf}{u} = (n-1)c\,\mathrm{cof.}\,\Phi - nf + \frac{(n-1)cc}{2nff}((n-1)f + c\,\mathrm{cof.}\,\Phi)\,\mathrm{fin.}\,\Phi^2$$
$$+ \frac{c^4}{8n^3f^4}((nn-1)^2f + (n^3-1)c\,\mathrm{cof.}\,\Phi)\,\mathrm{fin.}\,\Phi^4$$

ſeu

seu

$$\frac{cf}{u} = (n-1)c - nf + \frac{(n-1)c(c-f)(c+nf)}{2nff}\Phi\Phi$$
$$+ \frac{(n-1)(c-f)((nn+n+1)c^3 + 2nn(n+1)ccf + 3n(n-1)cff - n^3f^3)}{24n^3f^4}\Phi^4.$$

Problema 2.

26. Si prout in problemate praecedente inuenimus, radius MO in vitrum missus per superficiem sphaericam BN iterum in aerem erumpat, definire punctum V, vbi is cum axe concurret.

Solutio.

Ponatur interuallum AB$=d$; sitque superficiei sphaericae BN centrum in D eiusque radius DB$=$DN$=g$. Quoniam igitur positio radii incidentis MNO datur, ponamus BO$=b$, et angulum BON$=\psi$, et breuitatis ergo interuallum DO$=b+g=e$. Cum nunc in triangulo DON dentur latera DN$=g$, DO$=e$ cum angulo BON$=\psi$, reperitur

$$\sin. DNM = \frac{e}{g}\sin.\psi = \frac{e}{g}\psi - \frac{e}{6g}\psi^3, \text{ hincque}$$

$$DNM = \frac{e}{g}\psi + \frac{e(ee-gg)}{6g^3}\psi^3 \text{ et}$$

$$ODN = \frac{e-g}{g}\psi + \frac{e(ee-gg)}{6g^3}\psi^3$$

Sed hic sin. DNM est sinus incidentiae, cui respondet angulus refractionis VNd, cuius sinus propterea est ad illum vt $n : 1$, vnde fit

$$\sin. VNd = \frac{ne}{g}\psi - \frac{ne}{6g}\psi^3$$

hincque

$$VNd = \frac{ne}{g}\psi + \frac{ne(nnee - gg)}{6g^3}\psi^3$$

a quo ablato angulo ODN relinquitur angulus

$$DVN = \frac{(n-1)e+g}{g}\psi + \frac{e((n^3-1)ee-(n-1)gg)}{6g^3}\psi^3$$

ergo

$$\text{fin. } DVN = \frac{(n-1)e+g}{g}\psi + \frac{3n(n-1)e^3-3(n-1)^2eeg-4(n-1)egg-g^3}{6g^3}\psi^3$$

Cum nunc fit

$$\text{fin. } DVN : g = \text{fin. } VNd : DV \text{ erit}$$

$$\frac{g}{DV} = \frac{(n-1)e+g+\frac{1}{6gg}(3n(n-1)e^3-3(n-1)^2eeg-4(n-1)egg-g^3)\psi^2}{ne-\frac{1}{6}ne\psi^2}$$

quae expreffio reducitur ad hanc formam:

$$\frac{g}{DV} = \frac{(n-1)e+g}{ne} + \frac{(n-1)(e-g)(ne+g)\psi^2}{2ngg}$$

vnde reciproce oritur

$$DV = \frac{neg}{(n-1)e+g} - \frac{n(n-1)ee(e-g)(ne+g)}{2g((n-1)e+g)^2}\psi^2$$

et

$$BV = \frac{g(e-g)}{(n-1)e+g} - \frac{n(n-1)ee(e-g)(ne+g)}{2g((n-1)e+g)^2}\psi^2$$

Puncto autem V inuento notandum eft effe angulum

$$BVN = \frac{(n-1)e+g}{g}\psi + \frac{e((n^3-1)ee-(n-1)gg)}{6g^3}\psi^3;$$

Coroll. I.

27. Cum fit
$e = b+g$ erit $(n-1)e+g = (n-1)b+ng$,
quo

quo valore reſtituto habebimus.

$$BV = \frac{bg}{(n-1)b+ng} - \frac{n(n-1)b(b+g)^2(nb+(n+1)g)}{2g((n-1)b+ng)^2}\psi^2 \text{ et}$$

$$BVN = \frac{(n-1)b+ng}{g}\psi + \frac{(b+g)((n^2-1)(b+g)^2-(n-1)gg)}{6g^3}\psi^3$$

Coroll. 2.

28. Hae formulae etiam ex praecedentibus erui poſſunt, ſi pro litteris n, a, f ſcribantur $\frac{1}{n}$, $-b$, et $-g$, quoniam hoc modo caſus praecedentis problematis ad hunc reducitur. Praeterea autem, qui angulus ibi erat ϕ hic eſt ψ.

Coroll. 3.

29. Quia in praecedente problemate inuenimus tam lineam AO quam angulum AOM erit hoc problemate ad radium illum refractum MO accommodato

$$BO = b = \frac{naf}{(n-1)a-f} - d - \frac{(n-1)a(a+f)^2(a+(n+1)f)}{2nf((n-1)a-f)^2}\phi\phi$$

et

$$\psi = \frac{(n-1)a-f}{nf}\phi + \frac{(n-1)(a+f)((nn+n+1)a(a+2f)+(n+1)ff)}{6n^3f^3}\phi^3$$

Problema 3.

30. Propoſita lente vitrea MABN faciebus ſphaericis AM et BN terminata, ſi a puncto quocunque E in eius axe poſito incidat in eam radius EM, definire punctum V, in quo is poſt geminam refractionem iterum cum axe lentis ſit concurſurus. Tab. I. Fig. 2.

Solutio.

Confideremus lentem vt vtrinque conuexam, fitque faciei anterioris AM radius $AC = f$, pofterioris vero BN radius $BD = g$, ipfius lentis autem craffities $AB = d$. Lens porro fit vitrea, ita vt fi in eam radius lucis ex aere incidat, fit finus incidentiae ad finum refractionis vt n ad 1. Iam recta iungens centra vtriusque faciei C et D erit axis lentis, in quo reperiatur punctum lucidum E ante lentem in diftantia $AE = a$, vnde fub angulo $AEM = \Phi$ in lentem incidat radius EM, qui prima refractione ita inflectatur, vt productus cum axe concurrat in O. Quodfi iam ex iis, quae problemate primo funt inuenta, ponamus

$$BO = \frac{naf}{(n-1)a-f} - d - \frac{(n-1)a(a+f)^2(a+(n+1)f)}{2nf((n-1)a-f)^3}\Phi\Phi = b$$

$$\text{et ang. } BON = \frac{(n-1)a-f}{nf}\Phi = \psi$$

in valore enim anguli ψ negligere licet terminum Φ^3 inuoluentem, quoniam calculum tantum ad fecundam poteftatem ipfius ψ extendimus; his pofitis in problemate fecundo inuenimus fore.

$$BV = \frac{bg}{(n-1)b+ng} - \frac{n(n-1)b(b+g)^2(nb+(n+1)g)}{2g((n-1)b+ng)^3}\psi\psi$$

et

$$BVN = \frac{(n-1)b+ng}{g}\psi.$$

Totum ergo negotium huc redit, vt ifthic pro b et ψ valo-

valores assignatos substituamus, quod quo facilius fieri possit, statuamus

$$b = P - Q\Phi\Phi \text{ et } \psi = R\Phi \text{ vt sit}$$

$$P = \frac{naf}{(n-1)a-f} - d = \frac{naf-(n-1)ad+df}{(n-1)a-f}$$

$$Q = \frac{(n-1)a(a+f)^2(a+(n+1)f)}{2nf((n-1)a-f)^2} \text{ et } R = \frac{(n-1)d-f}{nf}$$

Hinc erit:

$$\frac{bg}{(n-1)b+ng} = \frac{Pg - Qg\Phi\Phi}{(n-1)P+ng-(n-1)Q\Phi\Phi} = \frac{Pg}{(n-1)P+ng} - \frac{nQgg\Phi\Phi}{((n-1)P+ng)^2}$$

at in altero membro sufficit pro b scribere P: ex quo obtinebimus

$$BV = \frac{Pg}{(n-1)P+ng} - \frac{nQgg\Phi\Phi}{((n-1)P+ng)^2} - \frac{n(n-1)PRR(P+g)^2(nP+(n+1)g)}{2g((n-1)P+ng)^3}\Phi\Phi$$

$$\text{et } BVN = \frac{(n-1)P+ng}{g}R\Phi$$

Pro his autem substitutionibus notandum est fore:

$$(n-1)P+ng = \frac{n(n-1)a(f+g)-nfg-(n-1)^2ad+(n-1)df}{(n-1)a-f}$$

$$P+g = \frac{naf+(n-1)ag-fg-(n-1)ad+df}{(n-1)a-f}$$

$$nP+(n+1)g = \frac{nnaf+(nn-1)ag-(n+1)fg-n(n-1)ad+ndf}{(n-1)a-f}$$

vnde concluditur:

$$BV = \frac{nafg-(n-1)adg+dfg}{n(n-1)a(f+g)-nfg-(n-1)^2ad+(n-1)df}$$

$$- \frac{(n-1)agg(a+f)^2(a+(n+1)f)}{2f(n(n-1)a(f+g)-nfg-(n-1)^2ad+(n-1)df)^2}\Phi\Phi$$

$$- \frac{(n-1)(naf-(n-1)ad+df)(naf+(n-1)ag-fg-(n-1)ad+df)^2(nnaf+(nn-1)ag-(n+1)fg-n(n-1)ad+nd)}{2nffg(n(n-1)a(f+g)-nfg-(n-1)^2ad+(n-1)df)^3}\Phi\Phi$$

$$\text{et } BVN = \frac{n(n-1)a(f+g)-nfg+(n-1)df-(n-1)^2ad}{nfg}\Phi.$$

Coroll. 1.

31. Si angulus Φ prorſus euaneſcat, punctum V cadet in imaginem principalem, cuius ſi a lente diſtantia dicatur $= \alpha$ erit

$$\alpha = \frac{nafg-(n-1)adg+dfg}{n(n-1)a(f+g)-nfg-(n-1)^2ad+(n-1)df}$$

quae ſi vt data ſpectetur, hac aequatione relatio inter f et g definitur, vt haec diſtantia principalis oriatur.

Coroll. 2.

32. Quare ſi diſtantia obiecti ante lentem ſit $= a$, eiusque imago principalis poſt lentem ad diſtantiam $= \alpha$ proiici debeat huic aequationi ſatisfieri oportet.

$$\begin{aligned} n(n-1)a\alpha(f+g) - nafg + (n-1)adg - (n-1)^2 a\alpha d &= 0 \\ - n\alpha fg + (n-1)\alpha df \quad - dfg & \end{aligned}$$

Coroll. 3.

33. Hanc autem imaginis principalis diſtantiam α in calculum introducendo expreſſiones noſtrae inuentae haud mediocriter contrahentur. Cum enim ſit $\alpha = \frac{Pg}{(n-1)P+ng}$, erit hinc $P = \frac{n\alpha g}{g-(n-1)\alpha}$

et porro

$$(n-1)P + ng = \frac{ngg}{g-(n-1)\alpha}; \quad P + g = \frac{gg+\alpha g}{g-(n-1)\alpha} = \frac{g(\alpha+g)}{g-(n-1)\alpha}$$

$$nP + (n+1)g = \frac{g(\alpha+(n+1)g)}{g-(n-1)\alpha}$$

vnde

vnde facta substitutione sequentes determinationes simpliciores assequemur.

$$BV = \alpha - \frac{(n-1)a(\alpha+f)^2(g-(n-1)\alpha)^2(\alpha+(n+1)f)}{2nnfgg((n-1)a-f)^2}\Phi\Phi$$

$$-\frac{(n-1)\alpha(\alpha+g)^2((n-1)a-f)^2(\alpha+(n+1)g)}{2nnffg(g-(n-1)\alpha)}\Phi\Phi$$

et

$$BVN = \frac{((n-1)a-f)g}{(g-(n-1)\alpha)f}\Phi.$$

Scholion.

34. Non solum hae formulae multo sunt breuiores et concinniores, quam primo inuentae, sed etiam perspicuus ordo in iis obseruatur quo litterae α et g cum litteris a et f ita connexae sunt, vt permutationem admittant. Nullum igitur est dubium, quin si statim distantiam α in calculum introduxissemus, via breuiori ad eas peruenire licuisset. Ceterum quia hae formulae posteriores non amplius crassitiem lentis $AB = d$ inuoluunt, euidens est eas ex primo inventis nasci, si introducta distantia α crassicies d eliminetur, seu eius loco hic valor surrogetur.

$$d = \frac{n(n-1)a\alpha(f+g) - n(a+\alpha)fg}{(n-1)^2 a\alpha - (n-1)(ag+\alpha f) + fg} = n\frac{(n-1)a\alpha(f+g) - (a+\alpha)fg}{((n-1)a-f)((n-1)\alpha-g)}$$

qui labor autem ne suscipi quidem mereretur, nisi iam ante de eximio eius vsu certiores essemus facti. In sequentibus igitur his elegantioribus formulis vtamur, quoad negotium adhuc succinctius expedire didicerimus.

Proble-

Problema 4.

35. Propofita lente quacunque faciebus fphaericis terminata, fi obiectum in data ab ea diftantia fit conftitutum, pro data lentis apertura fpatium diffufionis affignare.

Solutio.

Tab. I. Fig. 1. Concipiamus lentem vtrinque conuexam, fitque faciei anterioris MAM radius $= f$, pofterioris NBN $= g$, lentisque craffities AB $= d$. Sit porro MM lentis huius apertura, cuius femidiameter fit $= x$; atque in axe lentis expofitum fit obiectum vel faltem punctum lucidum E, cuius a lente ponatur diftantia AE $= a$. Iam primo quaeramus eius imaginem principalem, quae cadat in F atque fupra (31) invenimus fore

$$BF = \frac{nafg - (n-1)adg + dfg}{n(n-1)a(f+g) - nfg - (n-1)^2 ad + (n-1)df}$$

Vocetur ergo haec diftantia $= \alpha$, et fi radii EM circa oram aperturae tranfeant, erit angulus AEM $= \Phi = \frac{x}{a}$, quo valore in fuperioribus formulis fubftituto prodibit diftantia imaginis extremae f a lente fcilicet:

$$Bf = \alpha - \frac{(n-1)a(a+f)^2(g-(n-1)\alpha)^2(a+(n+1)f)}{2nnfgg((n-1)a-f)^2} \cdot \frac{xx}{aa}$$
$$- \frac{(n-1)\alpha(\alpha+g)^2((n-1)a-f)^2(\alpha+(n+1)g)}{2nnffg(g-(n-1)\alpha)^2} \cdot \frac{xx}{aa}$$

ex quo colligitur fpatium diffufionis quaefitum:

$$Ff = + \frac{(n-1)a(a+f)^2(g-(n-1)\alpha)^2(a+(n+1)f)}{2nnfgg((n-1)a-f)^2} \cdot \frac{xx}{aa}$$
$$+ \frac{(n-1)\alpha(\alpha+g)^2((n-1)a-f)^2(\alpha+(n+1)g)}{2nnffg(g-(n-1)\alpha)^2} \cdot \frac{xx}{aa}$$

ac

ac praeterea angulus $Bf N$ erit

$$Bf N = \frac{((n-1)a-f)g}{(g-(n-1)\alpha)f} \cdot \frac{x}{a}.$$

Coroll. 1.

36. Quoties ergo spatium diffusionis hoc modo expressum est positiuum, imago extrema propius ad lentem cadit quam principalis seu est $Bf < BF$. Contra autem si ista expressio valorem obtineat n gatiuum, imago extrema a lente longius erit remota principali.

Coroll. 2.

37. Patet hinc etiam spatium diffusionis cum apertura ita crescere, vt sit quadrato semidiametri aperturae proportionale, sequetur ergo ipsam aperturae rationem.

Coroll. 3.

38. Spatium diffusionis etiam hoc modo exprimi potest

$$Ff = \begin{cases} \dfrac{+(n-1)a(1+\frac{\alpha}{f})^2(1-\frac{(n-1)\alpha}{g})^2(n+1+\frac{\alpha}{f})}{2nn(\frac{(n-1)a}{f}-1)^2} \cdot \dfrac{xx}{aa} \\ \dfrac{+(n-1)\alpha(1+\frac{\alpha}{g})^2(\frac{(n-1)a}{f}-1)^2(n+1+\frac{\alpha}{g})}{2nn(1-\frac{(n-1)\alpha}{g})^2} \cdot \dfrac{xx}{aa} \end{cases}$$

et angulus

$$Bf N = \frac{\frac{(n-1)a}{f}-1}{1-\frac{(n-1)\alpha}{g}} \cdot \frac{x}{a}.$$

Coroll. 4.

39. Quia hae formulae introducendis litterarum valoribus reciprocis redditae ſunt ſimpliciores, in hunc modum etiam aequatio (§. 32) exhibita tractetur quae per $aadfg$ diuiſa abit in hanc formam.

$$n(n-1)\frac{1}{d}\left(\frac{1}{f}+\frac{1}{g}\right)-n\frac{1}{d}\left(\frac{1}{a}+\frac{1}{\alpha}\right)+(n-1)\left(\frac{1}{\alpha f}+\frac{1}{ag}\right)-(n-1)^2\frac{1}{fg}-\frac{1}{a\alpha}=0$$

ſeu

$$\frac{n}{d}\left((n-1)a\alpha\left(\frac{1}{f}+\frac{1}{g}\right)-a-\alpha\right)=\left(\frac{(n-1)a}{f}-1\right)\left(\frac{(n-1)\alpha}{g}-1\right)$$

quae commodior erit tam ad relationem inter a et α, quam inter f et g definiendam.

Scholion.

40. Scilicet ſi propoſita lente obiectum in variis diſtantiis exponatur ac pro ſingulis diſtantiam imaginis principalis definire velimus, aequatio hoc modo referatur:

$$\frac{1}{a\alpha}-(n-1)\left(\frac{1}{\alpha f}+\frac{1}{ag}\right)+n\left(\frac{1}{da}+\frac{1}{d\alpha}\right)-n(n-1)\left(\frac{1}{df}+\frac{1}{dg}\right)+(n-1)^2\frac{1}{fg}=0$$

quae per factores ita adornari poterit

$$\left(\frac{1}{a}-\frac{(n-1)}{f}+\frac{n}{d}\right)\left(\frac{1}{\alpha}-\frac{(n-1)}{g}+\frac{n}{d}\right)=\frac{nn}{dd}$$

Vnde patet productum ex his duobus factoribus ſemper eſſe idem. Cuius reſolutio quo facilius inſtituatur, capiantur numeri μ et ν ita, vt eorum ſumma ſit $=1$, ſcilicet $\mu+\nu=1$ ac ſtatuatur

$$\frac{1}{a}-\frac{(n-1)}{f}+\frac{n}{d}=\frac{-\mu n}{\nu d} \text{ erit } \frac{1}{a}=\frac{n-1}{f}-\frac{n}{\nu d}$$

$$\frac{1}{\alpha}-\frac{(n-1)}{g}+\frac{n}{d}=\frac{-\nu n}{\mu d} \text{ erit } \frac{1}{\alpha}=\frac{n-1}{g}-\frac{n}{\mu d}$$

Quare

Quare si distantia obiecti $AE = a$ ita capiatur, vt sit $\frac{1}{a} = \frac{n-1}{f} - \frac{n}{\nu d}$, distantia imaginis principalis $BF = \alpha$ ita se habebit, vt sit $\frac{1}{\alpha} = \frac{n-1}{g} - \frac{n}{\mu d}$.

Eodem modo si dentur binae distantiae $AE = a$ et $BF = \alpha$ cum crassitie lentis $AB = d$, radii facierum f et g ita debent esse comparati, vt sit

$$\frac{1}{f} = \frac{1}{(n-1)a} + \frac{n}{\nu(n-1)d} \text{ et } \frac{1}{g} = \frac{1}{(n-1)\alpha} + \frac{n}{\mu(n-1)d}$$

id quod infinitis modis praestari potest, cum numeri μ et ν pro arbitrio accipi queant, dummodo eorum summa $\mu + \nu$ aequetur vnitati. Si hoc modo etiam in formulis pro spatio diffusionis inuentis omnes litterae in denominatores detrudantur, reperietur:

$$Ff = \frac{(n-1)\alpha\alpha xx}{2nn}\left\{\frac{(\frac{1}{a}+\frac{1}{f})^2(\frac{1}{\alpha}-\frac{(n-1)}{g})^2(\frac{n+1}{a}+\frac{1}{f})}{(\frac{n-1}{f}-\frac{1}{a})^2} + \frac{(\frac{1}{\alpha}+\frac{1}{g})^2(\frac{n-1}{f}-\frac{1}{a})^2(\frac{n+1}{\alpha}+\frac{1}{g})}{(\frac{1}{\alpha}-\frac{(n-1)}{g})^2}\right\}$$

et

$$\text{angulus } BfN = \frac{\frac{n-1}{f}-\frac{1}{a}}{\frac{1}{\alpha}-\frac{(n-1)}{g}} \cdot \frac{x}{a} = \frac{\frac{n}{\nu d}}{\frac{-n}{\mu d}} \frac{x}{a} = -\frac{\mu}{\nu} \cdot \frac{x}{a}$$

Cum autem sit

$$\frac{n-1}{f} - \frac{1}{a} = \frac{n}{\nu d} \text{ et } \frac{1}{\alpha} - \frac{(n-1)}{g} = \frac{-n}{\mu d} \text{ erit}$$

$$Ff = \frac{(n-1)\alpha\alpha xx}{2nn}\left(\frac{\nu\nu}{\mu\mu} \cdot \frac{1}{a} + \frac{1}{f}\right)^2\left(\frac{n+1}{a} + \frac{1}{f}\right) + \frac{\mu\mu}{\nu\nu} \cdot \left(\frac{1}{\alpha} + \frac{1}{g}\right)^2\left(\frac{n+1}{\alpha} + \frac{1}{g}\right)$$

et

$$\text{angulus } BfN = -\frac{\mu x}{\nu \alpha} = \frac{\mu x}{(\mu-1)\alpha}.$$

 Proble-

Problema 5.

Tab. I. Fig. 1.

41. Datis diſtantiis obiecti ante lentem $AE = a$ et imaginis principalis poſt lentem $BF = \alpha$ vna cum lentis craſſitie $AB = d$, definire omnes lentes ſatisfacientes, ſimulque pro ſingulis ſpatium diffuſionis Ff.

Solutio.

Modo vidimus ſi lentis faciei anterioris AM radius ponatur $= f$, poſterioris $BN = g$, lente vt conuexa vtrinque ſpectata, hos duos radios ita comparatos eſſe debere, vt ſit

$$\frac{n-1}{f} = \frac{1}{a} + \frac{n}{\nu d} \text{ et } \frac{n-1}{g} = \frac{1}{\alpha} + \frac{n}{\mu d}$$

ſumtis pro μ et ν numeris quibuscunque, vt ſit $\mu + \nu = 1$; vnde infinitae lentes quaeſito ſatisfacientes obtinentur. Deinde ſi huius lentis ſemidiameter aperturae ponatur $= x$, ſpatium diffuſionis Ff ita exprimi poteſt vt ſit

$$Ff = \frac{\alpha\alpha xx}{2nn(n-1)^2}\left(\frac{\nu\nu}{\mu\mu}\left(\frac{n-1}{a} + \frac{n-1}{f}\right)^2\left(\frac{nn-1}{a} + \frac{n-1}{f}\right) + \frac{\mu\mu}{\nu\nu}\left(\frac{n-1}{\alpha} + \frac{n-1}{g}\right)^2\left(\frac{nn-1}{\alpha} + \frac{n-1}{g}\right)\right)$$

et angulus $BfN = \frac{\mu x}{(\mu - 1)\alpha}$.

Quod ſi iam hic pro $\frac{n-1}{f}$ et $\frac{n-1}{g}$ valores aſſignati ſubſtituantur ſpatium diffuſionis erit

$$Ff = \frac{n\alpha\alpha xx}{2(n-1)^2}\left(\frac{\nu\nu}{\mu\mu}\left(\frac{1}{a} + \frac{1}{\nu d}\right)^2\left(\frac{n}{a} + \frac{1}{\nu d}\right) + \frac{\mu\mu}{\nu\nu}\left(\frac{1}{\alpha} + \frac{1}{\mu d}\right)^2\left(\frac{n}{\alpha} + \frac{1}{\mu d}\right)\right)$$

ſeu

$$Ff = \frac{n\alpha\alpha xx}{2(n-1)}\left\{\begin{array}{l} + \frac{\nu\nu}{\mu\mu}\left(\frac{n}{a^3} + \frac{2n+1}{\nu aad} + \frac{n+2}{\nu\nu add} + \frac{1}{\nu^3 d^3}\right) \\ + \frac{\mu\mu}{\nu\nu}\left(\frac{n}{\alpha^3} + \frac{2n+1}{\mu\alpha\alpha d} + \frac{n+2}{\mu\mu\alpha dd} + \frac{1}{\mu^3 d^3}\right)\end{array}\right\}$$

Si

Si crassities lentis fuerit valde parua, ne confusio fiat enormis, necesse est pro μ et ν sumi numeros vehementer magnos, alterum scilicet positiuum alterum negatiuum. Cum igitur sit $\mu d + \nu d = d$, statuatur $\mu d = \frac{d-k}{2}$ et $\nu d = \frac{d+k}{2}$, vt sit

$$\frac{n-1}{f} = \frac{1}{\alpha} + \frac{2n}{k+d} \text{ et } \frac{n-1}{g} = \frac{1}{\alpha} - \frac{2n}{k-d} \text{ seu}$$

$$f = \frac{(n-1)\alpha(k+d)}{2n\alpha + k + d} \text{ et } g = \frac{(n-1)\alpha(k-d)}{k-d-2n\alpha}$$

hincque obtinetur spatium diffusionis

$$Ff = \frac{n\alpha\alpha xx}{2(n-1)^2} \left\{ \begin{array}{l} +\left(\frac{k+d}{k-d}\right)^2\left(\frac{n}{\alpha}+\frac{2}{k+d}\right)\left(\frac{1}{\alpha}+\frac{2}{k+d}\right)^2 \\ +\left(\frac{k-d}{k+d}\right)^2\left(\frac{n}{\alpha}-\frac{2}{k-d}\right)\left(\frac{1}{\alpha}-\frac{2}{k-d}\right)^2 \end{array} \right\}$$

in quibus formulis paruitas crassitiei lentis $AB = d$ nullum negotium facessit: tum vero est

$$\text{angulus } BfN = \frac{k-d}{k+d} \cdot \frac{x}{\alpha}.$$

Coroll. 1.

42. Propositis ergo binis distantiis $AE = \alpha$ et $BF = \alpha$ vna cum crassitie lentis $AB = d$, infinitis modis lentes idoneae parari possunt cum pro k quantitates pro lubitu siue positiuae siue negatiuae assumi queant.

Coroll. 2.

43. Cum semidiameter aperturae x faciem anteriorem respiciat, et angulus BfN seu BFN sit $= \frac{k-d}{k+d} \cdot \frac{x}{\alpha}$, existente distantia $BF = \alpha$, manifestum est semidiametrum

trum aperturae posterioris faciei esse debore $=\frac{k-d}{k+d}.x$ vel saltem non minorem.

Coroll. 3.

44. Quodsi lentis crassities tam sit parua, vt ea prae k contemni queat, formulae nostrae fient simpliciores

$$f=\frac{(n-1)\alpha k}{k+2n\alpha} \text{ et } g=\frac{(n-1)\alpha k}{k-2n\alpha}$$

et spatium diffusionis

$$Ff=\frac{n\alpha\alpha xx}{2(n-1)^3}\left(\left(\frac{n}{\alpha}+\frac{2}{k}\right)\left(\frac{1}{\alpha}+\frac{2}{k}\right)^2+\left(\frac{n}{\alpha}-\frac{2}{k}\right)\left(\frac{1}{\alpha}-\frac{2}{k}\right)^2\right)$$

angulusque $BfN=\frac{x}{\alpha}$.

Scholion.

45. Sufficiat haec de spatio diffusionis in genere quaecunque fuerit lentis crassities, tradidisse, cum non obstante his satis concinnis transformationibus calculus nimium fieret molestus, si in determinatione spatii diffusionis rationem crassitiei lentis habere vellemus. Licebit autem, vt modo vidimus, crassitiem lentis negligere non solum, quando ipsa est per se valde exigua verum etiam dummodo prae quantitate k fuerit perparua. Atque hinc etiam in sequentibus facile iudicare poterimus, vtrum crassitiem lentis recte in calculo contemserimus, nec ne? quouis enim casu consideretur quantitas pro k assumta, quae si multoties fuerit maior quam d error nullus erit pertimescendus;

ſcendus; contra vero ſi k non multum ſuperet d multum aberrabitur, quantumuis exigua fuerit craſſities ipſa per ſe.

Vt fiat Ff minimum, definiri poteſt conueniens valor ipſius k hoc modo. Ponatur $\frac{k-d}{k+d} = z$ et pro minimo peruenitur ad hanc aequationem:

$$0 = 2z^4\left(\frac{n}{\alpha}+\frac{1}{d}\right)\left(\frac{1}{\alpha}+\frac{1}{d}\right)^2$$
$$-\frac{2nz^3}{\alpha d}\left(\frac{1}{\alpha}+\frac{1}{d}\right)-\frac{z^3}{d}\left(\frac{1}{\alpha}+\frac{2}{d}\right)^2$$
$$+\frac{2nz}{ad}\left(\frac{1}{a}+\frac{1}{d}\right)+\frac{z}{d}\left(\frac{1}{a}+\frac{2}{d}\right)^2$$
$$-2\left(\frac{n}{a}+\frac{1}{d}\right)\left(\frac{1}{a}+\frac{1}{d}\right)^2$$

vnde valor ipſius z erui debet.

Problema 6.

46. Neglecta lentis craſſitie, ſi detur cum obiecti ante lentem diſtantia $AE = a$, tum imaginis principalis poſt lentem diſtantia $BF = \alpha$, eam definire lentem quae pro data apertura minimam pariat diffuſionem.

Solutio.

Poſitis radiis faciei anterioris $AM = f$ et poſterioris $BN = g$ vtraque vt connexa ſpectata, vidimus omnes lentes datis diſtantiis a et α conuenientes his formulis determinari: ſi ſcilicet pro k ſcribamus $2k$

$$\frac{n-1}{f} = \frac{1}{a}+\frac{n}{k} \text{ et } \frac{n-1}{g} = \frac{1}{\alpha}-\frac{n}{k}$$

deno-

denotante k quantitatem quamcunque. Tum autem ſpatium diffuſionis ita exprimi eſt repertum

$$Ff=\frac{n\alpha\alpha xx}{2(n-1)^2}\left(\left(\frac{n}{a}+\frac{1}{k}\right)\left(\frac{1}{a}+\frac{1}{k}\right)^2+\left(\frac{n}{\alpha}-\frac{1}{k}\right)\left(\frac{1}{\alpha}-\frac{1}{k}\right)^2\right.$$

quae expreſſio ad hanc reducitur formam:

$$Ff=\frac{na\alpha xx}{2(n-1)^2}\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(n\left(\frac{1}{aa}-\frac{1}{a\alpha}+\frac{1}{\alpha\alpha}\right)+\frac{2n+1}{k}\left(\frac{1}{a}-\frac{1}{\alpha}\right)+\frac{n+2}{kk}\right)$$

Quaeſtio igitur huc reducitur, vt definiatur quantitas k qua huic expreſſioni valor minimus concilietur; cui quidem requiſito ſatisfacit valor

$$\frac{1}{k}=\frac{-(2n+1)}{2(n+2)}\left(\frac{1}{a}-\frac{1}{\alpha}\right) \text{ ſeu } \frac{1}{k}=\frac{2n+1}{2(n+2)}\left(\frac{1}{\alpha}-\frac{1}{a}\right)$$

quo ſubſtituto habebitur:

$$\frac{n-1}{f}=\frac{4+n-2nn}{2(n+2)a}+\frac{n(2n+1)}{2(n+2)\alpha} \text{ et } \frac{n-1}{g}=\frac{4+n-2nn}{2(n+2)\alpha}+\frac{n(2n+1)}{2(n+2)a}.$$

et ſpatium diffuſionis quod eſt minimum:

$$Ff=\frac{na\alpha xx}{2(n-1)^2}\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(n\left(\frac{1}{aa}-\frac{1}{a\alpha}+\frac{1}{\alpha\alpha}\right)-\frac{(2n+1)^2}{4(n+2)}\left(\frac{1}{a}-\frac{1}{\alpha}\right)^2\right)$$

At eſt

$$n\left(\frac{1}{aa}-\frac{1}{a\alpha}+\frac{1}{\alpha\alpha}\right)=n\left(\frac{1}{a}-\frac{1}{\alpha}\right)^2+\frac{n}{a\alpha} \text{ ideoque}$$

$$Ff=\frac{na\alpha xx}{2(n-1)^2}\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\frac{4n-1}{4(n+2)}\left(\frac{1}{a}-\frac{1}{\alpha}\right)^2+\frac{n}{a\alpha}\right)$$

quae expreſſio etiam in hanc formam transfunditur

$$Ff=\frac{na\alpha xx}{2(n-1)^2}\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\frac{4n-1}{4(n+2)}\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2+\frac{(n-1)^2}{(n+2)a\alpha}\right)$$

ſeu

$$Ff=\frac{n(4n-1)a\alpha xx}{8(n-1)^2(n+2)}\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2+\frac{4(n-1)^2}{(4n-1)a\alpha}\right)$$

Coroll.

Coroll. 1.

47. Vt igitur lens minimum ſpatium diffuſionis producat, eam ita formari neceſſe eſt vt ſit

$$f = \frac{2(n-1)(n+2)a\alpha}{n(2n+1)a + (4+n-2nn)\alpha} \quad \text{et}$$

$$g = \frac{2(n-1)(n+2)a\alpha}{n(2n+1)\alpha + (4+n-2nn)a}$$

neglecta ſcilicet lentis craſſitie, quae quidem recte negligitur, ſi modo fuerit vehementer parua prae quantitate $k = \frac{2(n+2)a\alpha}{(2n+1)(a-\alpha)}$.

Coroll. 2.

48. Si igitur ſit $a = \alpha$, ſeu $BF = AE$ craſſities lentis, quantacunque fuerit, nihil turbat in ſpatio diffuſionis. Pro hoc autem caſu erit $f = g = (n-1)a$ et ſpatium diffuſionis ipſum $Ff = \frac{nnxx}{(n-1)^2 a}$. At quo magis diſtantiae a et α a ſe inuicem diſcrepant, ſeu minor fuerit quantitas $\frac{a\alpha}{a-\alpha}$, eo magis haec determinatio ob lentis craſſitiem fiet erronea.

Coroll. 3.

49. Spatium autem hoc confuſionis minimum Ff pluribus modis exhiberi poteſt, inter quos commodiſſimum eligi conuenit; ſunt autem praecipui:

I. $Ff = \frac{n(4n-1)\alpha\alpha xx}{8(n-1)^2(n+2)} \left(\frac{1}{a} + \frac{1}{\alpha}\right)\left(\left(\frac{1}{a} + \frac{1}{\alpha}\right)^2 + \frac{4(n-1)^2}{(4n-1)a\alpha}\right)$

II. $Ff = \frac{n(4n-1)\alpha\alpha xx}{8(n-1)^2(n+2)} \left(\frac{1}{a} + \frac{1}{\alpha}\right)\left(\left(\frac{1}{a} - \frac{1}{\alpha}\right)^2 + \frac{4n(n+2)}{(4n-1)a\alpha}\right)$

III. $Ff = \frac{n(4n-1)\alpha\alpha xx}{8(n-1)^2(n+2)} \left(\frac{1}{a} + \frac{1}{\alpha}\right)\left(\left(\frac{1}{aa} + \frac{1}{\alpha\alpha}\right) + \frac{2(2nn+1)}{(4n-1)a\alpha}\right)$

IV. $Ff = \frac{n(4n-1)\alpha\alpha xx}{8(n-1)^2(n+2)} \left(\frac{1}{a} + \frac{1}{\alpha}\right)\left(\left(\frac{1}{aa} - \frac{1}{a\alpha} + \frac{1}{\alpha\alpha}\right) + \frac{(2n+1)^2}{(4n-1)a\alpha}\right)$,

Coroll. 4.

50. Cum ergo hoc ſpatium diffuſionis ſit minimum, ſi lenti alia quaecunque figura tribuatur, ita tamen, vt obiecti ad diſtantiam $AE = a$ remoti imaginem principalem in diſtantia $BF = \alpha$ exhibeat, ſpatium diffuſionis erit maius, quam hic inuenimus.

Scholion.

51. Quia $n : 1$ denotat rationem refractionis ex aere in vitrum, haec autem pro radiorum natura eſt variabilis, conueniet pro n medium valorem aſſumi, qui eſt $n = \frac{31}{20}$; hinc ergo erit

$$n - 1 = \tfrac{11}{20};\ n + 2 = \tfrac{71}{20};\ 2n + 1 = \tfrac{41}{10};\ 4 + n - 2nn = \tfrac{149}{200},$$

hincque

$$\frac{n(2n+1)}{2(n-1)(n+2)} = \frac{1271}{781} = 1,627401$$

$$\frac{4+n-2nn}{2(n-1)(n+2)} = \frac{149}{781} = 0,190781$$

vnde lens minimam confuſionem pariens ita definietur

$$\frac{1}{f} = \frac{149}{781a} + \frac{1271}{781\alpha} = \frac{0,190781}{a} + \frac{1,627401}{\alpha}$$

$$\frac{1}{g} = \frac{149}{781\alpha} + \frac{1271}{781a} = \frac{0,190781}{\alpha} + \frac{1,627401}{a}$$

ſeu

$$f = \frac{781a\alpha}{149\alpha + 1271a} = \frac{a\alpha}{0,190781\alpha + 1,627401a}$$

et

$$g = \frac{781a\alpha}{149a + 1271\alpha} = \frac{a\alpha}{0,190781a + 1,627401\alpha}.$$

Pro

Pro ſpatio autem diffuſionis ipſo definiendo ob $4n-1=\frac{26}{5}$

erit $\frac{n(4n-1)}{8(n-1)^2(n+2)}=\frac{8060}{8591}=0,938191$

et $\frac{4(n-1)^2}{4n-1}=\frac{121}{520}=0,232692$

hincque

$$Ff=0,938191\,\alpha\alpha xx\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2+\frac{0,232692}{a\alpha}\right)$$

vnde reliquae formulàe facile deducuntur. Ceterum hic monendum eſt, quoniam valor $n=\frac{31}{20}$ ex experimentis eſt concluſus, neque ipſe pro omnibus radiorum generibus valet, ſuperfluum fore in praxi hos numeros inuentos nimis ſtudioſe obſeruare; quin etiam ipſa natura minimi aliquam aberrationem permittit. Nihilo vero minus has fractiones decimales ad tot figuras producere viſum eſt, quo facilius quantum ab hac hypotheſi aberretur, dignoſci queat.

Problema 7.

52. Neglecta lentis craſſitie, ſi detur cum obiecti ante lentem diſtantia $AE=a$, tum imaginis principalis poſt lentem diſtantia $BF=\alpha$, eam definire lentem, quae pro data apertura non minimum, ſed datum pariat ſpatium diffuſionis Ff.

Solutio.

Lente vtrinque vt conuexa ſpectata, ſit faciei anterioris radius $=f$, poſterioris $=g$; atque vt ex data

 diſtan-

diſtantia obiecti $AE = a$, data oriatur imaginis principalis diſtantia $= \alpha$ neceſſe eſt ſit in genere

$$\frac{n-1}{f} = \frac{1}{a} + \frac{n}{k} \text{ et } \frac{n-1}{g} = \frac{1}{\alpha} - \frac{n}{k}$$

vnde ſpatium diffuſionis fit

$$Ff = \frac{n\alpha\alpha xx}{2(n-1)^2}\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(n\left(\frac{1}{aa}-\frac{1}{a\alpha}+\frac{1}{\alpha\alpha}\right)+\frac{2n+1}{k}\left(\frac{1}{a}-\frac{1}{\alpha}\right)+\frac{n+2}{kk}\right)$$

Cum autem ſpatium diffuſionis minimum repertum ſit

$$\frac{n(4n-1)\alpha\alpha xx}{8(n-1)^2(n+2)}\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2+\frac{4(n-1)^2}{(4n-1)a\alpha}\right)$$

neceſſe eſt vt illud ſit maius: ſtatuatur ergo:

$$Ff = \frac{n(4n-1)\alpha\alpha xx}{8(n-1)^2(n+2)}\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2+\frac{4(n-1)^2}{(4n-1)a\alpha}+S\right)$$

et habebitur iſta aequatio:

$$4n(n+2)\left(\frac{1}{aa}-\frac{1}{a\alpha}+\frac{1}{\alpha\alpha}\right)+\frac{4(n+2)(2n+1)}{k}\left(\frac{1}{a}-\frac{1}{\alpha}\right)+\frac{4(n+2)^2}{kk}$$
$$= (4n-1)\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2+\frac{4(n-1)^2}{a\alpha}+(4n-1)S$$

quae in hanc formam redigitur.

$$(2n+1)^2\left(\frac{1}{a}-\frac{1}{\alpha}\right)^2+\frac{4(n+2)(2n+1)}{k}\left(\frac{1}{a}-\frac{1}{\alpha}\right)+\frac{4(n+2)^2}{kk} = (4n-1)S$$

vnde radice extracta fit

$$(2n+1)\left(\frac{1}{a}-\frac{1}{\alpha}\right)+\frac{2(n+2)}{k} = \sqrt{(4n-1)S}$$

$$\text{et } \frac{1}{k} = \frac{-(2n+1)}{2(n+2)}\left(\frac{1}{a}-\frac{1}{\alpha}\right)+\frac{\sqrt{(4n-1)S}}{2(n+2)}$$

Quare erit

$$\frac{n-1}{f} = \frac{1}{a}-\frac{n(2n+1)}{2(n+2)}\left(\frac{1}{a}-\frac{1}{\alpha}\right)+\frac{n}{2(n+2)}\sqrt{(4n-1)S}$$

$$\frac{n-1}{g} = \frac{1}{\alpha}+\frac{n(2n+1)}{2(n+2)}\left(\frac{1}{a}-\frac{1}{\alpha}\right)-\frac{n}{2(n+2)}\sqrt{(4n-1)S}$$

ſiue

fiue

$$f=\frac{2(n-1)(n+1)a\alpha}{(4+n-2nn)\alpha+n(2n+1)a+na\alpha\sqrt{(4n-1)S}}$$

$$g=\frac{2(n-1)(n+1)a\alpha}{(4+n-2nn)a+n(2n+1)\alpha-na\alpha\sqrt{(4n-1)S}}$$

Oportet ergo pro S fumi quantitatem pofitiuam, atque vt cum reliqua parte, cui jungitur, fit homogenea, ponamus

$$S=(\lambda-1)\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2$$

vbi λ denotat numerum vnitate maiorem; atque effici poterit, vt fpatium diffufionis ita exprimatur

$$Ff=\frac{n(4n-1)\alpha\alpha xx}{4(n-1)^2(n+1)}\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\lambda\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2+\frac{4(n-1)^2}{(4n-1)a\alpha}\right)$$

quod cum vt datum fpectetur, numerus λ pro dato affumi poterit, ac lens hunc effectum producens ita determinabitur.

$$f=\frac{2(n-1)(n+1)a\alpha}{(4+n-2nn)\alpha+n(2n+1)a+n(a+\alpha)\sqrt{(4n-1)(\lambda-1)}}$$

$$g=\frac{2(n-1)(n+1)a\alpha}{(4+n-2nn)a+n(2n+1)\alpha-n(a+\alpha)\sqrt{(4n-1)(\lambda-1)}}$$

quod ergo cum fignum radicale ambiguo illatum inuoluat, duplici modo fieri poterit.

Coroll. I.

53. Omnes igitur lentes, quae obiecti ad diftantiam $AE=a$ remoti, imaginem principalem in diftantia $BF=\alpha$ repraefentant, hanc habent proprietatem vt fit:

$$\frac{n-1}{f}+\frac{n-1}{g}=\frac{1}{a}+\frac{1}{\alpha} \quad \text{feu} \quad \frac{fg}{f+g}=\frac{(n-1)a\alpha}{a+\alpha}.$$

 et

et cum sit

$$n = \tfrac{31}{20} \text{ erit } \frac{fg}{f+g} = \frac{11\,a\alpha}{20(a+\alpha)} = \frac{0,55\,a\alpha}{a+\alpha}.$$

Coroll. 2.

54. Si ergo lens sit vtrinque aeque conuexa seu $f = g$, oportet esse $f = g = \frac{2(n-1)a\alpha}{a+\alpha} = \frac{11\,a\alpha}{10(a+\alpha)}$.

sin autem lens desideretur plano-conuexa vt sit $f = \infty$, capi debet

$$g = \frac{(n-1)a\alpha}{a+\alpha} = \frac{11\,a\alpha}{20(a+\alpha)} = \frac{0,55\,a\alpha}{a+\alpha}$$

At si lens debeat esse conuexo-plana seu $g = \infty$, capi oportet

$$f = \frac{(n-1)a\alpha}{a+\alpha} = \frac{11\,a\alpha}{20(a+\alpha)} = \frac{0,55\,a\alpha}{a+\alpha}$$

Scholion I.

55. Substituamus pro n valorem ipsi conuenientem $\frac{31}{20}$, et iam vidimus fore:

$$\frac{n(4n-1)}{8(n-1)^2(n+2)} = \frac{8060}{8591} = 0,938191$$

$$\frac{4(n-1)^2}{4n-1} = \frac{121}{520} = 0,232692$$

$$\frac{4+n-2nn}{2(n-1)(n+2)} = \frac{149}{781} = 0,190781$$

$$\frac{n(2n+1)}{2(n-1)(n+2)} = \frac{1271}{781} = 1,627401$$

nunc vero notari oportet esse:

$$\frac{n\sqrt{(4n-1)}}{2(n-1)(n+2)} = \frac{62\sqrt{130}}{781} = 0,905133$$

Quare

Quare si lens ita construatur vt sit

$$f = \frac{a\alpha}{0,190781\,a + 1,627401\,\alpha \pm 0,905133(a+\alpha)\sqrt{(\lambda-1)}}$$

$$g = \frac{a\alpha}{0,190781\,a + 1,627401\,\alpha \mp 0,905133(a+\alpha)\sqrt{(\lambda-1)}}$$

erit pro eius apertura, cuius semidiameter $= x$, spatium diffusionis

$$Ff = 0,938191\,a\alpha xx\left(\tfrac{1}{a}+\tfrac{1}{\alpha}\right)\left(\lambda\left(\tfrac{1}{a}+\tfrac{1}{\alpha}\right)^2 + \tfrac{0,232692}{a\alpha}\right)$$

Cum autem in posterum hi numeri frequentissime occurrant eorum loco ad abbreuiandum certis characteribus vtamur, ponamus ergo

$$\frac{n(4n-1)}{8(n-1)^2(n+2)} = 0,938191 = \mu.$$

$$\frac{4(n-1)^2}{4n-1} = 0,232692 = \nu$$

$$\frac{4+n-2nn}{2(n-1)(n+2)} = 0,190781 = \varrho$$

$$\frac{n(2n+1)}{2(n-1)(n+2)} = 1,627401 = \sigma$$

$$\frac{n\sqrt{(4n-1)}}{2(n-1)(n+2)} = 0,905133 = \tau$$

seu

$$\mu = \frac{1}{4(n+2)} + \frac{1}{4(n-1)} + \frac{1}{8(n-1)^2}$$

$$\varrho = \frac{1}{2(n-1)} + \frac{1}{n+2} - 1$$

$$\sigma = 1 + \frac{1}{2(n-1)} - \frac{1}{n+2}$$

$$\tau = \tfrac{1}{3}\left(\frac{1}{2(n-1)} + \frac{1}{n+2}\right)\sqrt{(4n-1)}$$

vnde pro quouis alio medio pellucido hi valores supputari possunt sicque

ſicque vt prodeat ſpatium diffuſionis

$$Ff = \mu\alpha\alpha xx\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\lambda\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2+\frac{\nu}{a\alpha}\right)$$

lentis conſtructio erit

$$f = \frac{a\alpha}{\varrho\alpha+\sigma a\pm\tau(a+\alpha)\sqrt{(\lambda-1)}}$$

$$g = \frac{a\alpha}{\varrho a+\sigma\alpha\mp\tau(a+\alpha)\sqrt{(\lambda-1)}}$$

Dummodo igitur λ fuerit numerus poſitiuus vnitate non minor talis lens duplici modo effici poterit. Caſu autem $\lambda=1$, quo ſpatium diffuſionis eſt minimum, vnica lens propoſito ſatisfaciens conſtrui poteſt.

Coroll. 3.

56. Si igitur lens ſit vtrinque aequaliter convexa, ideoque $f=g=\frac{2(n-1)a\alpha}{a+\alpha}=\frac{11}{10}\cdot\frac{a\alpha}{a+\alpha}$, in expreſſione noſtra pro ſpatio diffuſionis inuenta valor ipſius λ ex aequalitate inter f et g ſtatuta definietur, vnde fit

$$(\sigma-\varrho)(a-\alpha)=2\tau(a+\alpha)\sqrt{(\lambda-1)} \text{ hincque}$$

$$\lambda = 1+\frac{0{,}629795\,\alpha\alpha-1{,}259589\,a\alpha+0{,}629795\,aa}{(a+\alpha)^2}.$$

Coroll. 4.

57. Si lens capiatur plano conuexa vt ſit:

$$f=\infty \text{ et } g=\frac{(n-1)a\alpha}{a+\alpha}=\frac{11}{20}\cdot\frac{a\alpha}{a+\alpha}$$

pro ſpatio diffuſionis habebitur

$$\varrho\alpha+\sigma a=\pm\tau(a+\alpha)\sqrt{(\lambda-1)}$$

vnde in numeris colligitur

$$\lambda = 1+\frac{0{,}044427\,\alpha\alpha+0{,}757540\,a\alpha+3{,}252692\,aa}{(a+\alpha)^2}$$

Coroll.

Coroll. 5.

58. Si denique lens adhibeatur conuexo - plana, vt ſit:

$$g = \infty \text{ et } f = \frac{(n-1)a\alpha}{a+\alpha} = \frac{11}{20} \cdot \frac{a\alpha}{a+\alpha}$$

pro ſpatio diffuſionis inueniendo poni oportet

$$\varrho a + \sigma \alpha = \pm \tau (a+\alpha) \sqrt{(\lambda - 1)}$$

vnde in numeris colligimus:

$$\lambda = 1 + \frac{3,232697\alpha\alpha + 0,757640a\alpha + 0,044427aa}{(a+\alpha)^2}$$

Scholion 2.

59. Quod ad aperturam attinet iam initio animaduertimus, in ea maiores arcus comprehendi non debere, quam qui ſint principiis ſtabilitis conformes. Scilicet vt nullus angulus ſupra 30° gradus occurrat, anguli ACM et BDN certe minores 30 gradibus eſſe debent, cum anguli EMc et VNd ipſis ſint maiores, quorum alter cum quandoque ad duplum aſſurgere poſſit, poterimus hanc regulam ſtatuere, vt aperturae ſemidiameter x neque $\frac{1}{4} f$ neque $\frac{1}{4} g$ ſuperet. Verum quouis caſu ad ipſos angulos EMc et VNd, qui ſunt maximi, attendi conueniet, atque tanta apertura admitti poterit, vnde neuter horum angulorum 30 gradus ſuperans oriatur, ſi cautius procedere velimus, etiam angulos 20 gradibus minores cuitare poterimus, quo pacto apertura magis reſtringetur.

Tab. I. Fig. 2.

Problema 8.

Tab. I. Fig. 3. 60. Non negleċta lentis craſſitie AB ſi pro diſtantia obiecti $AE = a$ detur diſtantia imaginis principalis $BF = \alpha$, obieċtum autem parumper longius in e remoueatur, definire locum imaginis principalis f.

Solutio.

Poſita lentis craſſitie $AB = d$, radiisque faciei anterioris $AM = f$ et poſterioris $BN = g$, ſupra vidimus hos radios ita a binis diſtantiis a et α atque craſſitie lentis d pendere debere, vt ſit

$$\frac{n-1}{f} = \frac{1}{a} + \frac{2n}{k+d} \text{ et } \frac{n-1}{g} = \frac{1}{\alpha} - \frac{2n}{k-d}$$

denotante k quantitatem quamcunque. Hinc ergo cum ſit

$$\frac{k+d}{2n} = \frac{af}{(n-1)a-f} \text{ et } \frac{k-d}{2n} = \frac{\alpha g}{g-(n-1)\alpha}$$

erit eliminando k

$$\frac{d}{n} = \frac{af}{(n-1)a-f} - \frac{\alpha g}{g-(n-1)\alpha}.$$

Ponamus iam diſtantiam $AE = a$ creſcere particula $Ee = da$ ac per differentiationem inueniemus, quantum inde diſtantia imaginis $BF = \alpha$ immutetur: habebimus ſcilicet:

$$\frac{-ffda}{((n-1)a-f)^2} - \frac{ggd\alpha}{(g-(n-1)\alpha)^2} = 0$$

vnde elicimus

$$d\alpha = \frac{-ff(g-(n-1)\alpha)^2}{gg((n-1)a-f)^2}\, da = \frac{-\alpha\alpha}{aa}\, da \left(\frac{k+d}{k-d}\right)^2$$

Quare

Quare obiecto E per ſpatiolum minimum Ee longius a lente remoto, imago principalis ex F propius ad lentem admouebitur per ſpatiolum minimum Ff, ita vt ſit

$$Ff = \frac{\alpha\alpha}{aa}\left(\frac{k+d}{k-d}\right)^2 . Ee = \frac{ff(g-(n-1)\alpha)^2}{gg((n-1)a-f)^2} . Ee.$$

Coroll. 1.

61. Quia quantitas $\frac{\alpha\alpha}{aa}\left(\frac{k+d}{k-d}\right)^2$ neceſſario eſt poſitiua, euidens eſt ſi obiectum longius a lente remoueatur, imaginem ſemper propius ad lentem admoueri. Contra ergo etiam ſi obiectum propius ad lentem accedat, imago longius ab ea recedet.

Coroll. 2.

62. Si craſſities lentis d euaneſcat erit $Ff = \frac{\alpha\alpha}{aa}$. E$e$, ſin autem ea non euaneſcat fieri poteſt vt ſit vel $Ff > \frac{\alpha\alpha}{aa}$ Ee vel $Ff < \frac{\alpha\alpha}{aa}$. E$e$, prius eueniet ſi k ſit quantitas poſitiua, poſterius ſi negatiua. At ſi ſit vel $k = \infty$ vel $k = 0$, vtroque caſu erit $Ff = \frac{\alpha\alpha}{aa}$. E$e$ etiamſi craſſities lentis non euaneſcat.

Coroll. 3.

63. Cum in diſtantiis a et α mutatio minima fieri concipiatur, ſpatium diffuſionis nullam inde variationem ſubire cenſendum eſt: ſiue ergo obiectum in E ſiue e reperiatur, ac ſemidiameter aperturae

 lentis

lentis in facie anteriore fuerit $= x$, erit spatium diffusionis =

$$\frac{nx\alpha xx}{2(n-1)}\left\{\begin{array}{l}+\left(\frac{k+d}{k-d}\right)^2\left(\frac{n}{a}+\frac{2}{k+d}\right)\left(\frac{1}{a}+\frac{2}{k+d}\right)^2\\+\left(\frac{k-d}{k+d}\right)^2\left(\frac{n}{\alpha}-\frac{2}{k-d}\right)\left(\frac{1}{\alpha}-\frac{2}{k-d}\right)^2\end{array}\right\}$$

in facie autem posteriori semidiameter aperturae debet esse $=\frac{k-d}{k+d}\cdot x$.

Problema 9.

Tab. I. Fig. 3.

64. Rationem definire, quam habet magnitudo imaginis ad magnitudinem obiecti non neglecta lentis crassitie.

Solutio.

Sit obiecti ante lentem distantia $AE = a$, imaginis vero $BF = \alpha$ existente lentis crassitie $= d$; vbi quidem tantum imaginem principalem spectamus neglecto spatio diffusionis. Sit iam $E\varepsilon$ obiectum, cui tribuatur magnitudo quam minima $E\varepsilon = z$, normaliter axi lentis insistens; eiusque imago in $F\zeta$ exhibebitur, cuius magnitudo $F\zeta$ quaeritur. Cum igitur punctum ζ a puncto ε oriatur, ita vt radii ab ε emissi in ζ colligantur, consideretur radius quicunque εM, qui productus cum axe in e occurrat: et perinde est, ac si hic radius ex axis puncto e emanaret. Quare is post refractionem cum axe in f concurret, vt sit $Ff = \frac{\alpha\alpha}{aa}\left(\frac{k+d}{k-d}\right)^2 \cdot Ee$, indeque ad ζ perget: ex quo magnitudinem $F\zeta$ definire licebit. Statuatur in

in hunc finem $AM = x$, erit $BN = \frac{k-d}{k+d} \cdot x$; hincque colligemus has proportiones:

$$Ee : E\varepsilon = eA : AM = a : x$$

$$Ff : F\zeta = fB : BN = \alpha : \frac{k-d}{k+d} x$$

ob ſpatiola Ee et Ff quam minima: vnde habebimus

$$\frac{Ff}{Ee} : \frac{F\zeta}{E\varepsilon} = \frac{\alpha}{a} : \frac{k-d}{k+d} = \frac{\alpha x}{a x} \left(\frac{k+d}{k-d}\right)^2 : \frac{F\zeta}{E\varepsilon}$$

Concluditur ergo $\frac{F\zeta}{E\varepsilon} = \frac{\alpha}{a} \cdot \frac{k+d}{k-d}$, ex quo cum magnitudo obiecti $E\varepsilon$ poſita ſit $= z$, erit magnitudo imaginis $F\zeta = \frac{\alpha(k+d)}{a(k-d)} z$.

Coroll. 1.

65. Secundum hanc ergo rationem diameter obiecti immutatur. Vbi notandum eſt, ſi expreſſio $\frac{\alpha(k+d)}{a(k-d)}$ habeat poſitiuum valorem obiecti imaginem ſitu inuerſo repraeſentari, contra autem ſitu erecto, ſi $\frac{\alpha(k+d)}{a(k-d)}$ negatiuum valorem adipiſcatur.

Coroll. 2.

66. Si craſſities lentis euaneſcat, fit $F\zeta = \frac{\alpha}{a} z$. quo ergo caſu recta iungens puncta extrema ε et ζ tranſit per centrum lentis. At ſi craſſities in computum ducatur, recta $\varepsilon\zeta$ modo intra lentem modo extra, axem lentis ſecare poterit.

Scholion.

67. Sic igitur omnia, quae circa vnam lentem quamcunque noſſe oportet, expediuimus, vt etiam

craſſitiei rationem habuerimus. Ac primo quidem ad binas diſtantias lentis quaſi determinatrices ſpectari conuenit, quae ſunt obiecti diſtantia ante lentem $AE = a$ et imaginis principalis poſt lentem diſtantia $BF = \alpha$, quibus addi debet lentis craſſities $AB = d$. His autem conditionibus infinitae lentes ſatisfaciunt; ſi enim radius faciei anterioris AM dicatur $= f$, et poſterioris $BN = g$, vtraque tanquam conuexa conſiderata, conſtructio lentis his continetur formulis:

$$\frac{n-1}{f} = \frac{1}{a} + \frac{2n}{k+d} \quad \text{et} \quad \frac{n-1}{g} = \frac{1}{\alpha} - \frac{2n}{k-d}$$

ſeu

$$f = \frac{(n-1)a(k+d)}{k+d+2na} \quad \text{et} \quad g = \frac{(n-1)\alpha(k-d)}{k-d-2n\alpha}$$

exiſtente $n = \frac{31}{20}$; vbi k eſt quantitas arbitraria, hincque lentes innumerabiles quaeſito ſatisfacientes obtinentur.

Quod ſi iam obiecti diameter ponatur $= z$, erit imaginis principalis repraeſentatae diameter $= \frac{\alpha(k+d)}{a(k-d)} z$, quatenus imago ſitu inuerſo exhibita conſideratur.

Deinde ſi aperturae in facie anterioris lentis ſemidiameter ſit $= x$, ſpatium diffuſionis, quatenus ab imagine principali ad lentem porrigitur, ita exprimetur vt ſit:

$$\frac{n\alpha\alpha xx}{2(n-1)^2} \cdot \left\{ \begin{array}{l} + \left(\frac{k+d}{k-d}\right)^2 \left(\frac{n}{a} + \frac{2}{k+d}\right)\left(\frac{1}{a} + \frac{2}{k+d}\right)^2 \\ + \left(\frac{k-d}{k+d}\right)^2 \left(\frac{n}{\alpha} - \frac{2}{k-d}\right)\left(\frac{1}{\alpha} - \frac{2}{k-d}\right)^2 \end{array} \right\}$$

in facie autem poſteriori aperturae ſemidiameter debet eſſe $= \frac{k-d}{k+d} x$ vel ſaltem non minor. Quia ſpatium diffu-

diffusionis factorem habet $\alpha\alpha xx$, breuitatis gratia id ita $P\alpha\alpha xx$ indicabimus. Haecque in genere teneantur etiam crassitiei lentis ratione habita. At si crassities lentis euanescat, formulas magis euoluere licuit, scilicet si breuitatis gratia ponatur:

$$\mu = 0,938191; \ \varrho = 0,190781; \ \tau = 0,905133$$
$$\nu = 0,232692; \ \sigma = 1,627401; \ \lambda > 1 \text{ arbitr.}$$

sumaturque pro formatione lentis:

$$f = \frac{a\alpha}{\varrho\alpha + \sigma a \pm \tau(a+\alpha)\sqrt{(\lambda-1)}}$$
$$g = \frac{a\alpha}{\varrho a + \sigma\alpha \mp \tau(a+\alpha)\sqrt{(\lambda-1)}}$$

spatium diffusionis erit pro aperturae semidiametro x

$$\mu\alpha\alpha xx\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\lambda\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2+\frac{\nu}{a\alpha}\right)$$

et obiecti diametro existente $= z$ imaginis diameter erit $= \frac{\alpha}{a} z$. His ergo pro vna lente determinatis, videamus quomodo in combinatione duarum pluriumve lentium spatium diffusionis definiatur: vt deinceps in omnis generis instrumentis dioptricis siue Telescopiis siue microscopiis confusionem assignare, modumque eam diminuendi iuuestigare possimus.

CAPVT II.

CAPVT II.

DE DIFFVSIONE IMAGINIS PER PLVRES LENTES REPRAESENTATAE.

Problema I.

68.

Tab. I. Fig. 4.

Si loco obiecti adsit imago per spatium Ee diffusa, indeque radii per lentem PP aperturae indefinitae transmittantur, determinare spatium diffusionis Ff per hanc lentem productum.

Solutio.

Loco obiecti veri hic consideramus imaginem iam per aliam lentem repraesentatam, quae sit diffusa per spatium Ee, ita vt in E sit imago principalis per radios axi proximos formata in e autem imago extrema, radiis scilicet per marginem aperturae lentis praecedentis transmissis formata; qui radii cum axe constituant angulum $= \Phi$. Quare a puncto E nonnisi radii axi proximi in lentem PP emittuntur, a puncto e autem eiusmodi tantum radii, qui ad axem eA sub angulo $MeA = \Phi$ sint inclinati. Ponatur iam

iam diſtantia $EA = a$, prae qua ſpatium diffuſionis Ee vt valde paruum ſpectetur, lens autem PP ſit eiusmodi vt obiecti in E exiſtentis imaginem principalem referat in diſtantia $BF = \alpha$, exiſtente lentis craſſitie $AB = d$. Hanc ob rem ſi ponatur faciei anterioris AM radius $= f$, poſterioris $BN = g$, lente vt conuexa vtrinque ſpectata oportet eſſe:

$$f = \frac{(n-1)a(k+d)}{k+d+2na} \text{ et } g = \frac{(n-1)\alpha(k-d)}{k-d-2n\alpha}$$

vbi eſt $n = \frac{31}{20}$ et k quantitas arbitraria. Hinc autem vti demonſtrauimus, pro aperturae ſemidiametro $= x$ naſceretur ſpatium diffuſionis

$$\frac{n\alpha\alpha xx}{2(n-1)^2} \cdot \left\{ \begin{array}{l} +\left(\frac{k+d}{k-d}\right)^2\left(\frac{n}{a}+\frac{2}{k+d}\right)\left(\frac{1}{a}+\frac{2}{k+d}\right)^2 \\ +\left(\frac{k-d}{k+d}\right)^2\left(\frac{n}{\alpha}-\frac{2}{k-d}\right)\left(\frac{1}{\alpha}-\frac{2}{k-d}\right)^2 \end{array} \right\}$$

pro quo ſcribamus breuitatis gratia $P\alpha\alpha xx$. Iam puncti E, quia inde nonniſi radii axi proximi ad lentem emittuntur imago exhibebitur in F, vt ſit diſtantia $BF = \alpha$: videamus ergo quorſum imago puncti e debeat cadere. Ac ſi radii ex puncto e emiſſi eſſent axi proximi, quia id a lente magis eſt remotum quam E, eius imago propius ad lentem caderet puta in Φ, vt eſſet, ſicut §. 60 definiuimus

$$F\Phi = \frac{\alpha\alpha}{a a}\left(\frac{k+d}{k-d}\right)^2 . Ee.$$

in Φ ſcilicet caderet imago principalis, ſi obiectum eſſet in e. Sed quia ab e tantum radii eM ad axem angulo $AeM = \Phi$ inclinati emittuntur, hi lenti in

punctis M ab A interuallo $AO = eA.\Phi = a\Phi$ remotis occurrent, quandoquidem interuallum Ee prae distantia $Ae = a$ contemnimus, perinde igitur est ac si lenti apertura tribueretur, cuius semidiameter esset $= a\Phi$, et obiecti in e existentis imago extrema definiri deberet, quae cadat in f ita vt Φf sit spatium diffusionis obiecto in e existenti, et aperturae lentis, cuius semidiameter $= a\Phi$, conueniens. Hinc ergo erit interuallum $\Phi f = P\alpha\alpha\alpha\alpha\Phi\Phi$; et quia puncti E imago in F, puncti e autem imago in f exhibetur, erit spatium diffusionis per lentem PP productum

$$Ff = \frac{\alpha\alpha}{aa}\left(\frac{k+d}{k-d}\right)^2 . Ee + P\alpha\alpha\alpha\alpha\Phi\Phi.$$

In imagine autem extrema f radii Nf ita cum axe concurrent, vt sit

$$\text{angulus } BfN = \frac{k-d}{k+d} \cdot \frac{\alpha\Phi}{a}.$$

Coroll. I.

69. Si ergo imago iam diffusa per spatium Ee locum obiecti teneat respectu lentis PP, per hanc spatium diffusionis nouum producitur Ff, ita vt imago principalis cadat in F extrema vero in f: fierique poterit, vt hoc nouum spatium diffusionis Ff maius sit vel minus proposito Ee.

Coroll. 2.

70. Aperturam lentis PP vt indefinitam assumsi, manifestum autem est sufficere, dum eius semidiameter non sit minor quam $a\Phi$. Si enim esset minor

minor, radii ex puncto e emissi plane non per lentem transitum inuenirent, neque imago puncti e exprimeretur, sed imaginis Ff punctum extremum responderet imagini cuipiam intermediae spatii Ee.

Coroll. 3.

71. Si diameter obiecti seu potius imaginis in E sitae sit $= z$, tum imaginis inde per lentem PP in F formatae diameter erit $= \frac{\alpha(k+d)}{a(k-d)} z$ quae expressio si sit positiua simul declarat situm imaginis in F esse inuersum respectu eius, quae est in E.

Scholion.

72. Cum in hoc capite plures lentes sim consideraturus, pro cuiuslibet determinatione, spectandae sunt primo binae distantiae determinatrices, altera obiecti seu imaginis, a qua lens radios accipit ante lentem, altera imaginis inde per lentem repraesentatae post lentem: quae distantiae ex imaginibus principalibus aestimentur. Deinde cuiusque lentis crassities in computum est ducenda. Tertio cum his lens nondum prorsus determinetur, insuper pro quaque lente accedit distantia quaedam arbitraria hactenus per litteram k indicata. Quarto vero imprimis ratio haberi deberet, spatii diffusionis, quod cuique lenti pro data apertura conueniat. Quemadmodum autem ex binis distantiis determinatricibus, crassitie lentis et quantitate illa arbitraria k, cum binae lentis facies tum

 etiam

etiam ſpatium diffuſionis pro apertura, cuius ſemidiameter ponitur $= x$ definiatur, in praecedente capite fuſius eſt expoſitum. Hic igitur cum plures lentes ſint conſiderandae, pro ſingulis haec elementa ſequentibus litteris indicabo.

Pro Lente	Diſtantiae determinatrices obiecti	imaginis	Craſſities lentis.	Quantitas arbitraria	Spatium diffuſionis pro aperturae ſemidiametro x
Prima	a	α	v	k	$P\alpha\alpha xx$
Secunda	b	β	v'	k'	$Q\beta\beta xx$
Tertia	c	γ	v''	k''	$R\gamma\gamma xx$
Quarta	d	δ	v'''	k'''	$S\delta\delta xx$
Quinta	e	ε	v''''	k''''	$T\varepsilon\varepsilon xx$

Sin autem ratio refractionis in ſingulis lentibus diſcrepet, pro prima lente eam ponamus $= n$; pro ſecunda $= n'$; pro tertia $= n''$ etc.

Prima autem lens hic mihi perpetuo erit ea, quae obiecto eſt proxima indeque recedendo reliquas lentes ordine numero: ex quo ſimul habebuntur diſtantiae lentium, ſcilicet ſecundae a prima $= \alpha + b$, tertiae a ſecunda $= \beta + c$; quartae a tertia $= \gamma + d$; quintae a quarta $= \delta + e$ etc. quae diſtantiae ex ſua natura debent eſſe poſitiuae etiamſi ſingulae diſtantiae determinatrices praeter primam a, quippe quae ad ipſum obiectum neceſſario ante lentem primam conſtituendum refertur, ſint quandoque negatiuae. Craſſitiem lentium hic

hic littera v defigno, quia littera d inter diftantias determinatrices, fi numerus lentium vltra ternarium affurgat, reperitur. Pro craffitie autem et quantitate arbitraria iisdem litteris vtor, commatibus infcriptis diftinguendis ob penuriam litterarum diuerfarum. Ceterum obferuandum eft me omnes lentes tanquam fuper communi axe difpofitas affumere.

Problema 2.

73. Si radii ab obiecto $E\varepsilon$ emiffi per duas lentes PP et QQ transmittantur definire fpatium diffufionis Gg, a data apertura primae lentis oriundum; vt et magnitudinem imaginis principalis in G exhibitae.

Tab. I. Fig. 5.

Solutio.

Sit obiecti magnitudo $E\varepsilon = z$, eiusque imago principalis per primam lentem PP proiiciatur in $F\zeta$, at per ambas in $G\eta$ erunt diftantiae determinatrices pro lente prima PP, obiecti $EA = a$, imaginis $aF = \alpha$ pro lente fecunda QQ, obiecti $FB = b$, imagiuis $bG = \beta$ Deinde fit

pro lente PP, craffities $Aa = v$, quantitas arbitr: $= k$
pro lente QQ, craffities $Bb = v'$, quantitas arbitr: $= k'$
Denique pro apertura in anteriori facie vtriusque lentis cuius femidiameter fit $= x$

fpatium diffufionis primae lentis $PP = P\alpha\alpha xx$
- - - - - fecundae lentis $QQ = Q\beta\beta xx$.

 His

His poſitis pro imagine per primam lentem proiecta $F\zeta$ erit eius magnitudo $F\zeta = \frac{\alpha(k+v)}{a(k-v)}z$ pro inuerſa habenda, ſi haec expreſſio fuerit poſitiua. Deinde ſi lentis primae PP ſemidiameter aperturae in anteriori facie ponatur $= x$, erit ſpatium diffuſionis $Ff = P\alpha\alpha xx$ et inclinatio radiorum in f ad axem $= \frac{k-v}{k+v} \cdot \frac{x}{\alpha}$. tum vero in facie lentis PP poſteriori ſemidiameter aperturae non minor eſſe debet quam $\frac{k-v}{k+v}x$. Tota iam haec imago diffuſa per ſpatium Ff reſpectu alterius lentis QQ tanquam obiectum ſpectari debet, cuius proinde repraeſentatio Gg per propoſitionem praecedentem determinabitur. Erit autem hic $\Phi = \frac{k-v}{k+v} \cdot \frac{x}{\alpha}$, et ſpatium ibi expreſſum $Ee = P\alpha\alpha xx$; tum vero pro a, α, k, d et P hic ſcribi oportet b, β, k', v' et Q, vnde fiet ſpatium diffuſionis quaeſitum:

$$Gg = \frac{\beta\beta}{bb}\left(\frac{k'+v'}{k'-v'}\right)^2 P\alpha\alpha xx + \frac{bb\beta\beta}{\alpha\alpha}\left(\frac{k-v}{k+v}\right)^2 Qxx$$

ſiue

$$Gg = \frac{\beta\beta}{bb}\left(\frac{k'+v'}{k'-v'}\right)^2 P\alpha\alpha xx + \frac{bb}{\alpha\alpha}\left(\frac{k-v}{k+v}\right)^2 Q\beta\beta xx$$

Radiorum porro in g incidentium inclinatio ad axem eſt $\left(\frac{k-v}{k+v}\right)\left(\frac{k'-v'}{k'+v'}\right)\frac{bx}{\alpha\beta}$. Denique cum ſit $F\zeta = \frac{\alpha(k+v)}{a(k-v)}z$ erit magnitudo imaginis in G vt erutae conſideratae:

$$G\eta = \frac{\alpha\beta}{ab}\left(\frac{k+v}{k-v}\right)\left(\frac{k'+v'}{k'-v'}\right)z.$$

Coroll. 1.

74. Quod ad aperturam facierum lentis QQ attinet, ea maior eſſe debet ſpatio tranſitus radiorum; hinc ergo erit ſemidiameter aperturae

faciei

faciei anterioris $> \left(\frac{k-v}{k+v}\right)\frac{bx}{\alpha}$

faciei posterioris $> \left(\frac{k-v}{k+v}\right)\left(\frac{k'-v'}{k'+v'}\right)\frac{bx}{\alpha}$.

Coroll. 2.

75. Si ponatur pro lente prima PP radius faciei anterioris $= f$, et posterioris $= g$ erit posito $n : 1$

$$f = \frac{(n-1)a(k+v)}{k+v+2na} \text{ et } g = \frac{(n-1)\alpha(k-v)}{k-v-2n\alpha}$$

Similique modo si pro lente altera QQ radius faciei anterioris ponatur $= f'$, et posterioris $= g'$ erit

$$f' = \frac{(n-1)b(k'+v')}{k'+v'+2nb} \text{ et } g' = \frac{(n-1)c(k'-v')}{k'-v'-2nc}$$

omnibus scilicet faciebus vt conuexis consideratis.

Coroll. 3.

76. Pro spatio autem diffusionis ab vtraque lente productae erit quemadmodum inuenimus:

$$P = \frac{n^2}{2(n-1)^2} \cdot \begin{Bmatrix} +\left(\frac{k+v}{k-v}\right)^2\left(\frac{n}{\alpha}+\frac{2}{k+v}\right)\left(\frac{1}{\alpha}+\frac{2}{k+v}\right)^2 \\ +\left(\frac{k-v}{k+v}\right)^2\left(\frac{n}{\alpha}-\frac{2}{k-v}\right)\left(\frac{1}{\alpha}-\frac{2}{k-v}\right)^2 \end{Bmatrix}$$

similique modo

$$Q = \frac{n^2}{2(n-1)^2} \cdot \begin{Bmatrix} +\left(\frac{k'+v'}{k'-v'}\right)^2\left(\frac{n}{b}+\frac{2}{k'+v'}\right)\left(\frac{1}{b}+\frac{2}{k'+v'}\right)^2 \\ +\left(\frac{k'-v'}{k'+v'}\right)^2\left(\frac{n}{c}-\frac{2}{k'-v'}\right)\left(\frac{1}{c}-\frac{2}{k'-v'}\right)^2 \end{Bmatrix}$$

Non opus est vt omnibus lentibus eadem refractionis ratio $n:1$ tribuatur, sed simili modo pro pluribus lentibus poni potest n, n', n'', n''', etc. vt iam supra monuimus vnde nullum aliud discrimen nascitur, nisi

nisi quod in formulis pro f' et g' inuentis loco h scribi debeat n', et Q statui debeat.

$$= \frac{n'}{2(n'-1)^2} \left\{ \begin{array}{l} +\left(\frac{k'+v'}{k'-v'}\right)^2 \left(\frac{n'}{b} + \frac{2}{k'+v'}\right)\left(\frac{1}{b} + \frac{2}{k'+v'}\right)^2 \\ +\left(\frac{k'-v'}{k'+v'}\right)^2 \left(\frac{n'}{\mathfrak{b}} - \frac{2}{k'-v'}\right)\left(\frac{1}{\mathfrak{b}} - \frac{2}{k'-v'}\right)^2 \end{array} \right\}$$

Coroll. 4.

77. Sin autem crassities lentium euanescat, et pro quantitate arbitraria k, k' introducamus numerum λ, λ', erit

pro lente PP

$$f = \frac{a\alpha}{\varrho\alpha + \sigma a \pm \tau(a+\alpha)\sqrt{(\lambda-1)}};\ g = \frac{a\alpha}{\varrho a + \sigma\alpha \mp \tau(a+\alpha)\sqrt{(\lambda-1)}}$$

et

$$P = \mu\left(\frac{1}{a} + \frac{1}{\alpha}\right)\left(\lambda\left(\frac{1}{a} + \frac{1}{\alpha}\right)^2 + \frac{\nu}{a\alpha}\right)$$

at pro lente QQ

$$f' = \frac{b\mathfrak{b}}{\varrho\mathfrak{b} + \sigma b \pm \tau(b+\mathfrak{b})\sqrt{(\lambda'-1)}};\ g' = \frac{b\mathfrak{b}}{\varrho b + \sigma\mathfrak{b} \mp \tau(b+\mathfrak{b})\sqrt{(\lambda'-1)}}$$

$$Q = \mu\left(\frac{1}{b} + \frac{1}{\mathfrak{b}}\right)\left(\lambda'\left(\frac{1}{b} + \frac{1}{\mathfrak{b}}\right)^2 + \frac{\nu}{b\mathfrak{b}}\right).$$

In expressionibus autem inuentis formulae $\left(\frac{k-v}{k+v}\right)$ et $\left(\frac{k'-v'}{k'+v'}\right)$ abeunt in vnitatem.

Numerorum vero μ, ν, ϱ, σ, τ indoles in §. 55 exposita est.

Si refractio lentium differat etiam litterae μ, ν, ϱ, σ, τ diuersos valores pro singulis lentibus sortientur, quae litte-

litterae si etiam commatibus a prioribus distinguantur, vt sit

$$\rho' = \frac{1}{2(n'-1)} + \frac{1}{n'+2} - 1$$
$$\sigma' = 1 + \frac{1}{2(n'-1)} - \frac{1}{n'+2}$$
$$\tau' = \frac{1}{3}\left(\frac{1}{2(n'-1)} + \frac{1}{n'+2}\right)\sqrt{(4n'-1)}$$

pro secunda lente erit

$$f' = \frac{b\beta}{\rho'\beta + \sigma' b \pm \tau'(b+\beta)\sqrt{(\lambda'-1)}}$$
$$g' = \frac{b\beta}{\rho' b + \sigma'\beta \mp \tau'(b+\beta)\sqrt{(\lambda'-1)}}$$

et ob eandem rationem erit

$$\mu' = \frac{1}{4(n'+2)} + \frac{1}{4(n'-1)} + \frac{1}{2(n'+1)^2}$$
$$\text{et } \nu' = \frac{4(n'-1)^2}{4n'-1}$$

quod et de sequentibus lentibus est intelligendum, si forte diuersa refractionis lege fuerint praeditae.

Scholion.

78. Quo formulas in problemate inuentas magis contrahamus, vt cum ad plures lentes processerimus, succinctiores euadant, ponamus

$$\frac{k-v}{k+v} = i \text{ et } \frac{k'-v'}{k'+v'} = i'$$

ita vt hi numeri i et i' abeant in vnitatem euanescente lentium crassitie v et v'. Tum autem erit spatium diffusionis

$$Gg = \frac{1}{i'i'} \cdot \frac{\beta\beta}{bb} \cdot P\alpha\alpha xx + ii \cdot \frac{bb}{\alpha\alpha} \cdot Q\beta\beta xx$$

 et

et radiorum in g incidentium inclinatio ad axem $= ii' . \frac{bx}{\alpha\beta}$: porro magnitudo imaginis $G\eta = \frac{1}{ii'} . \frac{\alpha\beta}{ab} z$ Ac pro apertura ſingularum facierum erit vt ſequitur

Semidiam. aperturae lentis	faciei anterioris	faciei poſterioris
primae PP	x	ix
ſecundae QQ	$i . \frac{bx}{\alpha}$	$ii' . \frac{bx}{\alpha}$

aperturae autem hae praeter primam maiores eſſe debent aſſignatis: valores enim aſſignati ſufficerent, ſi obiectum eſſet merum punctum in axe poſitum; quando autem habet magnitudinem, radii ab eius terminis per primam faciem ingreſſi latius diuagantur, et ad ſequentes facies maiorem aperturam exigunt.

Problema 3.

Tab. II. Fig. 6.

79. Si radii ab obiecto $E\varepsilon$ emiſſi per tres lentes PP, QQ et RR refringantur definire ſpatium diffuſionis Hh ob datam aperturam lentis primae oriundum, vt et magnitudinem imaginis principalis in H exhibitae.

Solutio.

Poſita obiecti magnitudine $E\varepsilon = z$, cadat eius imago principalis per lentem primam PP in $F\zeta$, dehinc per ſecundam QQ in $G\eta$, tum vero per tertiam RR in $H\theta$. Erunt ergo diſtantiae determinatrices.

pro lente PP, obiecti $EA = a$, imaginis $aF = \alpha$
pro lente QQ, obiecti $FB = b$, imaginis $bG = \beta$
pro lente RR, obiecti $GC = c$; imaginis $cH = \gamma$

Deinde

Deinde fit

pro lente PP, craſſities $Aa = v$, quantitas arbitr: $= k$
pro lente QQ, craſſities $Bb = v'$, quantitas arbitr: $= k'$
pro lente RR craſſities $Cc = v''$, quantitas arbitr: $= k''$
ac ponatur breuitatis gratia:

$$\frac{k-v}{k+v} = i;\ \frac{k'-v'}{k'+v'} = i';\ \frac{k''-v''}{k''+v''} = i''$$

Denique pro quauis lente, ſi eſſet ſingularis, eiusque aperturae ſemidiameter eſſet $= x$, fit

ſpatium diffuſionis primae lentis $PP = P\alpha\alpha xx$
\- - - - ſecundae lentis $QQ = Q\beta\beta xx$
\- - - - tertiae lentis $RR = R\gamma\gamma xx$

Iam in praecedente problemate inuenimus fore ſpatium diffuſionis per duas lentes priores productum

$$Gg = \frac{1}{i'i'} \cdot \frac{\beta\beta}{bb} P\alpha\alpha xx + ii.\frac{bb}{\alpha\alpha} \cdot Q\beta\beta xx$$

et radiorum in g concurrentium inclinationem ad axem $= ii'.\frac{bx}{\alpha\beta}$ imaginisque in G magnitudinem $G\eta = \frac{1}{ii'} \cdot \frac{\alpha\beta}{ab} z$.

Haec igitur ſi ad problema 1. transferantur, ibi loco

$$Ee, \frac{\alpha\alpha}{aa}, \left(\frac{k+d}{k-d}\right)^2, Paa\alpha\alpha \text{ et } \Phi \text{ ſcribi debebit}$$

$$Gg; \frac{\gamma\gamma}{cc}; \frac{1}{i''i''}, Rcc\gamma\gamma \text{ et } ii'.\frac{bx}{\alpha\beta},$$

hincque ſpatium diffuſionis per tres lentes productum orietur.

$$Hb = \frac{1}{i'i'.i''i''} \cdot \frac{\beta\beta\gamma\gamma}{bb\,cc} P\alpha\alpha xx + \frac{ii}{i''i''} \cdot \frac{bb\gamma\gamma}{\alpha\alpha cc} \cdot Q\beta\beta xx$$
$$+ ii.\ i'i'.\ \frac{bbcc}{\alpha\alpha\beta\beta} \cdot R\gamma\gamma xx$$

at radiorum in b concurrentium inclinatio ad axem

 erit

erit $= i i^{\prime} i^{\prime\prime} . \frac{bcx}{\alpha \mathfrak{b} \gamma}$. Denique imaginis in H formatae magnitudo erit $H\theta = \frac{1}{i i^{\prime} i^{\prime\prime}} . \frac{\alpha \mathfrak{b} \gamma}{abc} z$ ad ſitum inuerſum relata.

Coroll. 1.

80. Quod ad aperturas ſingularum facierum attinet, eas ſequenti modo comparatas eſſe conuenit.

Semid. aperturae Lentis	faciei anterioris	faciei poſterioris
Primae P P	x	$i x$
Secundae Q Q	$i . \frac{bx}{\alpha}$	$i i^{\prime} . \frac{bx}{\alpha}$
Tertiae R R	$i i^{\prime} . \frac{bcx}{\alpha \mathfrak{b}}$	$i i^{\prime} i^{\prime\prime} . \frac{bcx}{\alpha \mathfrak{b}}$

his ſcilicet valoribus non debent eſſe minores.

Coroll. 2.

81. Si inclinatio radiorum in b concurrentium ad axem vocetur $= \Phi$. Cum ſit $\Phi = i i^{\prime} i^{\prime\prime} . \frac{bcx}{\alpha \mathfrak{b} \gamma}$ et $H\theta = \frac{1}{i i^{\prime} i^{\prime\prime}} . \frac{\alpha \mathfrak{b} \gamma}{abc} z$ erit $\Phi . H\theta = \frac{xz}{a}$: quae proprietas pro lentium numero quantumuis magno locum habet.

Coroll. 3.

82. Quemadmodum radii ſingularum facierum determinandi ſint, ex praecedentibus facile liquet:

Erit nempe pro	Radius faciei anterioris	Radius faciei poſterioris
Lente prima PP	$\frac{(n-1)a(k+v)}{k+v+2na}$	$\frac{(n-1)\alpha(k-v)}{k-v-2n\alpha}$
Lente ſecunda QQ	$\frac{(n-1)b(k^{\prime}+v^{\prime})}{k^{\prime}+v^{\prime}+2nb}$	$\frac{(n-1)\mathfrak{b}(k^{\prime}-v^{\prime})}{k^{\prime}-v^{\prime}-2n\mathfrak{b}}$
Lente tertia RR	$\frac{(n-1)c(k^{\prime\prime}+v^{\prime\prime})}{k^{\prime\prime}+v^{\prime\prime}+2nc}$	$\frac{(n-1)\gamma(k^{\prime\prime}-v^{\prime\prime})}{k^{\prime\prime}-v^{\prime\prime}-2n\gamma}$

exiſtente pro vitro $n = \frac{31}{20}$.

Coroll.

Coroll. 4.

83. Tum vero valores literarum P, Q, R ita ſe habebunt

$$P = \frac{n}{2(n-1)^2}\left(\frac{1}{ii}\left(\frac{n}{a}+\frac{2}{k+v}\right)\left(\frac{1}{a}+\frac{2}{k+v}\right)^2 + ii\left(\frac{n}{\alpha}-\frac{2}{k-v}\right)\left(\frac{1}{\alpha}-\frac{2}{k-v}\right)^2\right)$$

$$Q = \frac{n}{2(n-1)^2}\left(\frac{1}{i'i'}\left(\frac{n}{b}+\frac{2}{k'+v'}\right)\left(\frac{1}{b}+\frac{2}{k'+v'}\right)^2 + i'i'\left(\frac{n}{\beta}-\frac{2}{k'-v'}\right)\left(\frac{1}{\beta}-\frac{2}{k'-v'}\right)^2\right)$$

$$R = \frac{n}{2(n-1)^2}\left(\frac{1}{i''i''}\left(\frac{n}{c}+\frac{2}{k''+v''}\right)\left(\frac{1}{c}+\frac{2}{k''+v''}\right)^2 + i''i''\left(\frac{n}{\gamma}-\frac{2}{k''-v''}\right)\left(\frac{1}{\gamma}-\frac{2}{k''-v''}\right)^2\right)$$

quibus valoribus ſpatia diffuſionis definiuntur.

Si refractio differat:

litteris	tribuatur refr.
P	n
Q	n'
R	n''.

Coroll. 5.

84. Iuuabit etiam ſpatia diffuſionis, prout per vnam, duas ac tres lentes producuntur, inter ſe comparaſſe, quod ita commodiſſime fieri videtur.

$$Ff = \alpha\alpha xx.\ P$$

$$Gg = \beta\beta xx\left(\frac{1}{i'i'}.\frac{\alpha\alpha}{bb}P + ii.\frac{bb}{\alpha\alpha}Q\right)$$

$$Hh = \gamma\gamma xx\left(\frac{1}{i'i'i''i''}.\frac{\alpha\alpha\beta\beta}{bbcc}P + \frac{ii}{i''i''}.\frac{bb\beta\beta}{\alpha\alpha cc}Q + ii.i'i'.\frac{bbcc}{\alpha\alpha\beta\beta}R\right)$$

Scholion.

85. Hinc facile perſpicitur, quomodo determinatio ſpatii diffuſionis, ad plures lentes extendi debeat;

 vnde

vnde problema generale ſtatim ad numerum lentium quemcunque accommodabo, idque pro quacunque lentium craſſitie.

Problema 4.

86. Si radii ab obiecto $E\varepsilon$ emiſſi per lentes quotcunque PP, QQ, RR, SS etc. ſuper communi axe diſpoſitas refringantur, defin:re ſpatium diffuſionis a data apertura lentis primae oriundum vt et magnitudinem imaginis repraeſentatae.

Solutio.

Poſita obiecti magnitudine $E\varepsilon = z$ imagines eius principales contemplemur: cadat igitur eius imago principalis per lentem primam PP in $F\zeta$ deinde per ſecundam QQ in $G\eta$, tum per tertiam RR in $H\theta$, porro per quartam SS in $I\iota$, per quintam TT in $K\varkappa$ etc. Hinc pro ſingulis lentibus habebimus diſtantias determinatrices quas ita litteris indicemus.

craſſitiem

Pro lente PP obiecti $EA = a$, imaginis $aF = \alpha$; $Aa = v$

Pro lente QQ obiecti $FB = b$; imaginis $bG = \beta$; $Bb = v'$

Pro lente RR obiecti $GC = c$; imaginis $cH = \gamma$; $Cc = v''$

Pro lente SS obiecti $HD = d$; imaginis $dI = \delta$; $Dd = v'''$

Pro lente TT obiecti $IE = e$; imaginis $eK = \varepsilon$; $Ee = v''''$

etc.

Deinde

Deinde cum determinatio cuiusuis lentis non solum has distantias determinatrices cum crassitie cuiusque sed etiam quantitatem quandam arbitrariam inuoluat, a qua spatium diffusionis cuiusque pendet, ponamus si quaelibet lens esset solitaria eiusque aperturae semidiameter $= x$.

Pro lente	quant: arbitr:	Spatium Diffusionis
Prima PP	k	$P\alpha\alpha xx$
Secunda QQ	k'	$Q\beta\beta xx$
Tertia RR	k''	$R\gamma\gamma xx$
Quarta SS	k'''	$S\delta\delta xx$
Quinta TT	k''''	$T\varepsilon\varepsilon xx$
	etc.	

Hinc ergo constructio cuiusque lentis ita se habebit posito $n = \frac{31}{20}$: vel si refractio differat, cuique lenti suus tribuatur valor primae n, secundae n', tertiae n'' etc.

Erit nempe pro	Radius faciei anterioris	Radius faciei posterioris
Lente prima PP	$\frac{(n-1)\alpha(k+v)}{k+v+2n\alpha}$	$\frac{(n-1)\alpha(k-v)}{k-v-2n\alpha}$
Lente secunda QQ	$\frac{(n-1)\beta(k'+v')}{k'+v'+2n\beta}$	$\frac{(n-1)\beta(k'-v')}{k'-v'-2n\beta}$
Lente tertia RR	$\frac{(n-1)\gamma(k''+v'')}{k''+v''+2n\gamma}$	$\frac{(n-1)\gamma(k''-v'')}{k''-v''-2n\gamma}$
Lente quarta SS	$\frac{(n-1)\delta(k'''+v''')}{k'''+v'''+2n\delta}$	$\frac{(n-1)\delta(k'''-v''')}{k'''-v'''-2n\delta}$
Lente quinta TT	$\frac{(n-1)\varepsilon(k''''+v'''')}{k''''+v''''+2n\varepsilon}$	$\frac{(n-1)\varepsilon(k''''-v'''')}{k''''-v''''-2n\varepsilon}$
	etc.	

at

at ſi ponamus breuitatis gratia:

$$\frac{k-v}{k+v}=i;\ \frac{k'-v'}{k'+v'}=i';\ \frac{k''-v''}{k''+v''}=i'';\ \frac{k'''-v'''}{k'''+v'''}=i''';\ \frac{k''''-v''''}{k''''+v''''}=i''''\ \text{etc.}$$

pro ſpatiis diffuſionis habebimus hos valores:

$$P=\frac{n}{2(n-1)^2}\left(\frac{1}{ii}\left(\frac{n}{a}+\frac{2}{k+v}\right)\left(\frac{1}{a}+\frac{2}{k+v}\right)^2+ii\left(\frac{n}{\alpha}-\frac{2}{k-v}\right)\left(\frac{1}{\alpha}-\frac{2}{k-v}\right)^2\right)$$

$$Q=\frac{n}{2(n-1)^2}\left(\frac{1}{i'i'}\left(\frac{n}{b}+\frac{2}{k'+v'}\right)\left(\frac{1}{b}+\frac{2}{k'+v'}\right)^2+i'i'\left(\frac{n}{\beta}-\frac{2}{k'-v'}\right)\left(\frac{1}{\beta}-\frac{2}{k'-v'}\right)^2\right)$$

$$R=\frac{n}{2(n-1)^2}\left(\frac{1}{i''i''}\left(\frac{n}{c}+\frac{2}{k''+v''}\right)\left(\frac{1}{c}+\frac{2}{k''+v''}\right)^2+i''i''\left(\frac{n}{\gamma}-\frac{2}{k''-v''}\right)\left(\frac{1}{\gamma}-\frac{2}{k''-v''}\right)^2\right)$$

$$S=\frac{n}{2(n-1)^2}\left(\frac{1}{i'''i'''}\left(\frac{n}{d}+\frac{2}{k'''+v'''}\right)\left(\frac{1}{d}+\frac{2}{k'''+v'''}\right)^2+i'''i'''\left(\frac{n}{\delta}-\frac{2}{k'''-v'''}\right)\left(\frac{1}{\delta}-\frac{2}{k'''-v'''}\right)^2\right)$$

$$T=\frac{n}{2(n-1)^2}\left(\frac{1}{i''''i''''}\left(\frac{n}{e}+\frac{2}{k''''+v''''}\right)\left(\frac{1}{e}+\frac{2}{k''''+v''''}\right)^2+i''''i''''\left(\frac{n}{\varepsilon}-\frac{2}{k''''-v''''}\right)\left(\frac{1}{\varepsilon}-\frac{2}{k''''-v''''}\right)^2\right)$$

etc.

His conſtitutis pro magnitudine ſingularum imaginum habebimus:

ad ſitum

pro vna lente imaginem $F\zeta=\frac{1}{i}\cdot\frac{\alpha}{a}z$ inuerſum

pro duabus lentibus .... $G\eta=\frac{1}{ii'}\cdot\frac{\alpha\beta}{ab}z$ erectum

pro tribus lentibus .... $H\theta=\frac{1}{ii'i''}\cdot\frac{\alpha\beta\gamma}{abc}z$ inuerſum

pro quatuor lentibus .... $I\iota=\frac{1}{ii'i''i'''}\cdot\frac{\alpha\beta\gamma\delta}{abcd}z$ erectum

pro quinque lentibus .... $K\varkappa=\frac{1}{ii'i''i'''i''''}\cdot\frac{\alpha\beta\gamma\delta\varepsilon}{abcde}z$ inuerſum

etc.

Deni-

Denique dum aperturae lentium non ſint minores, quam ſequentes formulae exhibent:

Semid. aperturae Lentis	faciei anterioris	faciei poſterioris
primae PP	x	ix
ſecundae QQ	$i.\frac{bx}{\alpha}$	$ii'.\frac{bx}{\alpha}$
tertiae RR	$ii'.\frac{bcx}{\alpha\beta}$	$ii'i''.\frac{bcx}{\alpha\beta}$
quartae SS	$ii'i''.\frac{bcdx}{\alpha\beta\gamma}$	$ii'i''i'''.\frac{bcdx}{\alpha\beta\gamma}$
quintae TT	$ii'i''i'''.\frac{bcdex}{\alpha\beta\gamma\delta}$	$ii'i''i'''i''''.\frac{bcdex}{\alpha\beta\gamma\delta}$
	etc.	

erit vt ſequitur pro quolibet lentium numero:

I. Pro vna lente

Spatium diffuſionis $Ff = \alpha\alpha xx.\,P$
inclinatio radiorum in f concurrentium ad axem $= i.\frac{x}{\alpha}$

II. Pro duabus lentibus

Spatium diffuſionis
$$Gg = \beta\beta xx\left(\frac{1}{i'i'}\cdot\frac{\alpha\alpha}{bb}P + ii.\frac{bb}{\alpha\alpha}Q\right)$$
et radiorum in g concurrentium
inclinatio ad axem $= ii'.\frac{bx}{\alpha\beta}$

III. Pro tribus lentibus

Spatium, diffuſionis:
$$Hh = \gamma\gamma xx\left(\frac{1}{i'i'.i''i''}\cdot\frac{\alpha\alpha\beta\beta}{bbcc}P + \frac{ii}{i''i''}\cdot\frac{bb\beta\beta}{\alpha\alpha\alpha}\cdot Q + ii.i'i'.\frac{bbcc}{\alpha\alpha\beta\beta}R\right)$$
et radiorum in h concurrentium
inclinatio ad axem $= ii'i''.\frac{bcx}{\alpha\beta\gamma}$.

IV. Pro quatuor lentibus.

Spatium diffusionis:

$$Ii = \delta\delta xx \left\{ \begin{array}{l} \frac{i}{i'i'.i''i''.i'''i'''} \cdot \frac{\alpha\alpha\mathfrak{b}\mathfrak{b}\gamma\gamma}{bbccdd} P + \frac{ii'}{i''i''.i'''i'''} \cdot \frac{bb\mathfrak{b}\mathfrak{b}\gamma\gamma}{\alpha\alpha ccdd} Q \\ + \frac{ii.i'i'}{i'''i'''} \cdot \frac{bbcc\gamma\gamma}{\alpha\alpha\mathfrak{b}\mathfrak{b}dd} R + ii.i'i'.i''i'' \cdot \frac{bbccdd}{\alpha\alpha\mathfrak{b}\mathfrak{b}\gamma\gamma} S \end{array} \right\}$$

et radiorum in i concurrentium

inclinatio ad axem $= ii'i''i''' \cdot \frac{bcdx}{\alpha\mathfrak{b}\gamma\delta}$.

V. Pro quinque lentibus.

Spatium diffusionis:

$$Kk = \varepsilon\varepsilon xx \left\{ \begin{array}{l} + \frac{i}{i'i'.i''i''.i'''i'''.i''''i''''} \cdot \frac{\alpha\alpha\mathfrak{b}\mathfrak{b}\gamma\gamma\delta\delta}{bbccddee} P \\ + \frac{ii'}{i''i''.i'''i'''.i''''i''''} \cdot \frac{bb\mathfrak{b}\mathfrak{b}\gamma\gamma\delta\delta}{\alpha\alpha ccddee} Q \\ + \frac{ii.i'i'}{i'''i'''.i''''i''''} \cdot \frac{bbcc\gamma\gamma\delta\delta}{\alpha\alpha\mathfrak{b}\mathfrak{b}ddee} R \\ + \frac{ii.i'i'.i''i''}{i''''i''''} \cdot \frac{bbccdd\delta\delta}{\alpha\alpha\mathfrak{b}\mathfrak{b}\gamma\gamma ee} S \\ + ii.i'i'.i''i''.i'''i''' \cdot \frac{bbccddee}{\alpha\alpha\mathfrak{b}\mathfrak{b}\gamma\gamma\delta\delta} T \end{array} \right\}$$

et radiorum in k concurrentium

inclinatio ad axem $= ii'i''i'''i'''' \cdot \frac{bcdex}{\alpha\mathfrak{b}\gamma\delta\varepsilon}$.

Vnde progressio harum formularum ad plures adhuc lentes satis est manifesta. Si in lentibus ratio refractionis sit diuersa atque ad singulas lentes ordine his litteris indicetur n, n', n'', n''', etc.; haec diuersitas facile ad formulas hic inuentas accommodabitur. Primum enim haec correctio occurrit in formulis pro radiis lentium,

lentium, ita, vt quemadmodum formulae f et g pro prima lente numerum n inuolunt, ita pro secunda lente numerus n', pro tertia n'' et ita porro introducatur. Similem correctionem etiam requirunt valores litterarum P, Q, R, S etc. et loco litterae n, quae in valore P occurrit, in valoribus Q, R, S etc. scribi oportet n', n'', n''' etc.

Coroll. 1.

87. Si obiectum sit tantum punctum in axe positum, sufficit vt lentium aperturae sint tantae, quantas assignauimus sin autem obiectum habeat quandam magnitudinem, tum aperturae praeter primam eo magis mensuras assignatas superare debent, quo maior fuerit obiecti magnitudo z.

Coroll. 2.

88. In expressione spatii diffusionis quadratum semidiametri aperturae primae faciei xx primo multiplicatur per quadratum distantiae postremae imaginis ab vltima lentium: quae ergo si fuerit infinita, etiam spatium diffusionis fit infinitum.

Coroll. 3.

89. Ceteris ergo paribus, quotcunque fuerint lentes, spatium diffusionis semper est proportionale quadrato diametri aperturae primae faciei, hoc est ipsi huic aperturae. Vnde diametro aperturae primae

faciei ad femiffem redacto fpatium diffufionis quadruplo fiet minus.

Scholion.

90. Confiderauimus hic ftatim loca fingularum imaginum principalium tanquam data ex iifque ftructuram cuiusque lentis quantitatem arbitrariam introducendo determinauimus. Quod fi vero ipfae lentes fuerint datae ita vt tam radii ambarum facierum cuiusque quam craffities, vna cum earum interuallis cognofcantur tum ope formularum exhibitarum, viciffim diftantiae determinatrices innotefcunt. Sint fcilicet radii facierum anterioris et pofterioris primae lentis PP, f, g, fecundae lentis QQ, f', g'; tertiae lentis RR, f'', g'' etc. craffitie earum exiftente v, v' v'' etc. tum vero dentur diftantiae $a\mathrm{B}=\mathrm{F}$; $b\mathrm{C}=\mathrm{G}$; $c\mathrm{D}=\mathrm{H}$ etc. Praeterea autem diftantia obiecti ante lentem primam fit $\mathrm{AE}=a$, ac fequenti modo omnia elementa ad problema fuperius neceffaria elicientur;

$$\begin{cases} 1.\ \frac{n-1}{f}=\frac{1}{a}+\frac{2n}{k+v}, & \text{hinc reperitur } k \\ 2.\ \frac{n-1}{g}=\frac{1}{\alpha}-\frac{2n}{k-v}, & \text{hinc vero } \alpha \\ 3.\ \mathrm{F}=\alpha+b & \text{vnde } b=\mathrm{F}-\alpha \end{cases}$$

$$\begin{cases} 4.\ \frac{n-1}{f'}=\frac{1}{b}+\frac{2n}{k'+v'}, & \text{hinc reperitur } k' \\ 5.\ \frac{n-1}{g'}=\frac{1}{\mathfrak{E}}-\frac{2n}{k'-v'}, & \text{hinc vero porro } \mathfrak{E} \\ 6.\ \mathrm{G}=\mathfrak{E}+c & \text{vnde } c=\mathrm{G}-\mathfrak{E} \end{cases}$$

(7.

$$\begin{cases} 7.\ \frac{n-1}{f''} = \frac{1}{c} + \frac{2n}{k''+\nu''}, & \text{hinc reperitur } k'' \\ 8.\ \frac{n-1}{g''} = \frac{1}{\gamma} - \frac{2n}{k''-\nu''}, & \text{hinc vero } \gamma \\ 9.\ \mathrm{H} = \gamma + d & \text{vnde } d = \mathrm{H} - \gamma \end{cases}$$

etc.

Vtcunque ergo lentes datae fuerint dispositae super axe communi, si ante eas constituatur obiectum in data distantia $AE = a$, inde singulae distantiae determinatrices $\alpha, b, \mathfrak{C}, c, \gamma$ etc. cum arbitrariis k, k', k'', k''' etc. facile definiuntur ex iisque porro spatium diffusionis cum reliquis phaenomenis in solutione problematis commemoratis assignabitur. Operae pretium autem erit casum, quo crassities lentium vt euanescens spectatur, accuratius euoluisse.

Problema 5.

91. Si crassities lentium euanescat, et quotcunque huiusmodi lentes super communi axe fuerint dispositae ante quas existat obiectum $E\varepsilon$, definire spatium diffusionis, per quod imago erit dissipata, vt et magnitudinem imaginis.

Solutio.

Si obiecti magnitudo $E\varepsilon = z$, cuius imagines principales successiue cadant in $F\zeta$, $G\eta$, $H\theta$, $I\iota$, $K\varkappa$ etc. hincque pro singulis lentibus sequentes habebimus distantias determinatrices, cum imago per quamuis lentem repraesentata respectu lentis sequentis vicem obiecti gerat.

 distan-

		distantia	distantia
Pro lente PP		obiecti $EA = a$;	imaginis $aF = \alpha$
Pro lente QQ		obiecti $FB = b$;	imaginis $bG = \beta$
Pro lente RR		obiecti $GC = c$;	imaginis $cH = \gamma$
Pro lente SS		obiecti $HD = d$;	imaginis $dI = \delta$
Pro lente TT		obiecti $IE = e$;	imaginis $eK = \varepsilon$

etc.

Porro autem sint numeri arbitrarii vnitate maiores cuiusque lentis figuram determinantes, λ pro lente PP, λ' pro QQ, λ'' pro RR, λ''' pro SS, λ'''' pro TT, etc. ita vt ponendo breuitatis gratia

$$\rho = 0,190781, \sigma = 1,627401, \tau = 0,905133$$

Pro lente PP radius faciei $\begin{cases} \text{anter.} = \dfrac{a\alpha}{\rho\alpha + \sigma a \pm \tau(a+\alpha)\sqrt{(\lambda - 1)}} \\ \text{poster.} = \dfrac{a\alpha}{\rho a + \sigma\alpha \mp \tau(a+\alpha)\sqrt{(\lambda - 1)}} \end{cases}$

Pro lente QQ radius faciei $\begin{cases} \text{anter.} = \dfrac{b\beta}{\rho\beta + \sigma b \pm \tau(b+\beta)\sqrt{(\lambda' - 1)}} \\ \text{poster.} = \dfrac{b\beta}{\rho b + \sigma\beta \mp \tau(b+\beta)\sqrt{(\lambda' - 1)}} \end{cases}$

Pro lente RR radius faciei $\begin{cases} \text{anter.} = \dfrac{c\gamma}{\rho\gamma + \sigma c \pm \tau(c+\gamma)\sqrt{(\lambda'' - 1)}} \\ \text{poster.} = \dfrac{c\gamma}{\rho c + \sigma\gamma \mp \tau(c+\gamma)\sqrt{(\lambda'' - 1)}} \end{cases}$

Pro lente SS radius faciei $\begin{cases} \text{anter.} = \dfrac{d\delta}{\rho\delta + \sigma d \pm \tau(d+\delta)\sqrt{(\lambda''' - 1)}} \\ \text{poster.} = \dfrac{d\delta}{\rho d + \sigma\delta \mp \tau(d+\delta)\sqrt{(\lambda''' - 1)}} \end{cases}$

Pro lente TT radius faciei $\begin{cases} \text{anter.} = \dfrac{e\varepsilon}{\rho\varepsilon + \sigma e \pm \tau(e+\varepsilon)\sqrt{(\lambda'''' - 1)}} \\ \text{poster.} = \dfrac{e\varepsilon}{\rho e + \sigma\varepsilon \mp \tau(e+\varepsilon)\sqrt{(\lambda'''' - 1)}} \end{cases}$

etc.

Deinde

Deinde si quaelibet lens cum binis suis distantiis determinatricibus seorsim consideretur, eiusque aperturae semidiameter foret $= x$, posito $\mu = 0,938191$ et $\nu = 0,232692$ esset spatium diffusionis

$$\text{Lentis } PP = \mu\alpha\alpha xx\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\lambda\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2+\frac{\nu}{a\alpha}\right)$$

$$\text{Lentis } QQ = \mu\beta\beta xx\left(\frac{1}{b}+\frac{1}{\beta}\right)\left(\lambda'\left(\frac{1}{b}+\frac{1}{\beta}\right)^2+\frac{\nu}{b\beta}\right)$$

$$\text{Lentis } RR = \mu\gamma\gamma xx\left(\frac{1}{c}+\frac{1}{\gamma}\right)\left(\lambda''\left(\frac{1}{c}+\frac{1}{\gamma}\right)^2+\frac{\nu}{c\gamma}\right)$$

$$\text{Lentis } SS = \mu\delta\delta xx\left(\frac{1}{d}+\frac{1}{\delta}\right)\left(\lambda'''\left(\frac{1}{d}+\frac{1}{\delta}\right)^2+\frac{\nu}{d\delta}\right)$$

$$\text{Lentis } TT = \mu\varepsilon\varepsilon xx\left(\frac{1}{e}+\frac{1}{\varepsilon}\right)\left(\lambda''''\left(\frac{1}{e}+\frac{1}{\varepsilon}\right)^2+\frac{\nu}{e\varepsilon}\right)$$

etc.

His constitutis pro magnitudine singularum imaginum habebimus

Pro vna lente: $F\zeta = \frac{\alpha}{a}z$ situ inuerso

Pro duabus lentibus $G\eta = \frac{\alpha\beta}{ab}z$ situ erecto

Pro tribus lentibus $H\theta = \frac{\alpha\beta\gamma}{abc}z$ situ inuerso

Pro quatuor lentibus $I\iota = \frac{\alpha\beta\gamma\delta}{abcd}z$ situ erecto

Pro quinque lentibus $K\varkappa = \frac{\alpha\beta\gamma\delta\varepsilon}{abcde}z$ situ inuerso

etc.

At si semidiameter aperturae primae lentis PP ponatur $= x$, necesse est, vt reliquarum lentium aperturae superent sequentes valores

Semi-

Semidiameter aperturae
Lentis fecundae $QQ > \frac{b}{\alpha} x$
Lentis tertiae $RR > \frac{bc}{\alpha\beta} x$
Lentis quartae $SS > \frac{bcd}{\alpha\beta\gamma} x$
Lentis quintae $TT > \frac{bcde}{\alpha\beta\gamma\delta} x$
etc.

Hinc fpatium diffufionis pro quolibet lentium numero ita fe habebit.

I. Pro vna lente

fpatium diffufionis

$$Ff = \mu \alpha\alpha xx \left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\lambda\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2+\frac{\nu}{a\alpha}\right)$$

radiorum in f concurrentium
inclinatio ad axem $= \frac{x}{\alpha}$

II. Pro duabus lentibus

fpatium diffufionis

$$Gg = \mu \beta\beta xx \begin{cases} +\frac{\alpha\alpha}{bb}\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\lambda\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2+\frac{\nu}{a\alpha}\right) \\ +\frac{bb}{\alpha\alpha}\left(\frac{1}{b}+\frac{1}{\beta}\right)\left(\lambda'\left(\frac{1}{b}+\frac{1}{\beta}\right)^2+\frac{\nu}{b\beta}\right) \end{cases}$$

et radiorum in g concurrentium
inclinatio ad axem $= \frac{bx}{\alpha\beta}$

III. Pro tribus lentibus

fpatium diffufionis

$$Hh = \mu \gamma\gamma xx \begin{cases} +\frac{\alpha\alpha\beta\beta}{bbcc}\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\lambda\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2+\frac{\nu}{\alpha a}\right) \\ +\frac{bb\beta\beta}{\alpha\alpha cc}\left(\frac{1}{b}+\frac{1}{\beta}\right)\left(\lambda'\left(\frac{1}{b}+\frac{1}{\beta}\right)^2+\frac{\nu}{b\beta}\right) \\ +\frac{bbcc}{\alpha\alpha\beta\beta}\left(\frac{1}{c}+\frac{1}{\gamma}\right)\left(\lambda''\left(\frac{1}{c}+\frac{1}{\gamma}\right)^2+\frac{\nu}{c\gamma}\right) \end{cases}$$

et

et radiorum in b concurrentium
inclinatio ad axem $\frac{bcx}{\alpha\beta\gamma}$

IV. Pro quatuor lentibus

ſpatium diffuſionis

$$I\,i = \mu\,\delta\,\delta\,x\,x \begin{cases} +\frac{\alpha\alpha\beta\beta\gamma\gamma}{bbccdd}\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\lambda\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2+\frac{\nu}{a\alpha}\right) \\ +\frac{bb\beta\beta\gamma\gamma}{\alpha\alpha ccdd}\left(\frac{1}{b}+\frac{1}{\beta}\right)\left(\lambda'\left(\frac{1}{b}+\frac{1}{\beta}\right)^2+\frac{\nu}{b\beta}\right) \\ +\frac{bbcc\gamma\gamma}{\alpha\alpha\beta\beta dd}\left(\frac{1}{c}+\frac{1}{\gamma}\right)\left(\lambda''\left(\frac{1}{c}+\frac{1}{\gamma}\right)^2+\frac{\nu}{c\gamma}\right) \\ +\frac{bbccdd}{\alpha\alpha\beta\beta\gamma\gamma}\left(\frac{1}{d}+\frac{1}{\delta}\right)\left(\lambda'''\left(\frac{1}{d}+\frac{1}{\delta}\right)^2+\frac{\nu}{d\delta}\right) \end{cases}$$

et radiorum in i concurrentium
inclinatio ad axem $= \frac{bcdx}{\alpha\beta\gamma\delta}$

V. Pro quinque lentibus

ſpatium diffuſionis

$$K\,k = \mu\,\varepsilon\,\varepsilon\,x\,x \begin{cases} +\frac{\alpha\alpha\beta\beta\gamma\gamma\delta\delta}{bbccddee}\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\lambda\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2+\frac{\nu}{a\alpha}\right) \\ +\frac{bb\beta\beta\gamma\gamma\delta\delta}{\alpha\alpha ccddee}\left(\frac{1}{b}+\frac{1}{\beta}\right)\left(\lambda'\left(\frac{1}{b}+\frac{1}{\beta}\right)^2+\frac{\nu}{b\beta}\right) \\ +\frac{bbcc\gamma\gamma\delta\delta}{\alpha\alpha\beta\beta ddee}\left(\frac{1}{c}+\frac{1}{\gamma}\right)\left(\lambda''\left(\frac{1}{c}+\frac{1}{\gamma}\right)^2+\frac{\nu}{c\gamma}\right) \\ +\frac{bbccdd\delta\delta}{\alpha\alpha\beta\beta\gamma\gamma ee}\left(\frac{1}{d}+\frac{1}{\delta}\right)\left(\lambda'''\left(\frac{1}{d}+\frac{1}{\delta}\right)^2+\frac{\nu}{d\delta}\right) \\ +\frac{bbccddee}{\alpha\alpha\beta\beta\gamma\gamma\delta\delta}\left(\frac{1}{e}+\frac{1}{\varepsilon}\right)\left(\lambda''''\left(\frac{1}{e}+\frac{1}{\varepsilon}\right)^2+\frac{\nu}{e\varepsilon}\right) \end{cases}$$

et radiorum in k concurrentium
inclinatio ad axem $= \frac{bcdex}{\alpha\beta\gamma\delta\varepsilon}$
neque ergo caſus, quibus plures occurrunt lentes, vlla amplius laborant difficultate.

Si lentes ratione refractionis diſcrepent, ad easque referendae ſint litterae n, n', n'', n''', etc. formu-

 lae

lae in hoc problemate inuentae ſequenti modo facile ad hunc caſum latius patentem adcommodabuntur. Primo ſcilicet in formulis pro radiis facierum inuentis litterae ϱ, σ, et τ tantum ad primam lentem pertinent, earumque loco pro ſecunda lente ſcribi oportet ϱ', σ', et τ'; pro tertia autem ϱ'', σ'' et τ'' et ita porro. Praeterea vero ſpatia diffuſionis hinc aliquam mutationem requirunt, eritque ſpatium diffuſionis.

I. Pro vna lente

$$\alpha\alpha xx\left(\tfrac{1}{a}+\tfrac{1}{\alpha}\right)\mu\left(\lambda\left(\tfrac{1}{a}+\tfrac{1}{\alpha}\right)^2+\tfrac{\nu}{a\alpha}\right).$$

II. Pro duabus lentibus

$$\beta\beta xx\begin{cases}\frac{\mu\alpha\alpha}{bb}\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\lambda\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2+\frac{\nu}{a\alpha}\right)\\ +\frac{\mu' bb}{\alpha\alpha}\left(\frac{1}{b}+\frac{1}{\beta}\right)\left(\lambda'\left(\frac{1}{b}+\frac{1}{\beta}\right)^2+\frac{\nu'}{b\beta}\right).\end{cases}$$

III. Pro tribus lentibus

$$\gamma\gamma xx\begin{cases}+\frac{\mu\alpha\alpha\beta\beta}{bbcc}\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\lambda\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2+\frac{\nu}{a\alpha}\right)\\ +\frac{\mu' bb\beta\beta}{\alpha\alpha cc}\left(\frac{1}{b}+\frac{1}{\beta}\right)\left(\lambda'\left(\frac{1}{b}+\frac{1}{\beta}\right)^2+\frac{\nu'}{b\beta}\right)\\ +\frac{\mu'' bbcc}{\alpha\alpha\beta\beta}\left(\frac{1}{c}+\frac{1}{\gamma}\right)\left(\lambda''\left(\frac{1}{c}+\frac{1}{\gamma}\right)^2+\frac{\nu''}{c\gamma}\right).\end{cases}$$

IV. Pro quatuor lentibus

$$\delta\delta xx\begin{cases}+\frac{\mu\alpha\alpha\beta\beta\gamma\gamma}{bbccdd}\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\lambda\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2+\frac{\nu}{a\alpha}\right)\\ +\frac{\mu' bb\beta\beta\gamma\gamma}{\alpha\alpha ccdd}\left(\frac{1}{b}+\frac{1}{\beta}\right)\left(\lambda'\left(\frac{1}{b}+\frac{1}{\beta}\right)^2+\frac{\nu'}{b\beta}\right)\\ +\frac{\mu'' bbcc\gamma\gamma}{\alpha\alpha\beta\beta dd}\left(\frac{1}{c}+\frac{1}{\gamma}\right)\left(\lambda''\left(\frac{1}{c}+\frac{1}{\gamma}\right)^2+\frac{\nu''}{c\gamma}\right)\\ +\frac{\mu''' bbccdd}{\alpha\alpha\beta\beta\gamma\gamma}\left(\frac{1}{d}+\frac{1}{\delta}\right)\left(\lambda'''\left(\frac{1}{d}+\frac{1}{\delta}\right)^2+\frac{\nu'''}{d\delta}\right).\end{cases}$$

valores autem harum litterarum μ', ν', μ'', ν'' etc. iam ſupra definiuimus §. 77.

Coroll.

Coroll. 1.

92. Si lentis primae PP ponatur radius faciei anterioris $=f$ et posterioris $=g$ erit

$$\frac{1}{f}=\frac{\rho}{a}+\frac{\sigma}{\alpha}\pm\tau\left(\frac{1}{a}+\frac{1}{\alpha}\right)\sqrt{(\lambda-1)}$$

$$\frac{1}{g}=\frac{\rho}{\alpha}+\frac{\sigma}{a}\mp\tau\left(\frac{1}{a}+\frac{1}{\alpha}\right)\sqrt{(\lambda-1)}$$

vnde si detur distantia obiecti $EA=a$, primo inuenitur α ex hac aequatione

$$\frac{1}{f}+\frac{1}{g}=(\rho+\sigma)\left(\frac{1}{a}+\frac{1}{\alpha}\right)=1,818182\left(\frac{1}{a}+\frac{1}{\alpha}\right)=\frac{10}{11}\left(\frac{1}{a}+\frac{1}{\alpha}\right)$$

Inuenta autem distantia α numerus λ reperitur ex hac aequatione.

$$\frac{1}{f}-\frac{1}{g}+(\sigma-\rho)\left(\frac{1}{a}-\frac{1}{\alpha}\right)=2\tau\left(\frac{1}{a}+\frac{1}{\alpha}\right)\sqrt{(\lambda-1)}.$$

Coroll. 2.

93. Deinde si distantia secundae lentis a prima sit $=F$, ob $F=\alpha+b$ habetur $b=F-\alpha$; qua distantia b cognita, si pro lente secunda datus sit radius faciei anterioris $=f'$, et posterioris $=g'$ habebuntur iterum duae aequationes

$$\frac{1}{f'}=\frac{\rho}{b}+\frac{\sigma}{\beta}\pm\tau\left(\frac{1}{b}+\frac{1}{\beta}\right)\sqrt{(\lambda'-1)}$$

$$\frac{1}{g'}=\frac{\rho}{\beta}+\frac{\sigma}{b}\mp\tau\left(\frac{1}{b}+\frac{1}{\beta}\right)\sqrt{(\lambda'-1)}$$

ex quibus cum distantiam β tum numerum λ' definire licet. Similique modo ex forma sequentium lentium earumque distantia reliqua elementa innotescent.

Coroll. 3.

94. Si singulae lentes ad minimum spatium diffusionis fuerint accommodatae, erit $\lambda=1, \lambda'=1, \lambda''=1, \lambda'''=1$ etc. sin autem hae lentes alia forma fuerint praeditae, isti numeri erunt vnitate maiores.

Scholion.

95. Quo plures fuerint lentes eo pluribus constabit membris spatium diffusionis ab iis productum; neque tamen propterea aucto lentium numero spatium diffusionis necessario augetur. Cum enim quantitates α, b, $\mathfrak{b}$, c, γ, d, δ, etc. valores quoque negatiuos recipere queant, dummodo binarum summae $\alpha+b$; $\mathfrak{b}+c$; $\gamma+d$; $\delta+e$ etc. vtpote lentium distantiae maneant positiuae, fieri potest, vt vnum vel aliquot membra fiant negatiua, hincque spatium diffusionis diminuatur, quin etiam interdum prorsus euanescat, quo casu repraesentatio sine dubio erit perfectissima. Verum in instrumentis dioptricis ad visionem instructis veluti Telescopiis ac Microscopiis non tam hoc, quod definiuimus, spatium diffusionis quam confusio in ipsa visione orta spectari debet; quae autem etsi a spatio diffusionis plurimum differt, tamen ex eo definiri potest vti mox explicabimus. Ante autem conueniet lentes compositas seu multiplicatas considerare, cuiusmodi oriuntur, si duae pluresue lentes, quarum crassities tam est parua vt negligi queat, immediate iungantur quo quidem pacto instar lentium simplicium spectari possunt; verum tali coniunctione effici potest, vt spatium diffusionis multo fiat minus quam si lens simplex adhiberetur, atque adeo euanescat valorque numeri λ istiusmodi lenti compositae conveniens vnitate minor fit proditurus vnde maxima commoda ad confusionem diminuendam obtinebuntur.

CAPVT III.

CAPVT III.

DE LENTIBVS COMPOSITIS SEV MVLTIPLICATIS.

Definitio I.

96.

Lens duplicata oritur, si duae lentes super communi axe sibi immediate iungantur.

Crassitiem hic vtriusque lentis tanquam nullam assumo et quia distantia inter lentes nulla ponitur, crassities etiam lentis duplicatae pro nulla haberi poterit.

Coroll. I.

97. Binae ergo lentes PP et QQ sibi ad contactum fere coniunctae lentem duplicatam constituunt; de qua tamen notandum est, eius crassitiem minus tuto negligi posse quam vtriusque lentis simplicis seorsim sumtae. Si enim lentes immediate se in puncto contingerent; phaenomena colorum a *Newtono* obseruata essent metuenda tum vero etiam ostendemus, quomodo ratio distantiae inter binas lentes haberi possit.

Tab. II. Fig. 7.

Coroll. 2.

98. Si lentis anterioris PP diſtantiae determinatrices ſint a et α, lentis poſterioris vero QQ, b et $\mathfrak{G}$, neceſſe eſt vt ſit $\alpha + b = 0$ ſeu $\alpha = -b$. Tum vero obiecti ante lentem ad diſtantiam $AE = a$ poſiti imago principalis repraeſentabitur poſt lentem ad diſtantiam $G = \mathfrak{G}$.

Coroll. 3.

99. Erunt ergo a et $\mathfrak{G}$ quaſi diſtantiae determinatrices lentis duplicatae; ac ſumendo α vel b ad libitum infinita paria lentium pro his diſtantiis exhiberi poſſunt. Cum deinde vtraque lens praeterea numerum indefinitum λ recipiat inſuper infinita varietas locum habet.

Coroll. 4.

100. Quia craſſities pro nihilo reputatur aperturae in ſingulis faciebus eadem eſt ratio; ſcilicet ſi in prima facie ſemidiameter aperturae ſit $= x$, in reliquis quoque faciebus apertura eadem vel ſaltem non minor eſſe debet.

Scholion.

101. Non opus eſt, vt lentes plane ad contactum coniungantur quoniam forte refractionis lex turbari poſſet: hoc autem vel minima interpoſita diſtantia euitabitur id quod ad inſtitutum noſtrum ſufficit, cum etiam craſſities vtriusque non omnino ſit nulla.

Proble-

Problema I.

102. Omnes lentes duplicatas describere, quibus obiectum $E\varepsilon$ in data ante lentem distantia AE propositum post lentem in data distantia bG repraesentetur, simulque diffusionem imaginis Gg pro data lentis apertura definire.

Solutio.

Sit distantia obiecti $AE = a$, imaginis principalis $bG = \mathfrak{b}$, tum vero lentis primae PP distantiae determinatrices a, et α; lentis posterioris vero QQ, b et $\mathfrak{b}$; iam quia distantia lentium est nulla, erit $\alpha + b = 0$, seu $\alpha = -b$; Hinc si obiecti magnitudo sit $E\varepsilon = z$, erit imaginis principalis magnitudo $G\eta = \frac{\mathfrak{b}}{a} z$ pro situ inuerso. Cum porro vtraque lens infinitis modis formari possit, sit λ numerus arbitrarius pro prima PP et λ' pro secunda QQ, quarum ergo constructio ita se habebit:

Pro Lente

PP radius faciei $\begin{cases} \text{anterioris} = \dfrac{a\alpha}{\rho\alpha + \sigma a \pm \tau(a+\alpha)\sqrt{(\lambda-1)}} \\ \text{posterioris} = \dfrac{a\alpha}{\rho a + \sigma\alpha \mp \tau(a+\alpha)\sqrt{(\lambda-1)}} \end{cases}$

QQ radius faciei $\begin{cases} \text{anterioris} = \dfrac{b\mathfrak{b}}{\rho\mathfrak{b} + \sigma b \pm \tau(b+\mathfrak{b})\sqrt{(\lambda'-1)}} \\ \text{posterioris} = \dfrac{b\mathfrak{b}}{\rho b + \sigma\mathfrak{b} \mp \tau(b+\mathfrak{b})\sqrt{(\lambda'-1)}} \end{cases}$

Si ratio refractionis sit diuersa; pro secunda lente scribi debet ρ', σ' et τ', loco ρ, σ et τ.

Pro

Pro ſpatio autem diffuſionis Gg inueniendo, ſit ſemidiameter aperturae lentis duplicatae $= x$, eritque

$$Gg = \mu \mathfrak{E}\mathfrak{E} xx. \begin{cases} +(\frac{1}{a}+\frac{1}{\alpha})(\lambda(\frac{1}{a}+\frac{1}{\alpha})^2+\frac{\nu}{a\alpha}) \\ +(\frac{1}{b}+\frac{1}{\mathfrak{E}})(\lambda'(\frac{1}{b}+\frac{1}{\mathfrak{E}})^2+\frac{\nu}{b\mathfrak{E}}) \end{cases}$$

et radiorum in g concurrentium inclinatio ad axem $= \frac{x}{\mathfrak{E}}$. Si ratio refractionis diſcrepet; in parte ex lente ſecunda orta ſcribatur μ', ν', loco μ, ν.

Coroll. 1.

103. Huius ergo lentis duplicatae diſtantiae determinatrices ſunt a et $\mathfrak{E}$, praeterea vero duo numeri arbitrarii λ et λ', vna cum diſtantia α vel b eius perfectam determinationem conſtituunt, vnde in huiusmodi lentibus multo maior varietas locum habet, quam in lentibus ſimplicibus.

Coroll. 2.

104. Si ex eisdem diſtantiis determinatricibus a et $\mathfrak{E}$ adiungendo numero arbitrario λ° lens ſimplex conſtruatur, ea imaginem eadem magnitudine $G\eta = \frac{\mathfrak{E}}{a} z$ referet; ſed pro eadem apertura, cuius ſemidiameter $= x$ habebitur ſpatium diffuſionis

$$Gg = \mu \mathfrak{E}\mathfrak{E} xx (\frac{1}{a}+\frac{1}{\mathfrak{E}})(\lambda^{\circ}(\frac{1}{a}+\frac{1}{\mathfrak{E}})^2+\frac{\nu}{a\mathfrak{E}})$$

Coroll. 3.

105. Fieri ergo poterit vt lens duplicata modo maiorem modo minorem diffuſionem gignat. Lens autem

autem ſimplex minimam parit diffuſionem, ſi $\lambda^\circ = 1$; ergo tum demum lentes duplicatae ſimplicibus erunt praeferendae, cum adhuc minorem diffuſionem producent.

Scholion.

106. Concipi quidem ſemper poterit lens ſimplex eandem diffuſionem gignens ac lens duplicata, ſi pro numero λ° omnes valores admittamus; quomodocunque enim lens duplicata fuerit comparata, ſi ſpatium diffuſionis inde productum huic ex lente ſimplici nato aequale ſtatuatur, determinatus valor pro numero λ° elicitur: qui ſi fuerit poſitiuus et vnitate maior, realis lens ſimplex aequiualens exhiberi poterit, ſin autem prodeat vnitate minor, vel adeo negatiuus, lens ſimplex inter imaginaria erit referenda. Quando autem ſit $\lambda^\circ > 1$, euidens eſt lentem ſimplicem eundem plane effectum eſſe edituram ac duplicatam ideoque ſemper expediet lente ſimplici potius vti quam duplicata; quodſi vero prodierit $\lambda^\circ < 1$, quo caſu lens ſimplex ſit imaginaria, tum lentes duplicatae effectum praeſtabunt a ſimplicibus non expectandum, qui adeo cum inſigni hoc commodo, quod ſpatium diffuſionis futurum ſit minus, erit coniunctus. Talibus ergo lentibus duplicatis maximo cum ſucceſſu vti poterimus, eoque magis eae ſimplicibus erunt anteponendae, quo minor fuerit valor numeri λ° iis reſpondens.

Problema 2.

107. Data lente duplicata ad binas distantias determinatrices $AE = \alpha$ et $bG = \beta$ relata pro iisdem distantiis definire lentem simplicem, quae pro eadem apertura, eandem diffusionem imaginis producat.

Solutio.

Totum ergo negotium huc redit, vt spatium diffusionis Gg a lente simplici productum (104) aequale ponatur spatio diffusionis a lente duplicata orto, cuius expressio in problemate praecedente (101) est inuenta, indeque valor numeri λ° pro constructione lentis simplicis eliciatur. Quae inuestigatio quo commodius institui possit, ponamus:

$$\frac{1}{a} = \frac{f-1}{\alpha} + \frac{f}{\beta} \text{ eritque } \frac{1}{b} = \frac{-f+1}{\alpha} - \frac{f}{\beta}$$

ita vt loco quantitatis a vel b numerum f introducamus eritque

$$\frac{1}{a} + \frac{1}{\alpha} = f\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) \text{ et } \frac{1}{b} + \frac{1}{\beta} = (1-f)\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)$$

vnde spatium diffusionis a lente duplicata ortum prodit:

$$Gg = \mu\beta\beta xx \begin{cases} +f\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)\left(\lambda ff\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)^2 + \nu\left(\frac{f-1}{\alpha\alpha} + \frac{f}{\alpha\beta}\right)\right) \\ +(1-f)\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)\left(\lambda'(1-f)^2\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)^2 + \nu\left(\frac{1-f}{\alpha\beta} - \frac{f}{\beta\beta}\right)\right) \end{cases}$$

quae expressio reducitur ad hanc formam:

$$Gg = \mu\beta\beta xx\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)\left(\left(\lambda f^3 + \lambda'(1-f)^3\right)\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)^2 + \nu\left(\frac{f(f-1)}{\alpha\alpha} + \frac{1-2f+2ff}{\alpha\beta} + \frac{f(f-1)}{\beta\beta}\right)\right)$$

Verum postremum membrum $\frac{f(f-1)}{\alpha\alpha} + \frac{1-2f+2ff}{\alpha\beta} + \frac{f(f-1)}{\beta\beta}$ mutatur

mutatur in $f(f-1)(\frac{1}{a}+\frac{1}{b})^2+\frac{1}{ab}$, sicque habebimus pro lente duplicata:

$$Gg = \mu bbxx(\tfrac{1}{a}+\tfrac{1}{b})((\lambda f^3+\lambda'(1-f)^3-\nu f(1-f))(\tfrac{1}{a}+\tfrac{1}{b})^2+\tfrac{\nu}{ab})$$

quae forma iam facillime cum spatio diffusionis lentis simplicis comparatur indeque manifesto colligitur:

$$\lambda^{\circ} = \lambda f^3+\lambda'(1-f)^3-\nu f(1-f)$$

Cum autem loco quantitatum a et b numerum f introduxerimus constructio lentis duplicatae ita se habebit

Pro lente radius faciei

prima PP $\begin{cases} \text{anterioris} = \dfrac{ab}{(\rho-\sigma(1-f))b+\sigma fa \pm \tau f(a+b)\sqrt{(\lambda-1)}} \\ \text{posterioris} = \dfrac{ab}{(\sigma-\rho(1-f))a+\rho fb \mp \tau f(a+b)\sqrt{(\lambda-1)}} \end{cases}$

secunda QQ $\begin{cases} \text{anterioris} = \dfrac{ab}{(\sigma-\rho f)a+\rho(1-f)b \pm \tau(1-f)(a+b)\sqrt{(\lambda'-1)}} \\ \text{posterioris} = \dfrac{ab}{(\rho-\sigma f)b+\sigma(1-f)a \mp \tau(1-f)(a+b)\sqrt{(\lambda'-1)}} \end{cases}$

Tum vero inuento numero λ° constructio lentis simplicis aequiualentis erit

Radius faciei $\begin{cases} \text{anterioris} = \dfrac{ab}{\rho b+\sigma a \pm \tau(a+b)\sqrt{(\lambda^{\circ}-1)}} \\ \text{posterioris} = \dfrac{ab}{\rho a+\sigma b \mp \tau(a+b)\sqrt{(\lambda^{\circ}-1)}} \end{cases}$

Coroll. 1.

108. Quoties ergo fuerit

$$\lambda f^3+\lambda'(1-f)^3-\nu f(1-f) > 1$$

semper lens simplex parari potest duplicatae aequiualens

lens iisque ergo casibus praestabit lente simplici vel potius quam lente duplicata.

Coroll. 2.

109. Verum si fuerit

$$\lambda f^3 + \lambda'(1-f)^3 - \nu f(1-f) < 1$$

ob $\lambda^\circ < 1$, constructio lentis simplicis fit impossibilis, ac lens duplicata minorem pariet diffusionem quam per vllam lentem simplicem obtineri potest.

Coroll. 3

110. Si esset $f=0$, prodiret $\lambda^\circ=\lambda'$, et in lente duplicata anteriori nullam refractionem produceret ob facies parallelas, resque eodem rediret, ac si posterior sola adesset. Sin autem sumatur $f=1$, fit $\lambda^\circ=\lambda$, et lens posterior superflua; vtroque ergo casu nullum lucrum impetratur.

Coroll. 4.

111. Cum autem numeri λ et λ' vnitate nequeant esse minores, sitque $\nu=0,232692$, patet pro f nullum valorem inter limites 0 et 1 assumi posse, vnde fiat $\lambda^\circ=0$. At si f extra hos limites capiatur, vtique pro λ et λ' eiusmodi numeri vnitate maiores assignari poterunt, vt fiat $\lambda^\circ=0$.

Scholion 1.

112. Ratione ergo numeri f tres casus lentium duplicatarum considerari conueniet, prout vel f intra limi-

limites o et 1 continetur, vel fuerit $f > 1$ vel $f < o$. Primo casu euenire non potest, vt fiat $\lambda^{\circ} = o$; sed plurimum intererit eam determinasse lentem duplicatam, pro qua λ° minimum obtineat valorem qui quo magis infra vnitatem cadat, eo perfectior lens erit censenda, maioreque iure simplicibus anteferenda. Binis reliquis vero casibus $f > 1$ et $f < o$ eiusmodi adeo lentes duplicatae parari poterunt, quae praebeant $\lambda^{\circ} = o$, quae ergo pro perfectissimis essent habendae. Verum hic quoque ad praxin est respiciendum, quae cum semper a praeceptis Theoriae aberrare soleat, euenire potest, vt leui errore commisso numerus λ° non solum non euanescat, sed adeo vnitatem excedat quo casu vtique expediret lente vti simplici.

Scholion 2.

113. Cum tanti sit momenti rationem aberrationis a qua praxis vix liberari potest habere, eam etiam in lentibus simplicibus perpendi conueniet. Postulauimus autem pro datis distantiis determinatricibus lentem construi posse in qua numerus λ datum obtineat valorem dum ne sit vnitate minor; hic igitur obseruari oportet, quo maior fuerit λ, eo difficilius fore errorem euitare; si enim in constructione leuissimus error committatur alius eo magis diuersus valor pro λ orietur, quo maior fuerit λ. Verum e contrario cum vnitas sit minimus valor, quem λ recipere potest ex natura minimi liquet etiamsi in praxi a praescripta regula notabiliter recedatur tamen inde vix

 sensi-

ſenſibile diſcrimen in valorem λ eſſe redundaturum. Ex quo concludimus feliciſſimo cum ſucceſſu eiusmodi lentes ſimplices parari poſſe, pro quibus futurum ſit $\lambda = 1$, neque hic errores praxeos, niſi fuerint enormes, admodum eſſe pertimeſcendos. Deinde quo minus numerus λ vnitatem ſuperare debeat, eo certiores eſſe poterimus de ſucceſſu, ſed non eo gradu, quo caſu, $\lambda = 1$; at ſi opus ſit eiusmodi lente, pro qua valor ipſius λ debeat eſſe numerus ſatis magnus, difficillime per praxin ſatisfiet ac fortaſſe ingentem lentium numerum parare oportebit, antequam vna obtineatur ſcopo ſatisfaciens. Quamobrem ſi praxi conſulere velimus, vix alias lentes exigere debemus, niſi pro quibus numerus λ vel ſit vnitas ipſa, vel parumper maior. Sin autem ad inſigne aliquod commodum aliae lentes requirantur, labori non erit parcendum, cum fortaſſe non niſi poſt plurimos conatus irritos voti tandem compotes reddi queamus.

Problema 3.

114. Definire eam lentem duplicatam, pro qua, ſi numerus f intra limites 0 et 1 accipiatur numerus λ° minimum adipiscatur valorem.

Solutio.

Poſitis iisdem, quae in praecedentibus problematibus ſunt conſtituta, inuenimus eſſe

$$\lambda^\circ = \lambda f^3 + \lambda'(1-f)^3 - \nu f(1-f)$$

vbi cum f intra limites 0 et 1 aſſumi debeat, ambo

termini

termini λf^3 et $\lambda'(1-f)^3$ erunt positiui. Quare vt λ° omnium minimum valorem nanciscatur, necesse est vtrique numero λ et λ' minimum valorem cuius est capax tribui.

Sit ergo $\lambda = 1$ et $\lambda' = 1$, ac habebimus

$$\lambda^{\circ} = 1 - 3f + 3ff - \nu f + \nu ff = 1 - (3+\nu) f(1-f)$$

quae expressio vt minima reddatur, oportet fieri $f(1-f)$ maximum, id quod fit sumendo $f = \frac{1}{2}$; hincque oritur

$$\lambda^{\circ} = 1 - \tfrac{1}{4}(3+\nu) = \frac{1-\nu}{4} = 0,191827$$

Quare constructio huius lentis duplicatae ita se habebit:

Pro lente P P radius faciei $\begin{cases} \text{anterioris} = \dfrac{2\alpha\beta}{(2\varrho-\sigma)\beta+\sigma\alpha} \\ \text{posterioris} = \dfrac{2\alpha\beta}{(2\sigma-\varrho)\beta+\varrho\alpha} \end{cases}$

Pro lente QQ radius faciei $\begin{cases} \text{anterioris} = \dfrac{2\alpha\beta}{(2\sigma-\varrho)\alpha+\varrho\beta} \\ \text{posterioris} = \dfrac{2\alpha\beta}{(2\varrho-\sigma)\alpha+\sigma\beta} \end{cases}$

et si aperturae semidiameter sit $= x$, erit spatium diffusionis

$$Gg = \mu\beta\beta xx \left(\tfrac{1}{\alpha}+\tfrac{1}{\beta}\right)\left(0,191827 \left(\tfrac{1}{\alpha}+\tfrac{1}{\beta}\right)^2 + \tfrac{\nu}{\alpha\beta}\right)$$

Si eiusmodi vitro vtamur, pro quo est $n = 1,60 = \frac{8}{5}$ tum ob $\nu = \frac{4}{11}$, prodiret $\lambda^{\circ} = 0,183333$ ideoque haec vitri species adhuc minorem confusionem pareret.

Coroll.

Coroll. 1.

115. Si pro iisdem diſtantiis determinatricibus a et $\mathfrak{b}$ lens ſimplex minimam diffuſionem pariens conſtruatur quod fit ſumendo

radium faciei $\begin{cases} \text{anterioris} = \frac{a\mathfrak{b}}{\rho\mathfrak{b}+\sigma a} \\ \text{poſterioris} = \frac{a\mathfrak{b}}{\rho a+\sigma\mathfrak{b}} \end{cases}$

ſpatium diffuſionis foret

$$\mu\mathfrak{b}\mathfrak{b}xx\left(\frac{1}{a}+\frac{1}{\mathfrak{b}}\right)\left(\left(\frac{1}{a}+\frac{1}{\mathfrak{b}}\right)^2+\frac{v}{a\mathfrak{b}}\right).$$

Coroll. 2.

116. Apparet ergo a lente duplicata deſcripta multo minorem oriri diffuſionem, quam a lente ſimplici, etiamſi haec iam ad minimam diffuſionem ſit inſtructa: Cum enim ceterae partes ſint pares coefficiens membri $\left(\frac{1}{a}+\frac{1}{\mathfrak{b}}\right)^3$ plus quam quintuplo minor eſt in duplicata quam in ſimplici.

Coroll. 3.

117. Si ponamus $\lambda=1$ et $\lambda'=1$, vel ſaltem $\lambda'=\lambda$, minor valor pro λ° obtineri nequit, quam inuenimus, etiamſi pro f alios valores admittere velimus. Vnde ſi vtraque lens per ſe iam minimam diffuſionem pariat, pro lente duplicata valor ipſius λ° minor quam $0,191827$ fieri nequit.

Scholion.

118. Huiusmodi ergo lentes duplicatae maxime ſunt notatu dignae cum loco ſimplicium adhibitae multo

multo minorem diffusionem pariunt, ex quo in constructione Telescopiorum et Microscopiorum earum amplissimus erit vsus. Neque vero hac insigni proprietate sunt praeditae, sed etiam earum constructio in praxi minimis difficultatibus est obnoxia: propterea quod etsi a praescriptis regulis parumper aberretur, effectus tamen inde vix vllam mutationem patiatur. Siue enim in constructione vtriusque seorsim leuis error committatur, valores numerorum λ et λ' vnitatem haud sensibiliter excedent, siue in quantitate f valor iustus $f = \frac{1}{2}$ non exacte obseruetur, error vix sentietur, quoniam hi numeri ex natura minimi sunt eruti. Quomodocunque autem a regulis praescriptis aberretur, valor ipsius λ° inde paulisper maior prodibit. Veluti si eiusmodi errores committantur, vt sit $\lambda = 1 + \frac{1}{10}$; $\lambda' = 1 + \frac{1}{10}$ et $f = \frac{1}{2} \pm \frac{1}{50}$, prodibit $\lambda^{\circ} = 0,191827 + 0,02558$ seu $\lambda^{\circ} = 0,2174$, ita vt discrimen partem tantum quadragesimam vnitatis conficiat. Facile autem intelligitur, dummodo λ° prodeat minus quam $\frac{1}{4}$, quod nunquam non, facile praestari posse videtur, ab his lentibus duplicatis insignem vtilitatem expectandam esse. Etsi ergo eiusmodi lentes duplicatae confici possunt, pro quibus numerus λ° plane euanescat, ob harum lubricam constructionem illae istis anteferendae videntur, vt mox clarius patebit.

Problema 4.

119. Pro datis distantiis determinatricibus $AE = a$ et $bG = \mathfrak{b}$ eas lentes duplicatas inuenire, in quibus sit $\lambda^{\circ} = 0$.

Solutio.

Cum fieri nequeat $\lambda^{\circ}=0$, nisi numerus f extra limites 0 et 1 accipiatur, simulque numeri λ et λ' fuerint inaequales, ita vt alterutra saltem lens non debeat seorsim minimam diffusionem parere; ponamus esse vel $f > 1$ vel $f < 0$. Sit ergo primo $f=1+\xi$, et cum sit

$$\lambda^{\circ}=\lambda(1+\xi)^3-\lambda'\xi^3+\nu\xi(1+\xi)$$

vt fiat $\lambda^{\circ}=0$ oportet esse:

$$\lambda'=\lambda(1+\tfrac{1}{\xi})^3+\frac{\nu(1+\xi)}{\xi^2}$$

vnde λ' necessario vnitatem superabit, cuius valor ne prodeat nimis magnus, sumi conueniet $\lambda=1$, ita vt sit

$$\lambda'=(1+\tfrac{1}{\xi})^3+\frac{0,232602(1+\xi)}{\xi^2} \text{ et } \lambda=1.$$

Quicunque ergo valor ipsi ξ tribuatur, lens duplicata habetur, pro qua sit $\lambda^{\circ}=0$, ac propterea spatium diffusionis $=\mu bbxx(\frac{1}{a}+\frac{1}{b})\frac{\nu}{ab}$. Videamus nonnullos casus speciales.

$$f=\tfrac{3}{2};\ \xi=\tfrac{1}{2};\ \lambda=1 \text{ et } \lambda'=28,396152$$

$$f=2;\ \xi=1;\ \lambda=1 \text{ et } \lambda'=8,465384$$

$$f=3;\ \xi=2;\ \lambda=1 \text{ et } \lambda'=3,549519$$

$$f=4;\ \xi=3;\ \lambda=1 \text{ et } \lambda'=2,473789$$

$$f=5;\ \xi=4;\ \lambda=1 \text{ et } \lambda'=2,025841$$

$$f=6;\ \xi=5;\ \lambda=1 \text{ et } \lambda'=1,783846$$

etc.

Pro

Pro altero casu sit $f = -\xi$, ideoque

$$\lambda^{\circ} = -\lambda\xi^{3} + \lambda'(1+\xi)^{3} + \nu\xi(1+\xi)$$

vnde facto $\lambda^{\circ} = 0$ prodit

$$\lambda = \lambda'(1+\tfrac{1}{\xi})^{3} + \frac{\nu(1+\xi)}{\xi^{2}}.$$

Statui ergo conueniet $\lambda' = 1$, ac pro λ notentur casus sequentes.

$f = -\tfrac{1}{2}$; $\xi = \tfrac{1}{2}$; $\lambda = 28,396152$ et $\lambda' = 1$

$f = -1$; $\xi = 1$; $\lambda = 8,465384$ et $\lambda' = 1$

$f = -2$; $\xi = 2$; $\lambda = 3,549519$ et $\lambda' = 1$

$f = -3$; $\xi = 3$; $\lambda = 2,473789$ et $\lambda' = 1$

$f = -4$; $\xi = 4$; $\lambda = 2,025841$ et $\lambda' = 1$

$f = -5$; $\xi = 5$; $\lambda = 1,783846$ et $\lambda' = 1$

sicque patet infinitis modis huiusmodi lentes duplicatas parari posse, pro quibus sit $\lambda^{\circ} = 0$, et spatium diffusionis

$$Gg = \mu \mathfrak{B}\mathfrak{B} xx (\tfrac{1}{a} + \tfrac{1}{b}) \frac{\nu}{ab}.$$

Scholion I.

120. Si huiusmodi lentes accuratissime parari possent, nullum est dubium, quin praecedentibus essent anteferendae propterea quod diffusio iis adhuc magis diminuitur. Verum dolendum est, quod minimus error in earum constructione commissus omnem

fere vſum deſtruat. Quo hoc facilius diiudicare queamus, examinemus eum caſum, quo eſt

$$f=5,\ \lambda=1 \text{ et } \lambda'=2,025841$$

hincque

$$\lambda^{\circ}=\lambda f^{3}-\lambda'(f-1)^{3}+\nu f(f-1)=0.$$

Ponamus autem in conſtructione errorem eſſe commiſſum vt reuera non ſit $f=5$ ſed $f=5\frac{1}{10}$, dum numeri λ et λ' ſuos iuſtos valores obtineant: ob hunc autem vix vitandum errorem non ſit $\lambda^{\circ}=0$, ſed adeo $\lambda^{\circ}=-2,011$; ſicque haec lens duplicata ſimplicibus longe eſt poſtponenda, ſimili modo ſi f eſſet $=5$, ſed vel λ vel λ' tantillum a praeſcripto valore aberraret, enorme ſtatim diſcrimen in valorem ipſius λ° redundaret. Minus quidem error metuendus videtur in ea ſpecie, qua $\lambda=1$, $\lambda'=28,396152$ et $f=\frac{1}{2}$; ſed praeterquam, quod lens ſimplex poſterior difficillime parari queat, pro qua λ' praeciſe valorem aſſignatum conſequatur, talis lens ad vſum dioptricum plane eſt inepta, ob ingentem alterius faciei curuaturam. Quae cum ita ſint, quia tam exiguus error in paratione huiusmodi lentium commiſſus facit, vt λ° adeo ſupra vnitatem excreſcat, vix ſperare poterimus, vt vnquam talis lens duplicata perficiatur, pro qua λ° vsque ad $\frac{1}{7}$ diminuatur. In lentibus autem praecedentis generis ſucceſſus vix fallere poterit, niſi in praxi enormiter a praeſcripta regula aberretur: ex quo his ſolis lentibus duplicatis cum fructu vti licebit, dum

contra

contra eae, quas in praesente problemate descripsimus, penitus profligandae videntur.

Scholion 2.

121. Pro datis ergo binis distantiis determinatricibus a et $\mathfrak{b}$ semper eiusmodi lens duplicata parari potest, ex qua nascatur spatium diffusionis

$$Gg = \mu \mathfrak{b}\mathfrak{b} xx\left(\frac{1}{a}+\frac{1}{\mathfrak{b}}\right)\left(\lambda^{\circ}\left(\frac{1}{a}+\frac{1}{\mathfrak{b}}\right)^2+\frac{\nu}{a\mathfrak{b}}\right)$$

ita vt λ° numerum quemcunque denotare possit. Ad hoc enim satisfieri oportet huic aequationi:

$$\lambda^{\circ} = \lambda f^3 + \lambda'(1-f)^3 - \nu f(1-f)$$

id quod semper fieri potest, cum f plane ab arbitrio nostro pendeat, et numeri λ et λ' tantum non vnitate minores accipi debeant. Definitis autem his tribus numeris λ, λ' et f ita, vt λ° datum valorem obtineat, binae lentes simplices, ex quibus duplicata est componenda, secundum formulas §. 107 datas confici debent. Vbi quidem tenenda sunt ea, quae modo obseruauimus, in praxi eas lentes facillime obtineri, quando numeri λ et λ' vnitatem parum superant, f vero propemodum $\frac{1}{2}$ denotat, cum contra quo magis hi numeri ab istis terminis recedant, eo maius fit periculum ne effectus enormiter fallat. Ceterum cum diffusionem a lente duplicata oriundam ad eandem formam reduxerimus, qua diffusio lentis simplicis exprimitur, inde id commodi consequimur, vt simili modo diffusionem a lentibus magis multiplicatis ortum definire valeamus.

 Sup-

Supplementum I.

De lentibus duplicatis.

Si pro lente anteriori ratio refractionis fit $n : 1$, pro lente posteriori vero alia ratio $n' : 1$ locum habeat, problemata hic tractata sequenti modo resolui poterunt

Pro Problemate I.

Si pro numeris ϱ, σ, τ, qui ex n oriuntur, quaerantur simili modo ex n' valores ϱ', σ', et τ' erunt

Pro Lente

$$\text{P P radius faciei} \begin{cases} \text{anter.} = \dfrac{a\alpha}{\varrho\alpha + \sigma a \pm \tau(a+\alpha)\surd(\lambda - 1)} \\ \text{poster.} = \dfrac{a\alpha}{\varrho a + \sigma\alpha \mp \tau(a+\alpha)\surd(\lambda - 1)} \end{cases}$$

$$\text{QQ radius faciei} \begin{cases} \text{anter.} = \dfrac{b\mathfrak{b}}{\varrho'\mathfrak{b} + \sigma' b \pm \tau'(b+\mathfrak{b})\surd(\lambda' - 1)} \\ \text{poster.} = \dfrac{b\mathfrak{b}}{\varrho' b + \sigma'\mathfrak{b} \mp \tau'(b+\mathfrak{b})\surd(\lambda' - 1)} \end{cases}$$

et definitis simili modo valoribus μ', ν' ex ratione $n' : 1$ reperietur spatium diffusionis

$$\mathfrak{B}\,\mathfrak{b}\,x\,x \begin{cases} +\mu\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\lambda\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2+\frac{\nu}{a\alpha}\right) \\ +\mu'\left(\frac{1}{b}+\frac{1}{\mathfrak{b}}\right)\left(\lambda'\left(\frac{1}{b}+\frac{1}{\mathfrak{b}}\right)^2+\frac{\nu'}{b\mathfrak{b}}\right) \end{cases}$$

reliqua manent vt in problemate.

Pro Problemate II.

Quoniam hic duae vitri species occurrunt, ponamus lentem simplicem quaesitam ex alio quocunque vitri genere parari, cuius ratio refractionis fit $n^{\circ} : 1$, vnde prodeant numeri μ° et ν°; superfluum autem foret, numeros ϱ°, σ° et τ° computari, quoniam radii

dii facierum fiunt imaginarii, ita, vt eos exprimere non fit opus, et quoniam pro hac lente fimplici aequiualente numerus λ° eft introductus, erit huius lentis fpatium diffufionis

$$\mathfrak{E}\mathfrak{E}xx.\mu^\circ(\tfrac{1}{a}+\tfrac{1}{\mathfrak{E}})(\lambda^\circ(\tfrac{1}{a}+\tfrac{1}{\mathfrak{E}})^2+\tfrac{\nu^\circ}{a\mathfrak{E}})$$

quod vt cum fpatio diffufionis lentis duplicatae aequale fiat; ponatur, vti in problemate eft factum,

$$\tfrac{1}{\alpha}=\tfrac{f-1}{a}+\tfrac{f}{\mathfrak{E}} \text{ et } \tfrac{1}{b}=\tfrac{1-f}{a}-\tfrac{f}{\mathfrak{E}}$$

vt prodeat

$$\tfrac{1}{a}+\tfrac{1}{\alpha}=f(\tfrac{1}{a}+\tfrac{1}{\mathfrak{E}}) \text{ et } \tfrac{1}{b}+\tfrac{1}{\mathfrak{E}}=(1-f)(\tfrac{1}{a}+\tfrac{1}{\mathfrak{E}})$$

vnde peruenietur ad hanc aequationem

$$\mu f(\lambda f^2(\tfrac{1}{a}+\tfrac{1}{\mathfrak{E}})^2+\nu(\tfrac{f-1}{a^2}+\tfrac{f}{a\mathfrak{E}}))$$
$$+\mu'(1-f)(\lambda'.(1-f)^2(\tfrac{1}{a}+\tfrac{1}{\mathfrak{E}})^2+\nu'(\tfrac{1-f}{a\mathfrak{E}}-\tfrac{f}{\mathfrak{E}^2}))$$
$$=\mu^\circ(\lambda^\circ(\tfrac{1}{a}+\tfrac{1}{\mathfrak{E}})^2+\tfrac{\nu^\circ}{a\mathfrak{E}})$$

quae ita repraefentari poteft

$$(\tfrac{1}{a}+\tfrac{1}{\mathfrak{E}})^2(\mu\lambda f^3+\mu'\lambda'(1-f)^3)$$
$$+(\tfrac{f-1}{a}+\tfrac{f}{\mathfrak{E}})(\tfrac{\mu\nu f}{a}-\tfrac{\mu'\nu'(1-f)}{\mathfrak{E}})$$
$$=\mu^\circ\lambda^\circ(\tfrac{1}{a}+\tfrac{1}{\mathfrak{E}})^2+\tfrac{\nu^\circ\mu^\circ}{a\mathfrak{E}})$$

Vnde

$$\lambda^\circ=\tfrac{\mu\lambda f^3+\mu'\lambda'(1-f)^3}{\mu^\circ}+$$
$$\tfrac{aa\mathfrak{E}\mathfrak{E}}{\mu(a+\mathfrak{E})^2}((\tfrac{f-1}{a}+\tfrac{f}{\mathfrak{E}})(\tfrac{\mu\nu f}{a}-\tfrac{\mu'\nu'(1-f)}{\mathfrak{E}})-\tfrac{\nu^\circ\mu^\circ}{a\mathfrak{E}})$$

Deni-

Denique ſi ex ratione refractionis $n' : 1$ computentur numeri ρ', σ' et τ', radii facierum lentis duplicatae erunt

Pro Lente

P P radius faciei $\begin{cases} \text{anterioris} = \frac{\alpha\beta}{(\rho-\sigma(1-f))\beta+\sigma f\alpha+\tau f(\alpha+\beta)\sqrt{(\lambda-1)}} \\ \text{poſterioris} = \frac{\alpha\beta}{(\sigma-\rho(1-f))\beta+\rho f\alpha+\tau f(\alpha+\beta)\sqrt{(\lambda-1)}} \end{cases}$

QQ radius faciei $\begin{cases} \text{anter.} = \frac{\alpha\beta}{(\sigma'-\rho' f)\alpha+\rho'(1-f)\beta\pm\tau'(1-f)(\alpha+\beta)\sqrt{(\lambda'-1)}} \\ \text{poſter.} = \frac{\alpha\beta}{(\rho'-\sigma' f)\alpha+\sigma'(1-f)\beta+\tau'(1-f)(\alpha+\beta)\sqrt{\lambda'-1}} \end{cases}$

Ad Problema III.

Hoc problema non ſolum pro diuerſa refractione n et n' hic generalius pertractabo, ſed etiam rationem diſtantiae inter binas lentes habebo. Primum igitur vtramque lentem ad diſtantias determinatrices lentis duplicatae, α et β, reuocabo, ponendo

$$\frac{1}{a}+\frac{1}{\alpha}=f\left(\frac{1}{\alpha}+\frac{1}{\beta}\right) \text{ et } \frac{1}{b}+\frac{1}{\beta}=g\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)$$

vt ſit

$$\frac{1}{a}=\frac{f-1}{\alpha}+\frac{f}{\beta} \text{ et } \frac{1}{b}=\frac{g}{\alpha}+\frac{g-1}{\beta}$$

hincque

$$a=\frac{\alpha\beta}{f\alpha+(f-1)\beta} \text{ et } b=\frac{\alpha\beta}{(g-1)\alpha+g\beta}$$

ideoque diſtantia lentium

$$a+b=\frac{\alpha\beta(f+g-1)(\alpha+\beta)}{(f\alpha+(f-1)\beta)((g-1)\alpha+g\beta)}$$

quae ſi deberet eſſe $=0$, capi oporteret $g=1-f$ ſed ſi diſtantiam aliquam inter lentes admittamus, ſta-

tuamus

tuamus $f+g-1=\omega$, denotante ω fractionem quandam minimam, siue positiuam siue negatiuam vt distantia lentium prodeat positiua et valde parua. Cum hinc igitur sit $g=1+\omega-f$; erit lentium distantia

$$a+b=\frac{a\mathfrak{b}(a+\mathfrak{b})\omega}{(fa+(f-1)\mathfrak{b})((\omega-f)a+(1+\omega-f)\mathfrak{b})}$$

Spatium autem diffusionis nunc ita exprimetur:

$$\mathfrak{b}\mathfrak{b}xx.\left(\tfrac{1}{a}+\tfrac{1}{\mathfrak{b}}\right)\begin{cases}+\lambda\mu f^3\left(\tfrac{1}{a}+\tfrac{1}{\mathfrak{b}}\right)^2+\lambda'\mu' g^3\left(\tfrac{1}{a}+\tfrac{1}{\mathfrak{b}}\right)^2\\ +\tfrac{\mu\nu f}{a}\left(\tfrac{f-1}{a}+\tfrac{f}{\mathfrak{b}}\right)\\ +\tfrac{\mu'\nu' g}{\mathfrak{b}}\left(\tfrac{g}{a}+\tfrac{g-1}{\mathfrak{b}}\right)\end{cases}$$

quae cum in hoc problemate ita tractari debeat, vt tam f, quam g positiue sumantur, et haec formula minima reddatur; euidens est, litteris λ et λ' minimos valores tribui debere, scilicet $\lambda=1$ et $\lambda'=1$; deinde pro hoc casu minimi f et g conuenienter definiantur, vbi quia $f+g-1=\omega$ ideoque constans in differentiatione, habebimus $dg=-df$; vnde obtinebimus hanc aequationem

$$\left(3\mu f^2-3\mu' g^2\right)\left(\tfrac{1}{a}+\tfrac{1}{\mathfrak{b}}\right)^2+\tfrac{\mu\nu}{a}\left(\tfrac{2f-1}{a}+\tfrac{2f}{\mathfrak{b}}\right)$$
$$-\tfrac{\mu'\nu'}{\mathfrak{b}}\left(\tfrac{2g}{a}+\tfrac{2g-1}{\mathfrak{b}}\right)=0$$

cui proxime satisfit, ponendo $f=g=\frac{1+\omega}{2}$ quibus valoribus substitutis spatium diffusionis ipsum minimum, erit proxime

$$\frac{\mathfrak{b}^2x^2(1+\omega)}{8}\left(\tfrac{1}{a}+\tfrac{1}{\mathfrak{b}}\right)\begin{cases}\left(\tfrac{1}{a}+\tfrac{1}{\mathfrak{b}}\right)^2(1+\omega)^2(\mu+\mu')\\ +\tfrac{2\mu\nu}{a}\left(\tfrac{\omega-1}{a}+\tfrac{1+\omega}{\mathfrak{b}}\right)\\ +\tfrac{2\mu'\nu'}{\mathfrak{b}}\left(\tfrac{1+\omega}{a}+\tfrac{\omega-1}{\mathfrak{b}}\right)\end{cases}$$

 Tum

Tum vero distantia lentium erit

$$a+b=\frac{4ab(a+b)\omega}{((1+\omega)a+(\omega-1)b)((\omega-1)a+(1+\omega)b)}$$
$$=\frac{4ab(a+b)\omega}{-(1-\omega^2)(a^2+b^2)+2(1+\omega^2)ab}$$

cuius denominator cum sit negatiuus ob ω minimum; necesse est, fractionem ω sumi debere negatiuam; hincque adeo spatium diffusionis minus reddetur.

Coroll.

Si ergo distantia obiecti a fuerit infinita seu $a=\infty$, habebitur primo distantia lentium $=\frac{-2b\omega}{1-\omega^2}$; et secundo spatium diffusionis

$$\frac{x^2(1+\omega)}{4b}\begin{cases}(1+\omega)^2(\mu+\mu') \\ -2\mu'\nu'(1-\omega)\end{cases}$$

quod cum ω debeat esse negatiuum, non mediocriter minus erit, quam si distantia lentium esset nulla.

Ad Problema IV.

In hoc problemate etiam distantiam lentium non neglegamus; factaque reductione, vt ante, statuamus spatium diffusionis plane euanescens; id quod fieri nequit, nisi altera litterarum f et g sit negatiua, quod cum etiam fiat, quando confusio a diuersa radiorum refrangibilitate oriunda ad nihilum redigi debet, vt infra videbimus; hic casus multo magis euolutionem meretur. Ponatur igitur $g=-\zeta f$, vbi ζ ex illa

illa conditione determinatur, vt primo pro diſtantia lentium ſit

$$\alpha+b=\frac{-a\mathfrak{b}(a+\mathfrak{b})(f-\zeta f-1)^2}{(fa+(f-1)\mathfrak{b})((\zeta f+1)a+\zeta f\mathfrak{b})}$$

et poſito $f+g-1=\omega$, prodeat $f-\zeta f=1+\omega$ ideoque

$$f=\frac{1+\omega}{1-\zeta} \text{ et } g=\frac{-\zeta(1+\omega)}{1-\zeta} \text{ ſicque}$$

$$\alpha+b=\frac{-a\mathfrak{b}(a+\mathfrak{b})\omega(1-\zeta)^2}{((1+\omega)a+(\omega+\zeta)\mathfrak{b})((1+\omega\zeta)a+\zeta(1+\omega)\mathfrak{b})}$$

ac ſi in denominatore ω reiiciatur, erit

$$\alpha+b=\frac{-a\mathfrak{b}(a+\mathfrak{b})(1-\zeta)^2\omega}{(a+\zeta\mathfrak{b})}$$

ſicque patet ω negatiue capi debere.
Poſito autem $g=-\zeta f$, fit ſpatium diffuſionis

$$\mathfrak{b}^2 x^3 \left(\tfrac{1}{a}+\tfrac{1}{\mathfrak{b}}\right)\begin{cases}(\lambda\mu f^3-\lambda'\mu'\zeta^3 f^3)\left(\tfrac{1}{a}+\tfrac{1}{\mathfrak{b}}\right)^2\\ +\frac{\mu\nu f}{a}\left(\frac{f-1}{a}+\frac{f}{\mathfrak{b}}\right)\\ -\frac{\mu'\nu\zeta f}{\mathfrak{b}}\left(\frac{-\zeta f}{a}-\frac{(\zeta f+1)}{\mathfrak{b}}\right)\end{cases}$$

ad nihilum redigendum; vnde ſequitur fore

$$\lambda'=\frac{\lambda\mu}{\mu'\zeta^3}+\frac{\mu\,\nu\,a\,\mathfrak{b}^2}{\mu'\zeta^3 f^2(a+\mathfrak{b})^2}\left(\frac{f-1}{a}+\frac{f}{\mathfrak{b}}\right)$$
$$+\frac{\nu\,a^2\,\mathfrak{b}}{\zeta^2 f^2(a+\mathfrak{b})^2}\left(+\frac{\zeta f}{a}+\frac{\zeta f+1}{\mathfrak{b}}\right)$$

qui valor cum debeat eſſe maior vnitate, ſi forte eueniat, vt minor prodeat, tunc non λ' ſed λ definiri conueniet, vbi notandum eſt eſſe $f=\frac{1+\omega}{1-\zeta}$. Hincque per formulas ante datas facile eruuntur radii facierum vtriusque lentis.

Etſi formula ſuperius data pro ſpatio diffuſionis iam ad caſum, quo diſtantia lentium eſt nulla, eſt

adcommodata; tamen quia hic distantiam minimam assumimus, nullus inde error est metuendus.

Definitio 2.

122. *Lens triplicata est, quae consistit ex tribus lentibus simplicibus sibi immediate iunctis ad communem axem.*

Hic quidem etiam crassitiem negligo, etiamsi necessario maior sit quam in lentibus duplicatis In suplemento autem ostendetur, quomodo etiam distantiarum inter lentes ratio sit habenda.

Corollarium.

123. Potest ergo lens triplicata considerari, quasi composita ex lente duplicata et lente simplici, hocque duplici modo, prout vel binae anteriores, vel binae posteriores lentem duplicatam constituere concipiuntur.

Scholion.

124. Lentibus triplicatis tum demum vsum in praxi concedi conueniet, cum eadem commoda per lentes simplices vel duplicatas consequi non licet. Haec autem commoda in paruitate numeri λ consistunt, qui quamdiu vnitate fuerit maior, semper lente simplici vti praestat, cum vero circumstantiae in expressione diffusionis minorem numerum λ requirunt, ad lentes multiplicatas erit confugiendum. Quoniam igitur eiusmodi lentes duplicatas conficere docuimus, in

in quibus valor ipsius λ non solum ad nihilum vsque, sed adeo ad negatiua diminui queat, vsus lentium triplicatarum superfluus videtur. Verum iam obseruauimus in praxi aegre eiusmodi duplicatas lentes parari posse, pro quibus valor ipsius λ minor sit quam $0,191827$, propterea quod si leuissimus error committatur, omnis labor irritus reddatur. His igitur casibus imprimis, quando minori valore numeri λ opus est, lentes triplicatae in vsum erunt vocandae: et quia respectu ad praxin habito non omnes aequo successu construere licet, errores ineuitabiles hic quoque imprimis spectari oportet, vt pateat quousque numerus λ cum successu diminui queat, ac si adhuc minore valore opus fuerit lentes adeo quadruplicatae erunt inducendae.

Problema 5.

125. Datis binis distantiis determinatricibus omnes lentes triplicatas definire, simulque spatium diffusionis ab iis ortum. Tab. II. Fig. 8.

Solutio.

Sit Ee obiectum, cuius magnitudo $= z$, et distantia a lente $AE = a$: tum primae lentis PP distantiae determinatrices sint a et α, secundae lentis QQ b et β; ac tertiae RR, c et γ. His positis quia lentes immediate iunctae sumuntur, erit $\alpha + b = 0$, $\beta + c = 0$, et a et γ distantiae determinatrices lentis triplicatae, ita vt α et β arbitrio nostro relinquantur.

 Si

Si igitur lens prima fola PP adeffet, imago repraefentaretur in $F\zeta$, vt effet $AF=\alpha$ et $F\zeta=\frac{\alpha}{a}z$: fi binae priores PP et QQ folae adeffent imago exhiberetur in $G\eta$, vt effet $AG=\mathfrak{C}$ et $G\eta=\frac{\mathfrak{C}}{a}z$; at per lentem triplicatam referetur in $H\theta$ vt fit $cH=\gamma$ et $H\theta=\frac{\gamma}{a}z$, pro fitu inuerfo ob $\frac{\alpha\mathfrak{C}}{bc}=1$. Hactenus fcilicet res perinde fe habet, ac fi in A haberetur lens fimplex ad diftantias determinatrices a et γ accommodata.

At fi ad fpatium diffufionis Hh refpiciamus figuram fingularum lentium in computum ducere debemus, quatenus praeter diftantias determinatrices numeri arbitrarii $\lambda, \lambda', \lambda''$ inuoluuntur; ex quibus facies fingularum lentium fupra §. 91 funt definitae. Indidem autem colligitur pro apertura cuius femidiameter $=x$, fpatium diffufionis ob $\frac{\alpha}{b}=-1$ et $\frac{\mathfrak{C}}{c}=-1$ fore:

$$Hh=\mu\gamma\gamma xx\begin{cases}+\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\lambda\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2+\frac{\nu}{\alpha a}\right)\\+\left(\frac{1}{b}+\frac{1}{\mathfrak{C}}\right)\left(\lambda'\left(\frac{1}{b}+\frac{1}{\mathfrak{C}}\right)^2+\frac{\nu}{b\mathfrak{C}}\right)\\+\left(\frac{1}{c}+\frac{1}{\gamma}\right)\left(\lambda''\left(\frac{1}{c}+\frac{1}{\gamma}\right)^2+\frac{\nu}{c\gamma}\right)\end{cases},$$

quae expreffio, vt ad formam vni lenti refpondentem reducatur ftatuamus:

$$\frac{1}{a}+\frac{1}{\alpha}=f\left(\frac{1}{a}+\frac{1}{\gamma}\right)$$

$$\frac{1}{b}+\frac{1}{\mathfrak{C}}=g\left(\frac{1}{a}+\frac{1}{\gamma}\right)$$

$$\frac{1}{c}+\frac{1}{\gamma}=h\left(\frac{1}{a}+\frac{1}{\gamma}\right)$$

et

et quia $\frac{1}{\alpha}+\frac{1}{b}=0$, et $\frac{1}{\mathfrak{b}}+\frac{1}{c}=0$, his aequationibus addendis adipiscimur: $f+g+h=1$. Porro vero erit

$$\frac{1}{\alpha}=f\left(\frac{1}{a}+\frac{1}{\gamma}\right)-\frac{1}{a};\quad \frac{1}{b}=\frac{1}{a}-f\left(\frac{1}{a}+\frac{1}{\gamma}\right)$$

$$\frac{1}{\mathfrak{b}}=(f+g)\left(\frac{1}{a}+\frac{1}{\gamma}\right)-\frac{1}{a};\quad \frac{1}{c}=\frac{1}{a}-(f+g)\left(\frac{1}{a}+\frac{1}{\gamma}\right)$$

siue ob $1=f+g+h$

$$\frac{1}{a}=\frac{f+g+h}{a};\quad \frac{1}{\alpha}=\frac{-g-h}{a}+\frac{f}{\gamma}$$

$$\frac{1}{b}=\frac{g+h}{a}-\frac{f}{\gamma};\quad \frac{1}{\mathfrak{b}}=-\frac{h}{a}+\frac{f+g}{\gamma}$$

$$\frac{1}{c}=\frac{h}{a}-\frac{f+g}{\gamma};\quad \frac{1}{\gamma}=\frac{f+g+h}{\gamma}$$

Vnde cum spatium diffusionis fiat.

$$Hh=\mu\gamma\gamma xx\left(\frac{1}{a}+\frac{1}{\gamma}\right)\left((\lambda f^3+\lambda' g^3+\lambda'' h^3)\left(\frac{1}{a}+\frac{1}{\gamma}\right)^2+\nu\left(\frac{f}{a\alpha}+\frac{g}{b\mathfrak{b}}+\frac{h}{c\gamma}\right)\right)$$

reducetur id ad hanc formam

$$Hh=\mu\gamma\gamma xx\left(\frac{1}{a}+\frac{1}{\gamma}\right)\left((\lambda f^3+\lambda' g^3+\lambda'' h^3-\nu(1-f)(1-g)(1-h))\left(\frac{1}{a}+\frac{1}{\gamma}\right)^2+\frac{\nu}{a\gamma}\right)$$

Radiorum autem in h concurrentium inclinatio ad axem erit $=\frac{x}{\gamma}$.

Coroll. 1.

126. Haec igitur lens triplicata idem producit spatium diffusionis quod produceret lens simplex ad easdem distantias determinatrices instructa, numero eius arbitrario (per litteram λ indicato) existente

$$\lambda f^3+\lambda' g^3+\lambda'' h^3-\nu(1-f)(1-g)(1-h)$$

vbi quidem est $f+g+h=1$.

Coroll.

Coroll. 2.

127. Quatenus ergo haec quantitas reddi poteſt minor non ſolum vnitate; ſed etiam fractione 0,191827 ita ſcilicet, vt praxis non enormi aberrationi ſit expoſita, eatenus lentibus triplicatis vſus erit concedendus.

Coroll. 3.

128. Si ſit vel $f=0$, vel $g=0$, vel $h=0$, vna lentium habebit facies parallelas, et lens triplicata aequiualebit duplicatae ac ſi duae litterarum f, g, h ſimul euaneſcant, tertia in vnitatem abeunte, caſus habebitur lentis ſimplicis.

Coroll. 4.

129. Si ſit $f=1$, ideoque $h=-g$, valor numeri λ pro lente triplicata erit $\lambda+(\lambda'-\lambda'')g^2$, ideoque ſi $\lambda''=\lambda'$ lens triplicata ſimplici aequiualebit, quod idem euenit ſi fuerit vel $g=1$ vel $h=1$.

Coroll. 5.

130. Sumtis autem pro f, g, h numeris idoneis ob $f+g+h=1$, conſtructio lentis triplicatae ex formulis §. 91 exhibitis eſt petenda ſumendo.

$$\frac{1}{a}=\frac{-1+f}{\alpha}+\frac{f}{\gamma};\quad \frac{1}{b}=\frac{1-f}{\alpha}-\frac{f}{\gamma}$$

$$\frac{1}{c}=\frac{-h}{\alpha}+\frac{1-h}{\gamma};\quad \frac{1}{e}=\frac{h}{\alpha}-\frac{1+h}{\gamma}$$

Scholion

Scholion 1.

131. Quemadmodum hinc expressio inuenta prodeat, notandum est fore $\frac{f}{a\alpha}+\frac{g}{b\beta}+\frac{b}{c\gamma}=$

$$+\tfrac{1}{a\alpha}(-f(1-f)-gb(1-f))$$
$$+\tfrac{1}{a\gamma}(+ff+g(1-f)(1-b)+fgb+bb)$$
$$+\tfrac{1}{\gamma\gamma}(-fg(1-b)-b(1-b))$$

sed $-f(1-f)-gb(1-f)=-(1-f)(f+gb)=-(1-f)(1-g)(1-b)$ ob $f=1-g-b$ ideoque $f+gb=(1-g)(1-b)$. Simili modo pro $\frac{1}{\gamma\gamma}$ est $-fg(1-b)-b(1-b)=-(1-b)(b+fg)=-(1-f)(1-g)(1-b)$ ob $b=1-f-g$. Denique pro $\frac{1}{a\gamma}$, quia est $ff+bb=(f+b)^2-2fb=(1-g)^2-2fb=1-2g+gg-2fb$, hoc valore substituto coefficiens ipsius $\frac{1}{a\gamma}$ erit

$$1-2g-2fb+gg+fgb+g(1-f)(1-b)=$$
$$1+(g+fb)(g-2)-g(1-f)(1-b)=1-2(1-f)(1-g)(1-b)$$

ob $g+fb=(1-f)(1-b)$. Consequenter colligitur

$$\frac{f}{a\alpha}+\frac{g}{b\beta}+\frac{b}{c\gamma}=-(1-f)(1-g)(1-b)(\tfrac{1}{a}+\tfrac{1}{\gamma})^2+\tfrac{1}{a\gamma}.$$

Scholion 2.

132. Hoc problema etiam ope praecedentium facilius sequenti modo resolui potest. Considerentur scilicet binae lentes PP et QQ iunctim sumtae tanquam lens duplicata ad distantias determinatrices a et β per numeros arbitrarios λ, λ' et f instructa, ac posito $\lambda f^3+\lambda'(1-f)^3-\nu f(1-f)=\lambda^{(2)}$, spatium diffusionis ex ea sola ortum erit

$$\mu\beta\beta xx(\tfrac{1}{a}+\tfrac{1}{\beta})(\lambda^{(2)}(\tfrac{1}{a}+\tfrac{1}{\beta})^2+\tfrac{\nu}{a\beta})$$

quae lens ſi iam in compoſitione cum tertia RR tanquam ſimplex tractetur exinde elicitur ſpatium diffuſionis perinde atque ex coniunctione duarum ſimplicium

$$Hb = \mu\gamma\gamma xx \begin{cases} +(\frac{1}{a}+\frac{1}{\mathfrak{b}})(\lambda^{(\prime)}(\frac{1}{a}+\frac{1}{\mathfrak{b}})^2+\frac{\nu}{a\mathfrak{b}}) \\ +(\frac{1}{c}+\frac{1}{\gamma})(\lambda''(\frac{1}{c}+\frac{1}{\gamma})^2+\frac{\nu}{c\gamma}) \end{cases}$$

vbi notandum eſt eſſe $\mathfrak{b}+c=0$, et conſtructio lentis duplicatae ex §. 107 erit petenda lentis vero ſimplicis RR ex diſtantiis determinatricibus $c=-\mathfrak{b}$ et γ vna cum numero arbitrario λ''. Ponatur iam $\frac{1}{\mathfrak{b}}=\frac{g-1}{a}+\frac{g}{\gamma}$ vt ſit $\frac{1}{c}=\frac{1-g}{a}-\frac{g}{\gamma}$, eritque ſpatium diffuſionis huius lentis triplicatae,

$$Hb=\mu\gamma\gamma xx(\tfrac{1}{a}+\tfrac{1}{\gamma})((\lambda^{(\prime)}g^3+\lambda''(1-g)^3-\nu g(1-g))(\tfrac{1}{a}+\tfrac{1}{\gamma})^2+\tfrac{\nu}{a\gamma})$$

Quare ſi pro lente triplicata ponatur

$$\lambda^{(\prime)}g^3+\lambda''(1-g)^3-\nu g(1-g)=\lambda^{(\prime)}$$

ita vt iam numerus g inſuper arbitrio noſtro relinquatur, habebitur ſpatium diffuſionis more hactenus recepto expreſſum

$$Hb=\mu\gamma\gamma xx(\tfrac{1}{a}+\tfrac{1}{\gamma})(\lambda^{(\prime)}(\tfrac{1}{a}+\tfrac{1}{\gamma})^2+\tfrac{\nu}{a\gamma})$$

Hinc iam id intelligitur, quod ex praecedente ſolutione minus patet; ſi numerus $\lambda^{(\prime)}$ fuerit vnitate maior loco binarum priorum lentium PP et QQ commodius vnicam ſimplicem adhiberi; ex quo eatenus tantum lentes triplicatae reſultare cenſendae ſunt, quatenus nume-

numerus $\lambda^{(2)}$ vnitate eſt minor. Vidimus autem ſucceſſum tuto ſperari non poſſe, niſi $\lambda^{(\,)}$ aequalis ſit fractioni 0, 191827, vel ea non multo maior; vnde ſi praxi conſulere velimus, ipſi $\lambda^{(\,)}$ minorem valorem tribui non conuenit atque ob eandem rationem numerus g intra terminos 0 et 1 accipi debebit; cuius valor imprimis ad praxin erit accommodatus ſi reddat numerum $\lambda^{(\,)}$ minimum, quia tum leues errores negotium minime turbant. Quodſi vero pro $\lambda^{(2)}$ valorem aſſignatum ſubſtituamus, obtinebimus

$$\lambda^{(\,)}=\lambda f^3 g^3+\lambda' g^3(1-f)^3+\lambda''(1-g)^3-\nu g^3 f(1-f)-\nu g(1-g)$$

Con iucet autem vtramque expreſſionem pro numero $\lambda^{(1)}$, quo ſpatium diffuſionis a lente triplicata ortum definitur, hic expoſuiſſe cum aliae concluſiones ex altera facilius deducantur. Etſi autem hinc omnes valores pro $\lambda^{(1)}$ obtineri poſſunt tamen eos tantum, qui prope minimum ſubſiſtunt ad praxin adhiberi conueniet.

Problema. 6.

133. Datis diſtantiis determinatricibus $AE=a$ et $cH=\gamma$ definire eam lentem triplicatam, quae minimum ſpatium diffuſionis producat.

Solutio 1.

Duplici modo hoc problema ſolui poteſt, prout ſpatium diffuſionis vel ita exprimitur, vti in ſolutione problematis praecedentis, vel in Scholio 2. Priori

ori modo numeros f, g, h ita determinari oportet, vt minima reddatur haec expressio:

$$\lambda f^3 + \lambda' g^3 + \lambda'' h^3 - \nu(1-f)(1-g)(1-h)$$

vbi notandum est esse $f+g+h=1$. Eius ergo differentiali nihilo aequali posito habebimus:

$$3\lambda ffdf + 3\lambda' ggdg + 3\lambda'' hhdh$$
$$+\nu df(1-g)(1-h) + \nu dg(1-f)(1-h) + \nu dh(1-f)(1-g) = 0.$$

Cum autem sit $dh = -df - dg$ erit

$$\left.\begin{array}{l} +3\lambda ffdf - 3\lambda'' hhdf + \nu df(1-g)(f-h) \\ +3\lambda' ggdg - 3\lambda'' hhdg + \nu dg(1-f)(g-h) \end{array}\right\} = 0$$

quia vero bina differentialia df et dg a se inuicem non pendent ambo membra huius aequationis seorsim euanescere debent, vnde ob $1-g=f+h$ et $1-f=g+h$ has duas nanciscimur aequationes:

$$3\lambda ff - 3\lambda'' hh + \nu ff - \nu hh = 0$$
$$3\lambda' gg - 3\lambda'' hh + \nu gg - \nu hh = 0$$

ex quibus elicimus:

$$f = h\sqrt{\tfrac{3\lambda''+\nu}{3\lambda+\nu}} \text{ et } g = h\sqrt{\tfrac{3\lambda''+\nu}{3\lambda'+\nu}}.$$

Cum autem sit

$$f+g+h=1 \text{ seu } \tfrac{1}{h} = 1 + \tfrac{f}{h} + \tfrac{g}{h} \text{ erit}$$
$$\tfrac{1}{h} = 1 + \sqrt{\tfrac{3\lambda''+\nu}{3\lambda+\nu}} + \sqrt{\tfrac{3\lambda''+\nu}{3\lambda'+\nu}}$$
$$\tfrac{1}{g} = 1 + \sqrt{\tfrac{3\lambda'+\nu}{3\lambda+\nu}} + \sqrt{\tfrac{3\lambda'+\nu}{3\lambda''+\nu}}$$
$$\tfrac{1}{f} = 1 + \sqrt{\tfrac{3\lambda+\nu}{3\lambda'+\nu}} + \sqrt{\tfrac{3\lambda+\nu}{3\lambda''+\nu}}$$

Quicun-

Quicunque ergo numeri λ, λ', λ'' in conſtructione ſingularum lentium fuerint vſurpati, hinc numeri f, g, et b determinantur, ex quibus ſpatium diffuſionis minimum reſultet. Pro lentium autem conſtructione hinc diſtantiae a, b, $\mathfrak{b}$, c ita definiuntur, vt ſit

$$\frac{1}{\alpha}=\frac{-1+f}{a}+\frac{f}{\gamma};\quad \frac{1}{b}=\frac{1-f}{a}-\frac{f}{\gamma}$$

$$\frac{1}{\mathfrak{b}}=\frac{-b}{a}+\frac{1-b}{\gamma};\quad \frac{1}{c}=\frac{b}{a}-\frac{1+b}{\gamma}$$

eſt vero:

$$f=\frac{\sqrt{(3\lambda'+\nu)(3\lambda''+\nu)}}{\sqrt{(3\lambda+\nu)(3\lambda'+\nu)}+\sqrt{(3\lambda+\nu)(3\lambda''+\nu)}+\sqrt{(3\lambda'+\nu)(3\lambda''+\nu)}}$$

$$g=\frac{\sqrt{(3\lambda+\nu)(3\lambda''+\nu)}}{\sqrt{(3\lambda+\nu)(3\lambda'+\nu)}+\sqrt{(3\lambda+\nu)(3\lambda''+\nu)}+\sqrt{(3\lambda'+\nu)(3\lambda''+\nu)}}$$

$$b=\frac{\sqrt{(3\lambda+\nu)(3\lambda'+\nu)}}{\sqrt{(3\lambda+\nu)(3\lambda'+\nu)}+\sqrt{(3\lambda+\nu)(3\lambda''+\nu)}+\sqrt{(3\lambda'+\nu)(3\lambda''+\nu)}}$$

Ex diſtantiis autem a, α, b, $\mathfrak{b}$, c, γ cum numeris λ, λ', λ'' lentes ipſae per formulas §. 91 exhibitas conſtruuntur.

Coroll. I.

134. Si pro hac lente triplicata ponatur

$$\lambda f^3+\lambda' g^3+\lambda'' b^3-\nu(1-f)(1-g)(1-b)=\lambda^{(3)}$$

ſubſtituendis his valoribus pro f, g, et b reperietur

$$\lambda^{(3)}=\frac{1}{3\left(\frac{1}{\sqrt{(3\lambda+\nu)}}+\frac{1}{\sqrt{(3\lambda'+\nu)}}+\frac{1}{\sqrt{(3\lambda''+\nu)}}\right)^2}-\frac{1}{3}\nu$$

vnde ſpatium diffuſionis fit

$$\mathrm{H}b=\mu\gamma\gamma xx\left(\frac{1}{a}+\frac{1}{\gamma}\right)\left(\lambda^{(3)}\left(\frac{1}{a}+\frac{1}{\gamma}\right)^2+\frac{\nu}{a\gamma}\right)$$

Coroll. 2.

135. Hoc autem spatium diffusionis omnium fiet minimum si numeris λ, λ', λ'' minimi valores, quos accipere possunt, tribuantur. Sit ergo $\lambda = 1$, $\lambda' = 1$, $\lambda'' = 1$, eritque

$$f = \tfrac{1}{3};\quad g = \tfrac{1}{3},\quad b = \tfrac{1}{3}$$

$$\text{et } \lambda^{(3)} = \frac{\nu+1}{27} - \frac{\nu}{3} = \frac{1-8\nu}{27} = 0,042165$$

ob 0, 232692; qui ergo valor multo est minor, quam casu lentium duplicatarum.

Coroll. 3.

136. Hoc porro casu erit $\frac{1}{a} = -\frac{2}{3\alpha} + \frac{1}{3\gamma}$; $\frac{1}{b} = \frac{2}{3\alpha} - \frac{1}{3\gamma}$; $\frac{1}{\mathfrak{b}} = \frac{-1}{3\alpha} + \frac{2}{3\gamma}$ et $\frac{1}{c} = \frac{1}{3\alpha} - \frac{2}{3\gamma}$; vnde constructio lentium ternarum simplicium ita se habebit.

Pro Lente

Prima radius faciei $\begin{cases} \text{anterioris} = \dfrac{3\alpha\gamma}{(3\varrho - 2\sigma)\gamma + \varrho\alpha} \\ \text{posterioris} = \dfrac{3\alpha\gamma}{(3\sigma - 2\varrho)\gamma + \varrho\alpha} \end{cases}$

Secunda radius faciei $\begin{cases} \text{anterioris} = \dfrac{3\alpha\gamma}{(2\varrho - \sigma)\gamma + (2\sigma - \varrho)\alpha} \\ \text{posterioris} = \dfrac{3\alpha\gamma}{(2\sigma - \varrho)\gamma + (2\varrho - \sigma)\alpha} \end{cases}$

Tertia radius faciei $\begin{cases} \text{anterioris} = \dfrac{3\alpha\gamma}{\varrho\gamma + (3\sigma - 2\varrho)\alpha} \\ \text{posterioris} = \dfrac{3\alpha\gamma}{\sigma\gamma + (3\varrho - 2\sigma)\alpha} \end{cases}$

Solutio altera Problematis.

137. Consideremus binas lentes priores PP et QQ vt lentem duplicatam ad distantias determinatrices a et $\mathfrak{b}$ ita

ita instructam vt posito $\lambda f^3 + \lambda'(1-f)^3 - \nu f(1-f) = \lambda^{(2)}$ spatium diffusionis inde oriundum sit

$$= \mu \mathfrak{b}\mathfrak{b} xx \left(\frac{1}{a} + \frac{1}{\mathfrak{b}}\right)\left(\lambda^{(2)}\left(\frac{1}{a} + \frac{1}{\mathfrak{b}}\right)^2 + \frac{\nu}{a\mathfrak{b}}\right).$$

sed pro constructione binarum lentium simplicium, ex quibus haec lens est composita, recordandum est esse

$$\frac{1}{\alpha} = \frac{-1+f}{a} + \frac{f}{\mathfrak{b}} \text{ et } \frac{1}{b} = \frac{1-f}{a} - \frac{f}{\mathfrak{b}}$$

Adiuncta iam tertia lente RR ad distantias determinatrices $c = -\mathfrak{b}$ et γ per numerum arbitrarium λ'' instructa si ponamus $\frac{1}{\mathfrak{b}} = \frac{-1+g}{a} + \frac{g}{\gamma} = -\frac{1}{c}$, et

$$\lambda^{(2)} g^3 + \lambda''(1-g)^3 - \nu g(1-g) = \lambda^{(3)}$$

erit spatium diffusionis

$$Hb = \mu \gamma\gamma xx \left(\frac{1}{a} + \frac{1}{\gamma}\right)\left(\lambda^{(3)}\left(\frac{1}{a} + \frac{1}{\gamma}\right)^2 + \frac{\nu}{a\gamma}\right)$$

quod vt fiat minimum, valor ipsius $\lambda^{(3)}$ minimus reddi debet, quaeratur ergo primo g dum $\lambda^{(2)}$ vt numerus datus spectatur et habebimus

$$3\lambda^{(2)} gg - 3\lambda''(1-g)^2 - \nu + 2\nu g = 0$$

vnde eliciuntur

$$g = \frac{-3\lambda'' - \nu + \sqrt{(3\lambda^{(2)} + \nu)(3\lambda'' + \nu)}}{3\lambda^{(2)} - 3\lambda''} \quad \text{siue}$$

$$g = \frac{\sqrt{(3\lambda'' + \nu)}}{\sqrt{(3\lambda^{(2)} + \nu)} + \sqrt{(3\lambda'' + \nu)}}$$

et hinc valor ipsius $\lambda^{(3)}$ erit

$$\lambda^{(3)} = \frac{1}{3\left(\frac{1}{\sqrt{(3\lambda^{(2)} + \nu)}} + \frac{1}{\sqrt{(3\lambda'' + \nu)}}\right)} - \frac{1}{3}\nu$$

vel

vel

$$\frac{1}{3\lambda^{(3)}+\nu}=\left(\frac{1}{\sqrt{(3\lambda^{(2)}+\nu)}}+\frac{1}{\sqrt{(3\lambda''+\nu)}}\right)^2$$

Sicque fit:

$$\frac{1}{\sqrt{(3\lambda^{(3)}+\nu)}}=\frac{1}{\sqrt{(3\lambda^{(2)}+\nu)}}+\frac{1}{\sqrt{(3\lambda''+\nu)}}$$

Simili modo si f ita definiatur, vt $\lambda^{(2)}$ fiat minimum reperietur:

$$f=\frac{\sqrt{(3\lambda'+\nu)}}{\sqrt{(3\lambda+\nu)}+\sqrt{(3\lambda'+\nu)}}$$

hocque valore substituto

$$\frac{1}{\sqrt{(3\lambda^{(2)}+\nu)}}=\frac{1}{\sqrt{(3\lambda+\nu)}}+\frac{1}{\sqrt{(3\lambda'+\nu)}}$$

Quare si numeris $\lambda, \lambda', \lambda''$, arbitrio nostro relictis bini numeri f et g ita definiantur, vt $\lambda^{(3)}$ consequatur valorem minimum erit

$$\frac{1}{\sqrt{(3\lambda^{(3)}+\nu)}}=\frac{1}{\sqrt{(3\lambda+\nu)}}+\frac{1}{\sqrt{(3\lambda'+\nu)}}+\frac{1}{\sqrt{(3\lambda''+\nu)}}$$

vnde idem valor pro $\lambda^{(3)}$ reperitur, quem ante inuenimus.

Coroll. 1.

138. Ex hac ergo solutione numeri f et g ita definiuntur vt sit

$$\frac{1}{f\sqrt{(3\lambda+\nu)}}=\frac{1}{\sqrt{(3\lambda+\nu)}}+\frac{1}{\sqrt{(3\lambda'+\nu)}} \quad \text{et}$$

$$\frac{1}{g\sqrt{(3\lambda^{(2)}+\nu)}}=\frac{1}{\sqrt{(3\lambda^{(2)}+\nu)}}+\frac{1}{\sqrt{(3\lambda''+\nu)}}$$

siue

ſiue hoc modo

$$\left(\frac{1}{f}-1\right)\frac{1}{\sqrt{(3\lambda+\nu)}}=\frac{1}{\sqrt{(3\lambda'+\nu)}} \text{ et}$$

$$\left(\frac{1}{g}-1\right)\left(\frac{1}{\sqrt{(3\lambda+\nu)}}+\frac{1}{\sqrt{(3\lambda'+\nu)}}\right)=\frac{1}{\sqrt{(3\lambda''+\nu)}}$$

Coroll. 2.

139. Ex inuentis minimis valoribus numerorum $\lambda^{(\prime)}$ et. $\lambda^{(\prime\prime)}$ numeri f et g etiam ita definiuntur vt ſit

$$\frac{1}{f\sqrt{(3\lambda+\nu)}}=\frac{1}{\sqrt{(3\lambda^{(\prime)}+\nu)}} \text{ ſeu } f=\sqrt{\left(\frac{3\lambda^{(\prime)}+\nu}{3\lambda+\nu}\right)} \text{ et}$$

$$\frac{1}{g\sqrt{(3\lambda^{(\prime)}+\nu)}}=\frac{1}{\sqrt{(3\lambda^{(\prime\prime)}+\nu)}} \text{ ſeu } g=\sqrt{\left(\frac{3\lambda^{(\prime\prime)}+\nu}{3\lambda^{(\prime)}+\nu}\right)}$$

Coroll. 3

140. Diſtantiae autem determinatrices ſingularum lentium ita per a et γ prodibunt expreſſae:

$$\frac{1}{\alpha}=-\frac{1}{b}=\frac{-1+fg}{a}+\frac{fg}{\gamma}$$

$$\frac{1}{\mathfrak{b}}=-\frac{1}{c}=\frac{-1+g}{a}+\frac{g}{\gamma}$$

vbi cum ſit

$$g=\sqrt{\frac{3\lambda^{(\prime\prime)}+\nu}{3\lambda^{(\prime)}+\nu}} \text{ notandum eſt eſſe } fg=\sqrt{\frac{3\lambda^{(\prime\prime)}+\nu}{3\lambda+\nu}}.$$

Coroll. 4.

141. Eliminando autem numero $\lambda^{(\prime)}$ erit

$$\frac{1}{\alpha}=-\frac{1}{b}=-\frac{1}{a}\sqrt{\frac{3\lambda^{(\prime\prime)}+\nu}{3\lambda'+\nu}}-\frac{1}{a}\sqrt{\frac{3\lambda^{(\prime\prime)}+\nu}{3\lambda''+\nu}}+\frac{1}{\gamma}\sqrt{\frac{3\lambda^{(\prime)}+\nu}{3\lambda+\nu}}$$

$$\frac{1}{\mathfrak{b}}=-\frac{1}{c}=-\frac{1}{a}\sqrt{\frac{3\lambda^{(\prime\prime)}+\nu}{3\lambda''+\nu}}+\frac{1}{\gamma}\sqrt{\frac{3\lambda^{(\prime\prime)}+\nu}{3\lambda+\nu}}+\frac{1}{\gamma}\sqrt{\frac{3\lambda^{(\prime\prime)}+\nu}{3\lambda'+\nu}}$$

ex quibus formulis, inuento iam valore minimo $\lambda^{(7)}$ singulae lentes commodissime determinantur.

Coroll. 5.

142. Si praeterea singulae lentes ita fuerint comparatae vt per se minimam confusionem pariant quod fit si $\lambda = \lambda' = \lambda'' = 1$, erit

$$\frac{1}{\sqrt{(3\lambda^{(2)}+\nu)}} = \frac{2}{\sqrt{(3+\nu)}} \text{ hincque } 3\lambda^{(2)}+\nu = \frac{3+\nu}{4},$$

vnde fit $\lambda^{(2)} = \frac{1-\nu}{4}$. Deinde vero habebitur:

$$\frac{1}{\sqrt{(3\lambda^{(3)}+\nu)}} = \frac{3}{\sqrt{(3+\nu)}} \text{ hincque } 3\lambda^{(3)}+\nu = \frac{3+\nu}{9}$$

ac propterea

$\lambda^{(3)} = \frac{3-8\nu}{27}$. Sin autem fuerit tantum $\lambda = \lambda' = \lambda''$ reperietur:

$$\lambda^{(2)} = \frac{3\lambda-3\nu}{12} \text{ et } \lambda^{(3)} = \frac{3\lambda-8\nu}{27}.$$

Coroll. 6.

143. Eodem autem casu, quo $\lambda = \lambda' = \lambda''$, ob $\sqrt{(3\lambda^{(3)}+\nu)} = \frac{1}{3}\sqrt{(3\lambda+\nu)}$, distantiae determinatrices pro lentibus simplicibus erunt:

$$\frac{1}{\alpha} = \frac{-2}{3a} + \frac{1}{3\gamma};\ \frac{1}{b} = \frac{2}{3a} - \frac{1}{3\gamma}$$

$$\frac{1}{\beta} = \frac{-1}{3a} + \frac{2}{3\gamma};\ \frac{1}{c} = \frac{1}{3a} - \frac{2}{3\gamma}$$

vnde eaedem formulae pro earum constructione nascuntur quae supra (136) sunt allatae nisi quod iam in denominatoribus membra $\pm \tau(a+\gamma)\sqrt{(\lambda-1)}$ adiungi debeant.

Scholi-

Scholion.

144. Ternarum ergo lentium ſimplicium idonea coniunctione effici poteſt, vt in expreſſione ſpatii diffuſionis $Hb = \mu \gamma \gamma x x (\frac{1}{a} + \frac{1}{\gamma})(\lambda^{(3)} (\frac{1}{a} + \frac{1}{\gamma})^2 + \frac{v}{a\gamma})$ numerus $\lambda^{(3)}$ fiat $= 0,042165$. In lentibus autem duplicatis vidimus minimum valorem numeri $\lambda^{(2)}$ eſſe $= 0,191827$: ſicque in triplicatis hic numerus fere quinquies minor reddi poteſt at is fere vicies quater eſt minor, quam per lentes ſimplices obtineri poteſt. Loquor hic autem de lentibus ex principio minimi petitis, quippe quae ad praxin maxime ſunt accommodatae, dum conſtructio leuibus erroribus non admodum turbatur. Quanquam enim lentes triplicatae perinde ac duplicatae parari poſſent, pro quibus numerus conueniens λ non ſolum nihilo aequalis, ſed etiam negatiuus. reſultaret tamen earum conſtructio tam eſt lubrica, vt minimus error totum laborem irritum reddat. Interim tamen periculum in triplicatis non tantum eſt quam duplicatis, vnde ſequens problema ſoluiſſe operae erit pretium.

Problema 7.

145. Pro datis diſtantiis determinatricibus a et γ eas definire lentes triplicatas, pro quibus valor ipſius $\lambda^{(3)}$ prorſus in nihilum abeat.

Solutio.

Conſideretur numerus $\lambda^{(2)}$ ex duabus prioribus

 lenti-

lentibus natus vt datus, et cum ſit ſecundum ſolutionem poſteriorem praecedentis problematis

$$\lambda^{(1)} = \lambda^{(2)} g^3 + \lambda''(1-g)^3 - \nu g(1-g)$$

definiri debet g ita vt iſta quantitas euaneſcat. Verum cum $\lambda^{(2)}$ commode nequeat minor effici quam $\frac{1-\nu}{4}$, ſtatuamus $\lambda^{(2)} = \frac{1-\nu}{4}$, fierique oportet:

$$0 = \tfrac{1}{4} g^3 + \lambda''(1-g)^3 - \nu g(1-\tfrac{1}{2}g)^2$$

ſed quia λ'' vnitate minor eſſe nequit, neceſſe eſt vt g capiatur vnitate maior euoluantur ergo quidam caſus.

I. $g = \frac{5}{4}$; $0 = \frac{125}{4\cdot 64} - \frac{\lambda''}{64} - \frac{5\cdot 9}{4\cdot 64}\nu$ et $\lambda'' = \frac{125-45\nu}{4}$

II. $g = \frac{6}{4}$; $0 = \frac{216}{4\cdot 64} - \frac{8\lambda''}{64} - \frac{6\cdot 4}{4\cdot 64}\nu$ et $\lambda'' = \frac{216-24\nu}{4\cdot 8} = \frac{27-3\nu}{4}$

III. $g = \frac{7}{4}$; $0 = \frac{343}{4\cdot 64} - \frac{27\lambda''}{64} - \frac{7\cdot 1}{4\cdot 64}\nu$ et $\lambda'' = \frac{343-7\nu}{4\cdot 27}$

IV. $g = \frac{8}{4}$; $0 = \frac{512}{4\cdot 64} - \frac{64\lambda''}{64}$ et $\lambda'' = 2$

V. $g = \frac{9}{4}$; $0 = \frac{729}{4\cdot 64} - \frac{125\lambda''}{64} - \frac{9\cdot 1}{4\cdot 64}\nu$ et $\lambda'' = \frac{729-9\nu}{4\cdot 125}$

VI. $g = \frac{10}{4}$; $0 = \frac{1000}{4\cdot 64} - \frac{216\lambda''}{64} - \frac{10\cdot 4}{4\cdot 64}\nu$ et $\lambda'' = \frac{1000-40\nu}{4\cdot 216} = \frac{125-5\nu}{4\cdot 27}$

In primo et ſecundo caſu fit valor ipſius λ'' nimis magnus, quam vt iſta lens commode in praxin recipi queat; ac ſi ipſi g multo maior tribuatur valor, leuis error ingentem effectum producit. Ponamus enim loco g per errorem ſumi $g+\omega$, et cum λ'' ex g rite fuerit definitum; fiet

$$\lambda^{(1)} = \omega\left(\tfrac{3}{4}gg - 3\lambda''(1-g)^2 - \nu(1-\tfrac{1}{2}g)(1-\tfrac{3}{2}g)\right)$$

et

et pro λ'' ſubſtituto valore:

$$\lambda^{(3)} = \frac{\omega}{4(1-g)}(3gg - \nu(4 - gg))$$

vnde patet, quo minus g vnitatem excedat, ac ſimul quo maius fuerit g, valorem ipſius $\lambda^{(3)}$ ob errorem ω eo fieri maiorem. Intelligitur autem hunc errorem fieri minimum ſi capiatur

$g = 1 + \sqrt{\frac{2 - 2\nu}{\nu + 2}}$, ex hoc autem valore elicitur,

$$\lambda'' = \frac{(1+\nu)\sqrt{2(1-\nu)(1+\nu)} + 2(1+\nu\nu)}{g(1-\nu)}$$

Capi ergo debebit $g = 1,84384$ vnde colligitur:

$$\lambda'' = \frac{g^3 - \nu g(2-g)^2}{4(g-1)^3} = 2,60372$$

et ſi hae menſurae exacte obſeruentur, fiet $\lambda^{(3)} = 0$. At ſi in valore ipſius g particula ω aberretur, vt ſit $g = 1,84384 + \omega$, prodibit ob hunc errorem:

$$\lambda^{(3)} = -2,981\,\omega,$$

ita ſi eſſet error $\omega = \pm \frac{1}{10}$, loco $\lambda^{(3)} = 0$, prodiret:

$\lambda^{(3)} = \mp 0,2981$ ideoque lens triplicata poſtponenda duplicatae, longe tamen praeferenda foret ſimplici.

In genere igitur pro quouis valore alio ipſius $\lambda^{(3)}$ idem commodum inueſtigemus: ac primo cum ſit

$$\lambda'' = \frac{\lambda^{(3)} g^3 + \nu g(g-1)}{(g-1)^3}$$

ſi loco iuſti valoris g capiatur $g + \omega$ fiet

$$\lambda^{(3)} = \omega(3\lambda^{(3)} g^2 - 3\lambda''(g-1)^2 - \nu + 2\nu g)$$

vbi ſi pro λ'' valor ſubſtituatur, erit

$$\lambda^{(3)} = \frac{\omega(\nu - (3\lambda^{(3)} + \nu)gg)}{g-1}$$

qui error vt minimus reddatur capi debet

$$g = 1 + \sqrt{\frac{3\lambda^{(2)}}{3\lambda^{(2)} + \nu}}$$

qui eſt valor maxime idoneus pro g ſumendus, ex quo elicitur

$$\lambda'' = \frac{1}{3\lambda^{(2)}}\left(\sqrt{(3\lambda^{(2)}+\nu)} + \sqrt{3\lambda^{(2)}}\right)\left(\tfrac{1}{3}(6\lambda^{(2)}+\nu)\sqrt{3\lambda^{(2)}} + (2\lambda^{(2)}+\nu)\sqrt{(3\lambda^{(2)}+\nu)}\right)$$

et ſumto per errorem $g+\omega$ pro g erit

$$\lambda^{(3)} = -2\omega\left(3\lambda^{(2)}+\nu+\sqrt{3\lambda^{(2)}(3\lambda^{(2)}+\nu)}\right) \text{ ſeu}$$

$$\lambda^{(3)} = -2\omega\left(\sqrt{3\lambda^{(2)}}+\sqrt{(3\lambda^{(2)}+\nu)}\right)\sqrt{(3\lambda^{(2)}+\nu)}$$

Quo minor igitur iam fuerit valor ipſius $\lambda^{(2)}$ eo minus erit error metuendus, vnde ſolutio ante ex valore $\lambda^{(2)} = \frac{1-\nu}{4}$ eruta prae ceteris eſt commendanda. Tum autem erit $f = \frac{1}{2}$; $\lambda = 1$, $\lambda' = 1$, $\lambda'' = 2,60372$ et $g = 1,84384$, vnde pro lentium conſtructione habemus,

$$\frac{1}{\alpha} = -\frac{1}{b} = \frac{-1+g}{2a} + \frac{g}{2\gamma}$$

$$\frac{1}{\beta} = -\frac{1}{c} = \frac{-1+g}{a} + \frac{g}{\gamma}$$

vnde ſingulae lentes per formulas §. 91 conſtruentur.

Coroll. 1.

146. Lens ergo prima PP conſtrui debet ex diſtantiis determinatricibus a et $\frac{2a\gamma}{ga-(2-g)\gamma}$ cum numero $\lambda = 1$

Lens vero ſecunda QQ ex diſtantiis determinatricibus

$\frac{-2a\gamma}{ga-(2-g)\gamma}$ cum numero $\lambda' = 1$

at

at lens tertia RR ex diſtantiis determinatricibus

$$\frac{-a\gamma}{ga-(1-g)\gamma} \text{ et } \gamma \text{ cum numero } \lambda'' = 2,60372$$

exiſtente $g = 1,84384$.

Coroll. 2.

147. Quoniam in formula ſpatium diffuſionis exprimente quae ob $\lambda^{(3)} = 0$ eſt $\mu\gamma\gamma xx(\frac{1}{a}+\frac{1}{\gamma})\frac{v}{a\gamma}$, reiecto factore primo $\mu\gamma\gamma xx$, cuius ratio in his inueſtigationibus non eſt habita, diſtantiae a et γ inter ſe permutari poſſunt; hinc etiam alia lens triplicata quaeſito aeque ſatisfaciens exhiberi poterit.

Coroll. 3.

148. Nempe pro hac altera lente triplicata lens prima PP conſtrui debet ex diſtantiis determinatricibus

$$a \text{ et } \frac{+a\gamma}{(1-g)a-g\gamma} \text{ cum numero } \lambda = 2,60372$$

Lens ſecunda QQ ex diſtantiis determinatricibus

$$\frac{-a\gamma}{(1-g)a-g\gamma} \text{ et } \frac{+2a\gamma}{(2-g)a-g\gamma} \text{ cum numero } \lambda' = 1$$

at lens tertia ex diſtantiis determinatricibus

$$\frac{-2a\gamma}{(2-g)a-g\gamma} \text{ et } \gamma \text{ cum numero } \lambda'' = 1$$

exiſtente vt ante $g = 1,84384$.

Coroll. 4.

149. Hinc ergo duas nacti ſumus lentes triplicatas pro diſtantiis a et γ, quae producunt ſpatium diffu-

diffusionis $Hb = \mu\gamma\gamma xx(\frac{1}{a}+\frac{1}{\gamma})\frac{v}{a\gamma}$. Atque hae inter infinitas alias eundem effectum praestantes hac gaudent praerogatiua, vt leuis error in constructione commissus scopum minime perturbet.

Coroll. 5.

150. Si in constructione harum lentium per errorem numerus g parumper maior accipiatur, quam 1, 84384, tum pro lente triplicata numerus $\lambda^{(3)}$ prodit nihilo minor seu negatiuus. Sin autem numerus g in praxi aliquantillum maior sumatur numerus $\lambda^{(3)}$ fit nihilo maior, sicque lens triplicata ad naturam duplicatarum accedet.

Coroll. 6.

151. Si ergo opus fuerit lente, pro qua numerus λ valorem habeat negatiuum, huic scopo satisfieri commode poterit per lentes descriptas triplicatas, dummodo pro g numerus aliquanto maior quam 1, 84384 assumatur. Scilicet si sumatur

$$g = 1,84384 + \omega \text{ fiet } \lambda^{(3)} = -2,981\,\omega.$$

Supplementum II.

De lentibus triplicatis

Si pro singulis lentibus refractio sit diuersa, pro prima $n : 1$ pro secunda $n' : 1$, pro tertia n''

$n'' : 1$, radiique facierum lentium ſequenti modo definiantur

	Diſt. determinatrices	Refractio et litterae inde pendentes	
I.	a et α	$n : 1,\ \mu,\ \nu,\ \varrho,\ \sigma,\ \tau.$	λ
II.	b et $\mathfrak{G}$	$n' : 1,\ \mu',\ \nu',\ \varrho',\ \sigma',\ \tau'.$	λ'
III.	c et γ	$n'' : 1,\ \mu'',\ \nu'',\ \varrho'',\ \sigma'',\ \tau''.$	λ''

ſcilicet ſi pro lente prima vocetur radius faciei anterioris $= F$; poſterioris $= G$; erit

$$\frac{1}{F} = \frac{\varrho}{a} + \frac{\sigma}{\alpha} \mp \tau\left(\frac{1}{a} + \frac{1}{\alpha}\right)\sqrt{\lambda - 1}$$

$$\text{et } \frac{1}{G} = \frac{\varrho}{\alpha} + \frac{\sigma}{a} \pm \tau\left(\frac{1}{a} + \frac{1}{\alpha}\right)\sqrt{\lambda - 1}$$

ſimilique modo pro reliquis lentibus, nempe pro ſecunda

$$\frac{1}{F'} = \frac{\varrho'}{b} + \frac{\sigma'}{\mathfrak{G}} \mp \tau'\left(\frac{1}{b} + \frac{1}{\mathfrak{G}}\right)\sqrt{\lambda' - 1}$$

$$\frac{1}{G'} = \frac{\varrho'}{\mathfrak{G}} + \frac{\sigma'}{b} \pm \tau'\left(\frac{1}{b} + \frac{1}{\mathfrak{G}}\right)\sqrt{\lambda' - 1}$$

tum vero pro tertia

$$\frac{1}{F''} = \frac{\varrho''}{c} + \frac{{}''}{\gamma} \mp \tau''\left(\frac{1}{c} + \frac{1}{\gamma}\right)\sqrt{\lambda'' - 1}$$

$$\frac{1}{G''} = \frac{\varrho''}{\gamma} + \frac{\sigma''}{c} \pm \tau''\left(\frac{1}{c} + \frac{1}{\gamma}\right)\sqrt{\lambda'' - 1}$$

Deinde quia diſtantiae lentium pro nihilo habentur ſcilicet $a + b = 0$ et $\mathfrak{G} + c = 0$, erit ſpatium diffuſionis

$$= \gamma\gamma xx \begin{cases} + \mu\left(\frac{1}{a} + \frac{1}{\alpha}\right)\left(\lambda\left(\frac{1}{a} + \frac{1}{\alpha}\right)^2 + \frac{\nu}{a\alpha}\right) \\ + \mu'\left(\frac{1}{b} + \frac{1}{\mathfrak{G}}\right)\left(\lambda'\left(\frac{1}{b} + \frac{1}{\mathfrak{G}}\right)^2 + \frac{\nu'}{b\mathfrak{G}}\right) \\ + \mu''\left(\frac{1}{c} + \frac{1}{\gamma}\right)\left(\lambda''\left(\frac{1}{c} + \frac{1}{\gamma}\right)^2 + \frac{\nu''}{c\gamma}\right) \end{cases}$$

 ſtatua-

ſtatuatur nunc

$$\frac{1}{a}+\frac{1}{\alpha}=f\left(\frac{1}{a}+\frac{1}{\gamma}\right)$$
$$\frac{1}{b}+\frac{1}{\beta}=g\left(\frac{1}{a}+\frac{1}{\gamma}\right)$$
$$\frac{1}{c}+\frac{1}{\gamma}=b\left(\frac{1}{a}+\frac{1}{\gamma}\right)$$

vt fiat

$$\frac{1}{\alpha}=\frac{f-1}{a}+\frac{f}{\gamma}=\frac{-1}{b}; \quad \text{et}$$
$$\frac{1}{c}=\frac{b}{a}+\frac{b-1}{\gamma}=\frac{-1}{\beta}$$

vnde fit

$$\frac{1}{b}+\frac{1}{\beta}=\frac{1-f-b}{a}+\frac{1-f-b}{\gamma}=g\left(\frac{1}{a}+\frac{1}{\gamma}\right)$$

hincque $f+g+b=1$; quibus valoribus ſubſtitutis erit primo pro radiis facierum

$$\frac{1}{P}=\frac{\rho+(f-1)\sigma}{a}+\frac{\sigma f}{\gamma}\mp\tau f\left(\frac{1}{a}+\frac{1}{\gamma}\right)\sqrt{\lambda-1}$$
$$\frac{1}{Q}=\frac{\sigma+(f-1)\rho}{a}+\frac{\rho f}{\gamma}\pm\tau f\left(\frac{1}{a}+\frac{1}{\gamma}\right)\sqrt{\lambda-1}$$
$$\frac{1}{P'}=\frac{(1-f)\rho'-b\sigma'}{a}+\frac{(1-b)\sigma'-f\rho'}{\gamma}\mp\tau' g\left(\frac{1}{a}+\frac{1}{\gamma}\right)\sqrt{\lambda'-1}$$
$$\frac{1}{Q'}=\frac{(1-f)\sigma'-b\rho'}{a}+\frac{(1-b)\rho'-f\sigma'}{\gamma}\pm\tau' g\left(\frac{1}{a}+\frac{1}{\gamma}\right)\sqrt{\lambda'-1}$$

et pro lente tertia

$$\frac{1}{P''}=\frac{b.\rho''}{a}+\frac{(b-1)\rho''+\sigma''}{\gamma}\mp\tau''.b\left(\frac{1}{a}+\frac{1}{\gamma}\right)\sqrt{\lambda''-1}$$
$$\frac{1}{Q''}=\frac{b.\sigma''}{a}+\frac{(b-1)\sigma''+\rho''}{\gamma}\pm\tau''.b\left(\frac{1}{a}+\frac{1}{\gamma}\right)\sqrt{\lambda''-1}$$

Spatium

Spatium vero diffuſionis iam ita exprimetur

$$\gamma\gamma\, xx\left(\tfrac{1}{a}+\tfrac{1}{\gamma}\right)\left\{\begin{array}{l}\left(\tfrac{1}{a}+\tfrac{1}{\gamma}\right)^2\left(\mu\lambda f^3+\mu'\lambda' g^3+\mu''\lambda''.b^3\right)\\ +\tfrac{\nu\mu f}{a}\left(\tfrac{f-1}{a}+\tfrac{f}{\gamma}\right)\\ +\nu'\mu' g\left(\tfrac{1-f}{a}-\tfrac{f}{\gamma}\right)\left(\tfrac{1-b}{\gamma}-\tfrac{b}{a}\right)\\ +\tfrac{\nu''\mu''.b}{\gamma}\left(\tfrac{b}{a}+\tfrac{b-1}{\gamma}\right)\end{array}\right.$$

Nunc igitur circa has lentes triplicatas ſequentia ſunt obſeruanda

I. Diuerſa media refringentia eum tantum in finem adhiberi ſolent, vt non ſolum ſpatium diffuſionis hic determinatum ad nihilum redigatur, ſed etiam confuſio a diuerſa radiorum refrangibilitate oriunda tollatur, quippe quod per lentes eiusdem refractionis obtineri nequit. Infra autem videbimus ad hanc conditionem implendam requiri, vt ſit $\zeta f+\eta g+\vartheta b=0$, exiſtente $\zeta=\frac{dn}{n-1}$; $\eta=\frac{dn'}{n'-1}$ et $\vartheta=\frac{dn''}{n''-1}$; ex quibus formis iam perſpicitur, ſi haec differentialia dn, dn' et dn'' eſſent ipſis $n-1$; $n'-1$; $n''-1$ proportionalia, vti *Newtonus* ſtatuerat; tum proditura eſſe $\zeta=\eta=\vartheta$ ſiue $f+g+b=0$; at iam vidimus, eſſe debere $f+g+b=1$; quare ſi *Newtoni* ſententia eſſet vera; tum ne quidem diuerſis refractionibus adhibendis diffuſioni a diuerſa refrangibilitate oriundae remedium adferri poſſet. Eatenus igitur tantum hoc incommodum vitari poterit, quatenus litterae ζ, η et ϑ ſunt diuerſae, ita, vt ſimul eſſe poſſit, et $f+g+b=1$ et $\zeta f+\eta g+\vartheta b=0$ ex quo perſpicuum eſt, quantitatum f, g et b vnam

vel adeo duas eſſe debere negatiuas ſicque hac conditione ſuperaddita caſus ille principalis, quo omnes tres litterae f, g et b poſitiuae ſunt aſſumtae hic locum inuenire nequit.

II. Vt ergo noſtrum ſpatium diffuſionis euaneſcat, ſatisfaciendum eſt huic aequationi

$$\left(\frac{1}{a}+\frac{1}{\gamma}\right)^2 (\mu\lambda f^3+\mu'\lambda' g^3+\mu''\lambda'' b^3)$$
$$+\frac{\nu\mu f}{a}\left(\frac{f-1}{a}+\frac{f}{\gamma}\right)$$
$$+\nu'\mu' g\left(\frac{1-f}{a}-\frac{f}{\gamma}\right)\left(\frac{1-b}{\gamma}-\frac{b}{a}\right)$$
$$+\frac{\nu''\mu'' b}{\gamma}\left(\frac{b}{a}+\frac{b-1}{\gamma}\right)=0$$

Vnde vel λ vel λ' vel λ'' quaeri poteſt, dummodo caueatur, ne valor vel vnitate minor vel nimis magnus prodeat, quia prius naturae repugnat; alterum autem, quia conſtructio lentium fieret nimis lubrica. Hoc autem praeſtito circa binas reliquas litteras λ nihil amplius definitur neque etiam circa litteras f, g et b, praeterquam quod ſupra allatis conditionibus $f+g+b=1$ et $\zeta f+\eta g+\vartheta b=0$ continetur, vnde ex data vna harum litterarum duae reliquae ſponte definiuntur ſequenti ſcilicet modo:

$$g=\frac{(\vartheta-\zeta)f-\vartheta}{\eta-\vartheta}; \text{ et}$$
$$b=\frac{(\zeta-\eta)f+\eta}{\eta-\vartheta}$$

III. In praxi autem huiusmodi lentes triplicatae ideo potiſſimum quaeruntur, vt loco lentis obiectiuae

ctiuae in telefcopiis fubftitui queant, pro quibus eft $a=\infty$. Statuamus ergo ftatim $a=\infty$ et pro radiis facierum fingularum lentium habebimus

$$\frac{1}{F}=-\frac{\sigma f}{\gamma}\mp\frac{\tau f}{\gamma}\sqrt{\lambda-1};\ \frac{1}{G}=\frac{\rho f}{\gamma}\pm\frac{\tau f}{\gamma}\sqrt{\lambda-1}$$

pro fecunda lente

$$\frac{1}{F'}=\frac{(1-b)\sigma'-f\rho'}{\gamma}\mp\frac{\tau' g}{\gamma}\sqrt{\lambda'-1};\ \frac{1}{G'}=\frac{(1-b)\rho'-f\sigma'}{\gamma}\pm\frac{\tau' g}{\gamma}\sqrt{\lambda'-1}$$

pro tertia lente

$$\frac{1}{F''}=-\frac{(b-1)\rho''+\sigma''}{\gamma}\mp\frac{\tau''.b}{\gamma}\sqrt{\lambda''-1};\ \frac{1}{G''}=\frac{(b-1)\sigma''+\rho''}{\gamma}\pm\frac{\tau''.b}{\gamma}\sqrt{\lambda''-1}$$

Spatium autem diffufionis tum ita exprimetur

$$\frac{x^2}{\gamma}(\mu\lambda f^3+\mu'\lambda' g^3+\mu''\lambda'' b^3-\nu'\mu'.fg(1-b)-\nu''\mu''.b(1-b)$$

Ita vt fatisfieri oporteat huic aequationi

$$\mu\lambda f^3+\mu'\lambda' g^3+\mu''\lambda'' b^3-\nu'\mu'.fg(1-b)-\nu''\mu''.b(1-b)=0.$$

Vnde vnus valorum λ, λ', λ'', qui ad vfum commodiffimus videtur, determinari debet.

IV. Si tantum lentibus vitreis vti velimus, fufficiet duas tantum vitri fpecies adhiberi; fi igitur ftatuamus, lentem tertiam et primam ex eadem vitri fpecie parari, vt $\mu''=\mu$; $\nu''=\nu$; $\rho''=\rho$; $\sigma''=\sigma$; $\tau''=\tau$ et $\zeta=\vartheta$ ob $n''=n$; pro litteris autem f, g et b hae determinationes habebuntur, 1°. ex aequatione

$$\zeta f+\eta g+\zeta b=0 \text{ fit } f+b=\frac{-\eta}{\zeta}.g,$$

quo valore fubftituto fiet

$$g\frac{(\zeta-\eta)}{\zeta}=1;\ g=\frac{\zeta}{\zeta-\eta} \text{ ideoque } f+b=\frac{-\eta}{\zeta-\eta};$$

 vnde

vnde patet, prouti littera g fuerit vel positiua vel negatiua, fore vicissim summam $f+b$ vel negatiuam vel positiuam et aequatio resoluenda iam erit

$$\mu(\lambda f^3+\lambda'' b^3)+\mu'\lambda' g^3-\frac{\mu'\nu'.\zeta}{\zeta-\eta}.f(1-b)-\mu\nu.b(1-b)=0$$

Hinc igitur elicitur

$$\mu'\lambda' g^3=-\mu(\lambda f^3+\lambda'' b^3)+\frac{\mu'\nu'.\zeta}{\zeta-\eta}.f(1-b)+\mu\nu.b(1-b)$$

cui facile erit pro casu quouis proposito satisfacere.

IV. Dum lens prima et tertia ex eadem vitri specie parantur, media constet ex aqua vel alia materia fluida, vt lens triplicata intra duas lentes vitreas contineat fluidum, et quia fluidum plerumque minorem refractionem patitur, quam vitrum; erit $n' < n$ indeque porro $\eta < \zeta$. Quia igitur pro hoc casu fit g positiuum seu lens aquea conuexa, lentes vitreae vel ambae vel vna saltem debent esse concauae.

Inprimis autem praeter determinationes iam inventas necesse est, vt radius faciei anterioris pro lente media aequalis et contrarius sit radio faciei posterioris lentis primae, eodemque modo radius faciei posterioris aequalis et contrarius radio faciei anterioris lentis tertiae vnde hae aequalitates nascentur

$$\frac{1}{F'}=\frac{-1}{G}, \text{ seu } F'+G=0; \text{ et } \frac{1}{G'}=\frac{-1}{F''} \text{ seu } F''+G'=0.$$

Ideoque satisfieri oportet istis aequationibus:

$$(1-b)\sigma'-f\varrho'\mp\tau' g.\sqrt{\lambda'-1}=-\varrho f\mp\tau f\sqrt{\lambda-1} \text{ et}$$

$$(1-b)\varrho'-f\sigma'\pm\tau'.g.\sqrt{\lambda'-1}=(1-b)\varrho-\sigma\mp\tau'' b\sqrt{\lambda''-1}$$

En

En igitur duas conditiones, quibus satisfieri oportet, vnde vel numeri λ et λ'' vel alter eorum cum alterutra litterarum f et b definiri debent; quem in finem probe obseruandum est, formulas $\sqrt{\lambda - 1}$ et $\sqrt{\lambda'' - 1}$ pro lubitu siue positiuas siue negatiuas assumi posse, neque a se inuicem pendere. Pro formula autem $\sqrt{\lambda' - 1}$ notandum est, si ea in priore aequatione positiue ponatur, in posteriore necessario negatiue sumi debere et viciſſim.

Problema 8.

152. Determinare eas lentes quadruplicatas ad datas distantias determinatrices $AE = a$ et $dI = \delta$ accommodatas, quae minimum spatium diffusionis Ii producant.

Tab. II. Fig. 9.

Solutio.

Prima lens PP ad distantias determinatrices $AE = a$ et $AF = \alpha$ cum numero λ construatur, secunda QQ ad distantias $b = -\alpha$ et $AG = \mathfrak{b}$ cum numero λ', tertia RR ad distantias $c = -\mathfrak{b}$ et $AH = \gamma$ cum numero λ'': et quarta SS ad distantias $d = -\gamma$ et $dI = \delta$ cum numero λ''' construatur: vbi scilicet crassitiem lentium vt euanescentem spectamus. Posito iam semidiametro aperturae lentis x, sola prima lens PP produceret spatium diffusionis

$$Ff = \mu \alpha \alpha x x \left(\tfrac{1}{a} + \tfrac{1}{\alpha}\right)\left(\lambda\left(\tfrac{1}{a} + \tfrac{1}{\alpha}\right)^2 + \tfrac{\nu}{a\alpha}\right)$$

Adiun-

Adiuncta autem secunda lente QQ, positoque

$$\frac{1}{a} = \frac{-1+f}{a} + \frac{f}{\beta} \text{ et } \frac{1}{b} = \frac{1-f}{a} - \frac{f}{\beta}$$

si breuitatis gratia statuamus

$$\lambda^{(2)} = \lambda f^3 + \lambda'(1-f)^3 - \nu f(1-f)$$

vidimus fore spatium diffusionis:

$$Gg = \mu\beta\beta xx\left(\frac{1}{a}+\frac{1}{\beta}\right)\left(\lambda^{(2)}\left(\frac{1}{a}+\frac{1}{\beta}\right)^2 + \frac{\nu}{a\beta}\right)$$

Adiungatur insuper tertia lens RR, numerusque g ita sumatur vt sit

$$\frac{1}{\beta} = \frac{-1+g}{a} + \frac{g}{\gamma} \text{ et } \frac{1}{c} = \frac{1-g}{a} - \frac{g}{\gamma}$$

ac si breuitatis ergo ponamus:

$$\lambda^{(3)} = \lambda^{(2)} g^3 + \lambda''(1-g)^3 - \nu g(1-g)$$

erit spatium diffusionis:

$$Hh = \mu\gamma\gamma xx\left(\frac{1}{a}+\frac{1}{\gamma}\right)\left(\lambda^{(3)}\left(\frac{1}{a}+\frac{1}{\gamma}\right)^2 + \frac{\nu}{a\gamma}\right)$$

Nunc denique adiungatur lens quarta SS, et numero h ita in calculum introducto, vt sit

$$\frac{1}{\gamma} = \frac{-1+h}{a} + \frac{h}{\delta} \text{ et } \frac{1}{d} = \frac{1-h}{a} - \frac{h}{\delta}$$

si simili modo ponamus

$$\lambda^{(4)} = \lambda^{(3)} h^3 + \lambda'''(1-h)^3 - \nu h(1-h)$$

erit spatium diffusionis a lente quadruplicata productum:

$$Ii = \mu\delta\delta xx\left(\frac{1}{a}+\frac{1}{\delta}\right)\left(\lambda^{(4)}\left(\frac{1}{a}+\frac{1}{\delta}\right)^2 + \frac{\nu}{a\delta}\right)$$

quod igitur minimum reddi debet. Hunc in finem considerentur numeri λ, λ', λ'' et λ''' vt dati, et quae-

rantur

mantur idonei valores pro numeris f, g, et h: atque vt valor $\lambda^{(4)}$ minimus euadat, necesse est quoque valores $\lambda^{(2)}$ et $\lambda^{(3)}$ minimos fieri. Incipiamus ergo a valore $\lambda^{(2)}$ qui minimus redditur fumendo

$$f = \frac{\sqrt{(3\lambda'+\nu)}}{\sqrt{(3\lambda+\nu)}+\sqrt{(3\lambda'+\nu)}} \text{ feu } \frac{1}{f} = 1+\sqrt{\frac{3\lambda+\nu}{3\lambda'+\nu}}$$

vnde fit

$$\frac{1}{\sqrt{(3\lambda^{(2)}+\nu)}} = \frac{1}{\sqrt{(3\lambda+\nu)}} + \frac{1}{\sqrt{(3\lambda'+\nu)}}.$$

Deinde numerus $\lambda^{(3)}$ minimum induet valorem capiendo

$$g = \frac{\sqrt{(3\lambda''+\nu)}}{\sqrt{(3\lambda^{(2)}+\nu)}+\sqrt{(3\lambda''+\nu)}} \text{ feu } \frac{1}{g} = 1+\sqrt{\frac{3\lambda^{(2)}+\nu}{3\lambda''+\nu}}$$

hincque colligitur

$$\frac{1}{\sqrt{(3\lambda^{(3)}+\nu)}} = \frac{1}{\sqrt{(3\lambda^{(2)}+\nu)}} + \frac{1}{\sqrt{(3\lambda''+\nu)}}$$

Denique numerus $\lambda^{(4)}$ ideoque et fpatium diffufionis Ii minimum efficietur fumendo

$$h = \frac{\sqrt{(3\lambda'''+\nu)}}{\sqrt{(3\lambda^{(3)}+\nu)}+\sqrt{(3\lambda'''+\nu)}} \text{ feu } \frac{1}{h} = 1+\sqrt{\frac{3\lambda^{(3)}+\nu}{3\lambda'''+\nu}}$$

vnde obtinetur

$$\frac{1}{\sqrt{(3\lambda^{(4)}+\nu)}} = \frac{1}{\sqrt{(3\lambda^{(3)}+\nu)}} + \frac{1}{\sqrt{(3\lambda'''+\nu)}}.$$

Quod si hic valores ante inuentos substituamus, nanciscemur

$$\frac{1}{\sqrt{(3\lambda^{(4)}+\nu)}}=\frac{1}{\sqrt{(3\lambda+\nu)}}+\frac{1}{\sqrt{(3\lambda'+\nu)}}+\frac{1}{\sqrt{(3\lambda''+\nu)}}+\frac{1}{\sqrt{(3\lambda'''+\nu)}}.$$

Pro praecedentibus vero erit

$$\frac{1}{\sqrt{(3\lambda^{(3)}+\nu)}}=\frac{1}{\sqrt{(3\lambda+\nu)}}+\frac{1}{\sqrt{(3\lambda'+\nu)}}+\frac{1}{\sqrt{(3\lambda''+\nu)}};$$

et

$$\frac{1}{\sqrt{(3\lambda^{(2)}+\nu)}}=\frac{1}{\sqrt{(3\lambda+\nu)}}+\frac{1}{\sqrt{(3\lambda'+\nu)}}.$$

Tum vero ex his porro consequimur:

$$f=\sqrt{\frac{3\lambda^{(2)}+\nu}{3\lambda+\nu}},\ g=\sqrt{\frac{3\lambda^{(3)}+\nu}{3\lambda^{(2)}+\nu}},\ b=\sqrt{\frac{3\lambda^{(4)}+\nu}{3\lambda^{(3)}+\nu}}$$

Superest ergo vt constructionem singularum lentium luculentius exponamus: et earum distantias determinatrices per solas propositas a et δ exprimamus: erit igitur

$$\frac{1}{\gamma}=\frac{-1+b}{a}+\frac{b}{\delta};\quad \frac{1}{d}=\frac{1-b}{a}-\frac{b}{\delta}$$

$$\frac{1}{\epsilon}=\frac{-1+gb}{a}+\frac{gb}{\delta};\quad \frac{1}{c}=\frac{1-gb}{a}-\frac{gb}{\delta}$$

$$\frac{1}{\alpha}=\frac{-1+fgb}{a}+\frac{fgb}{\delta};\quad \frac{1}{b}=\frac{1-fgb}{a}-\frac{fgb}{\delta}$$

Ex superioribus vero formulis colligitur:

$$b=\sqrt{\frac{3\lambda^{(4)}+\nu}{3\lambda^{(3)}+\nu}};\ g=\sqrt{\frac{3\lambda^{(3)}+\nu}{3\lambda^{(2)}+\nu}};\ f=\sqrt{\frac{3\lambda^{(2)}+\nu}{3\lambda+\nu}}$$

vnde

vnde fit

$$gh = \sqrt{\frac{3\lambda^{(4)}+\nu}{3\lambda^{(2)}+\nu}} \text{ et } fgh = \sqrt{\frac{3\lambda^{(4)}+\nu}{3\lambda+\nu}}.$$

et superiores valores ita exprimi poterunt

$$\frac{1}{\alpha} = -\frac{1}{b} = -\frac{\sqrt{(3\lambda^{(4)}+\nu)}}{a}\left(\frac{1}{\sqrt{(3\lambda'+\nu)}}+\frac{1}{\sqrt{(3\lambda''+\nu)}}+\frac{1}{\sqrt{(3\lambda'''+\nu)}}\right)+\frac{\sqrt{(3\lambda^{(4)}+\nu)}}{\delta}\cdot\frac{1}{\sqrt{(3\lambda+\nu)}}.$$

$$\frac{1}{\beta} = -\frac{1}{c} = -\frac{\sqrt{(3\lambda^{(4)}+\nu)}}{a}\left(\frac{1}{\sqrt{(3\lambda''+\nu)}}+\frac{1}{\sqrt{(3\lambda'''+\nu)}}\right)+\frac{\sqrt{(3\lambda^{(4)}+\nu)}}{\delta}\left(\frac{1}{\sqrt{(3\lambda+\nu)}}+\frac{1}{\sqrt{(3\lambda'+\nu)}}\right)$$

$$\frac{1}{\gamma} = -\frac{1}{d} = -\frac{\sqrt{(3\lambda^{(4)}+\nu)}}{a}\cdot\frac{1}{\sqrt{(3\lambda'''+\nu)}}+\frac{\sqrt{(3\lambda^{(4)}+\nu)}}{\delta}\left(\frac{1}{\sqrt{(3\lambda+\nu)}}+\frac{1}{\sqrt{(3\lambda'+\nu)}}+\frac{1}{\sqrt{(3\lambda''+\nu)}}\right).$$

Coroll. I.

153. Si pro lentibus simplicibus numeri λ, λ' λ'' et λ''' sumantur inter se aequales, fiet pro minimo spatio diffusionis:

$$\sqrt{(3\lambda^{(2)}+\nu)} = \tfrac{1}{2}\sqrt{(3\lambda+\nu)};\quad \lambda^{(2)} = \frac{3\lambda - 1\cdot3\nu}{3\cdot4}$$

$$\sqrt{(3\lambda^{(3)}+\nu)} = \tfrac{1}{3}\sqrt{(3\lambda+\nu)};\quad \lambda^{(3)} = \frac{3\lambda - 2\cdot4\nu}{3\cdot9}$$

$$\sqrt{(3\lambda^{(4)}+\nu)} = \tfrac{1}{4}\sqrt{(3\lambda+\nu)};\quad \lambda^{(4)} = \frac{3\lambda - 3\cdot5\nu}{3\cdot16}$$

hinc $f = \frac{1}{2}$; $g = \frac{2}{3}$; $h = \frac{3}{4}$, et pro constructione lentium simplicium:

$$\tfrac{1}{\alpha} = -\tfrac{3}{4a}+\tfrac{1}{4\delta};\quad \tfrac{1}{b} = +\tfrac{3}{4a}-\tfrac{1}{4\delta}$$

$$\tfrac{1}{\beta} = -\tfrac{2}{4a}+\tfrac{2}{4\delta};\quad \tfrac{1}{c} = +\tfrac{2}{4a}-\tfrac{2}{4\delta}$$

$$\tfrac{1}{\gamma} = -\tfrac{1}{4a}+\tfrac{3}{4\delta};\quad \tfrac{1}{d} = +\tfrac{1}{4a}-\tfrac{3}{4\delta}$$

Coroll. 2.

154. Hinc ex §. 91 ſequens quatuor lentium ſimplicium conſtructio obtinetur:

Pro lente radius faciei

Prima PP $\begin{cases} \text{anterioris} = \dfrac{4\alpha\delta}{(4\rho-3\sigma)\delta+\sigma\alpha\pm\tau(\alpha+\delta)\surd(\lambda-1)} \\ \text{poſterioris} = \dfrac{4\alpha\delta}{(4\sigma-3\rho)\delta+\rho\alpha\mp\tau(\alpha+\delta)\surd(\lambda-1)} \end{cases}$

Secunda QQ $\begin{cases} \text{anterioris} = \dfrac{4\alpha\delta}{(3\rho-2\sigma)\delta+(2\sigma-\rho)\alpha\pm\tau(\alpha+\delta)\surd(\lambda-1)} \\ \text{poſterioris} = \dfrac{4\alpha\delta}{(3\sigma-2\rho)\delta+(2\rho-\sigma)\alpha\mp\tau(\alpha+\delta)\surd(\lambda-1)} \end{cases}$

Tertia RR $\begin{cases} \text{anterioris} = \dfrac{4\alpha\delta}{(2\rho-\sigma)\delta+(3\sigma-2\rho)\alpha\pm\tau(\alpha+\delta)\surd(\lambda-1)} \\ \text{poſterioris} = \dfrac{4\alpha\delta}{(2\sigma-\rho)\delta+(3\rho-2\sigma)\alpha\pm\tau(\alpha+\delta)\surd(\lambda-1)} \end{cases}$

Quarta SS $\begin{cases} \text{anterioris} = \dfrac{4\alpha\delta}{\rho\delta+(4\sigma-3\rho)\alpha\pm\tau(\alpha+\delta)\surd(\lambda-1)} \\ \text{poſterioris} = \dfrac{4\alpha\delta}{\sigma\delta+(4\rho-3\sigma)\alpha\mp\tau(\alpha+\delta)\surd(\lambda-1)} \end{cases}$

Coroll. 3.

155. Si praeterea numeri λ, λ', λ'', λ''' vnitati aequales ſtatuantur, qui eſt valor minimus, quem recipere poſſunt erit: ob $\nu = 0,232692$

$$\lambda^{(2)} = \frac{3-3\nu}{3\cdot 4} = 0,191827$$

$$\lambda^{(3)} = \frac{3-8\nu}{3\cdot 9} = 0,042165$$

$$\lambda^{(4)} = \frac{3-15\nu}{3\cdot 16} = -0,010216$$

ſicque pro lente quadruplicata valor numeri $\lambda^{(4)}$ adeo infra nihilum deprimitur.

Coroll.

Coroll. 4.

156. Maiorem ergo valorem numeris $\lambda, \lambda', \lambda'', \lambda'''$ tribuendo effici poterit, vt valor ipsius $\lambda^{(*)}$ praecise nihilo aequalis prodeat; quippe hoc fiet sumendo $\lambda = 5\nu = 1,163460$. Hinc habebitur $\lambda - 1 = 0,163460$ et $\tau\sqrt{(\lambda - 1)} = 0,365947$, vnde singulae lentes simplices duplici modo per formulas exhibitas construi poterunt.

Scholion 1.

157. Si inter se comparemus hos duos casus, quibus est vel $\lambda^{(*)} = -0,010216$ vel $\lambda^{(*)} = 0$, videmus, etiamsi discrimen vix partem centesimam vnitatis superet, in constructione tamen lentium simplicium satis magnum discrimen deprehendi, cum denominatores formularum, (154) siue augeri siue diminui debeant quantitate: $0,365947\ (a + \delta)$; quae differentia maior est, quam errores, qui forte ab artifice non nimis rudi committi queant. Ex quo vicissim colligimus, etiamsi in constructione harum lentium quadruplicatarum ab artifice leues errores committantur, inde vix perceptibilem effectum in spatio diffusionis vel valore ipsius $\lambda^{(*)}$ esse metuendum, quam ob causam hae lentes imprimis ad praxin accommodatae videntur. Si scilicet opus fuerit lente, pro qua valor ipsius λ in nihilum abeat, multo magis his lentibus quadruplicatis Coroll. 4 descriptis erit vtendum, quam triplicatis, quas supra definiui. Quin etiam eas lentes quadruplicatas adhibere licebit,

in quibus eſt $\lambda^{(4)} = -0,010216$, quia hic numerus vix nihilo eſt minor; ac ſi in conſtructione a praeſcriptis menſuris aberretur, ille valor adhuc propius ad nihilum perducatur, ita vt hoc caſu adeo errores commiſſi ſcopo magis attingendo inſeruiant. Infra autem videbimus, plurimos dari caſus, quibus eiusmodi lentibus vti conueniat, pro quibus valor ipſius λ non ſolum ſit nihilo aequalis, ſed etiam tantillum infra nihilum deprimatur; tunc optimo cum ſucceſſu huiusmodi lentes quadruplicatas in vſum vocabimus. Sin autem eiusmodi lentes ſufficiant, pro quibus valor ipſius λ ſit $0,042165$ vel aliquantillum excedat, lentes triplicatae erunt commendandae, quarum conſtructionem ſupra (§. 136) dedimus; quemadmodum duplicatae negotium conficient ſi non opus fuerit minore valore ipſius λ quam $0,191827$. Tota autem inſtrumentorum dioptricorum perfectio in hoc maxime conſiſtit, vt lentes habeantur, pro quibus valor ipſius λ ſit quam minimus, cum eae ſint aptiſſimae ad confuſionem penitus tollendam; ex quo lentium quadruplicatarum hic deſcriptarum vſus erit ampliſſimus.

Scholion 2.

158. Si haec, quae de lentibus quadruplicatis hic tradidimus, attente conſiderentur, facile patebit, quomodo lentes quintuplicatae magisque multiplicatae ad vſum ſint accommodandae. Eiusmodi ſcilicet ſemper conſtructione erit opus, quae a natura minimi parumper

rumper recedat, quoniam hoc modo errores in praxi commissi effectum propositum minime perturbant. Cum autem vix vnquam vsu veniat, vt lentibus opus sit, pro quibus valor numeri λ magis vltra nihilum imminuatur, superfluum foret, constructionem lentium quintuplicatarum magisue multiplicatarum vlterius prosequi. Interim tamen iuuabit, si pro huiusmodi lentibus numeri λ lentium numero conuenienter signis $\lambda^{(5)}$ $^{(6)}$ etc. indicentur, eorum valores, quos ex natura minimi recipiunt, exposuisse. Sumamus omnes numeros λ, λ', λ'', λ''', λ'''', λ''''' etc. lentibus simplicibus respondentes vnitati aequales, et eorum qui lentibus multiplicatis conueniunt ex natura minimi, ita se habebunt.

Pro Lente

Solitaria . . . $\lambda^{(1)} = \frac{3 - 0\nu}{3 \cdot 1} = 1,000000$

Duplicata . . . $\lambda^{(2)} = \frac{3 - 3\nu}{3 \cdot 4} = 0,191827$

Triplicata . . . $\lambda^{(3)} = \frac{3 - 8\nu}{3 \cdot 9} = 0,042165$

Quadruplicata $\lambda^{(4)} = \frac{3 - 15\nu}{3 \cdot 16} = -0,010216$

Quintuplicata $\lambda^{(5)} = \frac{3 - 24\nu}{3 \cdot 25} = -0,034461$

Sextuplicata $\lambda^{(6)} = \frac{3 - 35\nu}{3 \cdot 36} = -0,047632$

Septuplicata $\lambda^{(7)} = \frac{3 - 48\nu}{3 \cdot 49} = -0,055573$

Octuplicata $\lambda^{(8)} = \frac{3 - 63\nu}{3 \cdot 64} = -0,060727$

Noncuplicata $\lambda^{(9)} = \frac{3 - 80\nu}{3 \cdot 81} = -0,064261$

Decuplicata $\lambda^{(10)} = \frac{3 - 99\nu}{3 \cdot 100} = -0,066788$

ac

ac si huiusmodi lentes in infinitum multiplicentur; valor numeri respondentis $\lambda^{(\infty)}$ erit $= -\frac{1}{3} = -0{,}07564$ ita vt nunquam infra hunc numerum deprimi possit: ex quo patet vix vnquam casum existere posse, quo lente saltem quintuplicata opus esset. Ceterum etiam in genere, quicunque alii valores praeter vnitatem, numeris λ, λ', λ'', λ''' etc. tribuantur, ex formulis superioribus numeri $\lambda^{(5)}$, $\lambda^{(6)}$ etc. facile deriuabuntur; quin etiam distantiae determinatrices singularum lentium simplicium indidem sine difficultate definientur, cum lex progressionis satis sit manifesta. Verum per totum hoc caput probe tenendum est, vbique crassitiem lentium tanquam euanescentem esse consideratam; in sequente autem capite iterum lentes in genere non neglecta crassitie sumus contemplaturi.

CAPVT

CAPVT IV.

DE CONFVSIONE VISIONIS NEC NON DE MAGNITVDINE APPARENTE ET CLARITATE.

Definitio I.

159.

Visio est distincta, si omnes radii, qui ex quolibet obiecti puncto in oculum ingrediuntur in fundo oculi super retina iterum in vnum punctum congregantur.

Scholion.

160. Ad visionem distinctam requiritur, vt obiecta in certa quadam ab oculo distantia reperiantur, quae distantia pro varia oculorum indole maxime solet esse diuersa, dum Myopes exiguam, ii qui oculis valent ingentem, ac presbytes non solum infinitam sed quandoque etiam negatiuam exigunt; cuiusmodi distantia cum in veris obiectis locum habere nequeat, ope perspicillorum sibi satisfacere solent. Quilibet ergo oculus ad certam quandam distantiam obiectorum est instructus quam eius distantiam iustam appellabo; vbi quidem insignis latitudo locum habet,

propterea quod ſtructura oculi ita eſt artificioſa, vt contractione ac elongatione quadam ſe ad diſtantias aliquanto maiores et minores accommodare poſſit. Quando ergo obiecta in diſtantia ab oculo iuſta reperiuntur, viſio eſt diſtincta, dum ſingula obiectorum puncta ſuper retina ſingulis punctis exprimuntur.

Definitio 2.

161. Viſio eſt confuſa, ſi radii ex quolibet obiecti puncto in oculum immiſſi non in vno retinae puncto congregantur, ſed per aliquod ſpatium retinam afficiunt.

Corollarium I.

162. Eo maior ergo erit confuſio, quo maius fuerit hoc ſpatium in retina, per quod radii ex eodem obiecti puncto emiſſi diſſipantur. Ex quo huius ſpatii magnitudo veram confuſionis menſuram ſuppeditabit.

Coroll. 2.

163. Viſio ergo erit confuſa, cum obiecti viſi diſtantia multum fuerit diuerſa ab oculi diſtantia iuſta. Paruum enim diſcrimen vel per ſe nullam confuſionem parit vel oculus ſe ad obiecti diſtantiam, ſua qua pollet, volubilitate accommodare valet.

Scholion.

164. Si obiecta per lentes diſtincte repraeſentarentur, viſio imaginum eadem lege teneretur atque ipſo-

ipſorum obiectorum; iuſta ſcilicet earum ab oculo diſtantia viſionem diſtinctam, admodum autem diuerſa confuſam produceret. Verum ſi imago per aliquod ſpatium fuerit diffuſa etiamſi ab oculo ad diſtantiam iuſtam ſit remota, inde tamen in viſione confuſio oriatur neceſſe eſt. Quod ſi nempe non obiecta ipſa, ſed earum imagines per vnam pluresue lentes repraeſentatas intueamur ob duplicem cauſam viſio poterit eſſe confuſa; altera, ſi diſtantia imaginis ab oculo multum fuerit diuerſa a diſtantia iuſta, altera vero ſi ipſa imago per aliquod ſpatium diffundatur. Priorem quidem cauſam tollere in noſtra eſt poteſtate ſiquidem lentes ita diſponere licet, vt imaginem in iuſta ab oculo diſtantia exhibeant; quam lentium diſpoſitionem propterea hic perpetuo aſſumamus. Quamobrem in hoc capite inueſtigare conſtitui quanta confuſio in viſione imaginum a ſpatio diffuſionis oriri debeat, eiusque quantitate determinata deinceps in hoc erit elaborandum quemadmodum lentes formatas ac diſpoſitas eſſe oporteat, vt confuſio inde in viſione nata datum limitem, quo adhuc eſt tolerabilis, non excedat. In genere quidem certum eſt, quo maius fuerit ſpatium diffuſionis, eo maiorem inde in viſionem induci debere confuſionem, verum tamen mox videbimus, confuſionem viſionis non eſſe ſpatio diffuſionis proportionalem, ſed aliam omnino legem ſequi, quam accurate determinaſſe maximi erit momenti, cum ex hoc fonte conſtructio omnium inſtrumentorum Dioptricorum ad viſionem accommodatorum ſit repetenda.

 Pro-

Problema I.

Tab. II. Fig. 10.

165. Si oculus aspiciat imaginem cuiuspiam obiecti ab vna pluribusue lentibus per spatium Ll diffusam, atque imago principalis L in iusta ab oculo distantia reperiatur, definire confusionem, qua visio huius imaginis afficietur.

Solutio.

In spatio diffusionis Ll, in quo punctum L a radiis per lentium medium, punctum l vero a radiis per lentium extremitates transmissis exhibeatur, considerari oportet primo eius magnitudinem Ll deinde radiorum in l concurrentium inclinationem ad axem. Supra autem vidimus has res a primae lentis apertura, cuius semidiameter sit $=x$, ita pendere vt sit ipsum spatium $Ll = \mathrm{V}xx$, et angulus $OlT = \mathfrak{V}x$ pro quouis scilicet lentium numero valores horum coefficientium V et $\mathfrak{V}$ determinauimus. Repraesentet iam circulus OV oculum, quem hic vt exiguam cameram obscuram spectare licet, sed ita perfectam, vt radios ex vno puncto emissos iterum in vno puncto colligat, etsi enim in oculo plures fiunt refractiones, tamen conditione illa seruata vnica lens earum loco considerari potest, quae sit in TOT, a qua retina remota sit interuallo $OV = u$. Cum nunc imago principalis L in debita ab oculo distantia, quae vocetur $OL = l$ existat, puncti L imago in oculo in ipsam retinam incidet, et in V distin-

cte

ſte depingetur: puncti vero l imago non in V, ſed ante retinam in v referetur, quod interuallum Vv, ſi lentis in TOT conceptae craſſities euaneſcat, ſecundum §. 62 ita exprimitur, vt ſit $Vv = \frac{OV^2}{OL^2}.\, Ll = \frac{uu}{ll}.\, Vxx.$ Cum autem radii punctum l formantes ad axem inclinati ſint angulo $OlT = \mathfrak{B}x$, ii per lentis oculi puncta TT ita intrabunt, vt neglecto interuallo Ll prae diſtantia $OL = l$, ſit $OT = l\mathfrak{B}x$, ex quo ii in puncto v concurrentes cum axe angulum facient $OvT = \frac{OT}{Ov} = \frac{OT}{OV} = \frac{l}{u}\mathfrak{B}x$. Hinc ergo vltra ad retinam pergentes ſuper ea circulum UU effingent cuius radius erit $VU = \frac{l}{u}\mathfrak{B}x.\, Vv = \frac{l}{u}.\, \mathfrak{B}x.\, \frac{uu}{ll}.\, Vx^2x$ et a punctis inter L et l mediis hoc ſpatium circulare ſuper retina replebitur. Quare imago per ſpatium Ll diffuſa ſuper retina circello repraeſentabitur, cuius radius $VU = \frac{u}{l}\mathfrak{B}\, Vx^2$ qui circellus veram confuſionis menſuram ſupeditat. Quodlibet ſcilicet obiecti punctum, quod lentibus per ſpatium Ll diffuſum exhibetur, in oculo ſuper retina non puncto ſed circello exprimetur cuius radius erit $= \frac{u}{l}.\, \mathfrak{B}.\, Vx^2$.

Coroll. I.

166. Videmus ergo confuſionem, qua viſio afficitur non ſolum a quantitate ſpatii diffuſionis $Ll = Vxx$, ſed inſuper ab angulo, quo radii in l concurrentes ad axem inclinantur, qui eſt $= \mathfrak{B}x$ pendere; radiumque circelli confuſionem metientis producto illius ſpatii per hunc angulum eſſe proportionalem.

Coroll. 2.

167. Cum igitur posito semidiametro aperturae primae lentis $=x$, spatium diffusionis sit vt eius quadratum xx; radius circelli confusionem metientis est vt eius cubus x^3. Ac si confusio ipsa areae huius circelli proportionalis aestimetur erit ea vt x^6, seu vt cubus aperturae primae lentis.

Coroll. 3.

168 Hinc ergo intelligitur quanti intersit per idoneam lentium dispositionem spatium diffusionis Vxx diminuisse non solum enim in eadem ratione qua spatium diffusionis Vxx seu quantitas V diminuitur sed adeo in ratione duplicata, ipsa confusio visa minor redditur.

Scholion.

169. Assumsi hic pupillam tam late patere, vt radios ab l diuergentes recipiat; at si apertura pupillae minor esset, radios a puncto l venientes nequidem caperet: hoc ergo casu res eodem rediret ac si contracta primae lentis apertura spatium diffusionis Ll eo vsque diminueretur quoad pupilla omnes radios ab imagine emissos recipere posset hocque casu manifestum est, confusionem minorem esse prodituram. Verum in sequentibus ostendetur, tam in Telescopiis quam Microscopiis hunc casum vix vnquam locum inuenire cum plerumque conus radiosus ex puncto l emissus circa ingressum in oculum multo

to

to tenuior fit quam pupillae apertura; quamobrem ne opus quidem est hunc casum etsi per se facile expediretur, hic expendere. Ceterum tametsi oculus non commode cum lente simplici cuius crassities evanescat, conferri possit hincque expressio spatii V v secundum §. 84. aliquantum diuersa prodire potuisset hic ad istam circumstantiam non attendimus propterea quod spatium V v secundum certam quandam rationem auctum vel diminutum prodiisset; Hic autem non adeo necesse est ipsam quantitatem absolutam confusionis cognouisse, dummodo rationem quam sequitur accurate definiuerimus. Cum enim nostrum calculum cum experientia contulerimus, terminum cognoscemus, quem si confusio nostro more expressa superauerit, intolerabilis euadat; hincque perpetuo sufficiet confusionem simili modo expressam infra hunc terminum reduxisse. Interim tamen notasse iuvabit, per oculi conformationem confusionem adhuc multo minorem effici posse.

Problema 2.

Tab. II. Fig. 11.

170. Positis iisdem, quae in praecedente problemate sunt assumta, eam definire oculi conformationem, qua minima confusio percipiatur.

Solutio.

Oculus omnis facultate praeditus est sese aliquantillum aliter conformandi, vt etiam obiecta, quorum distantia a iusta non nimis differt, distincte videat:

videat: quod quomodocunque perficiatur, id ita fieri concipere licet, ac si retina ad pupillam seu potius eam lentem, quam oculi loco consideramus, propius admoueretur seu longius ab ea detorqueretur. Cum ergo ante retinam in V sitam simus contemplati, hic primo sumamus retinam per ipsum punctum v transire, ac manifestum est super ea punctum l distincte expressum iri. Verum quia punctum L alios radios nisi axi proximos non emittit etiam punctum L sine vlla confusione in v repraesentabitur; ita vt tantum puncta media inter L et l confusionem sint paritura. Consideretur ergo quoduis punctum intermedium λ, quod esset extremum in spatio confusionis, si semidiameter aperturae primae lentis minor foret quam x, qui ergo ponatur $=z$: eritque $L\lambda = Vzz$, et inclinatio radiorum in λ concurrentium ad axem $=\mathfrak{B}z$: hinc istius puncti imago intra oculum refe-retur in υ, vt sit $V\upsilon = \frac{uu}{ll}Vzz$, ideoque ob $Vv = \frac{uu}{ll}Vxx$ erit interuallum $v\upsilon = \frac{uu}{ll}V(xx-zz)$: et radiorum in υ conuergentium inclinatio ad axem $=\frac{l}{u}\mathfrak{B}z$; ex quo circelli super retina in v existentis radiis puncti λ expressi semidiameter erit $=\frac{u}{l}\mathfrak{B}Vz(xx-zz)$ qui euanescit, vti iam monuimus, siue sit $z=0$, siue $z=x$. Quaeratur iam ille valor ipsius z, quo ille circellus fiat maximus; id quod eueniet, si $xx-3zz=0$ seu $z=\frac{x}{\sqrt{3}}$; sicque circelli confusionem hoc casu metientis semidiameter erit $=\frac{2u}{3l\sqrt{3}}.\mathfrak{B}Vx^3$, quod est multo minus quam casu praecedente, quo retina erat in V.

At

At si retina aliquantillum a v versus V remoueatur, hic circellus adhuc minor effici poterit. Ponatur enim spatium $VS = s$, vt sit $vS = \frac{uu}{ll} Vxx - s$, et $S\upsilon = s - \frac{uu}{ll} Vzz$. Punctum ergo l, cuius effigies in u exhibetur, super retina S circello referetur, cuius radius est $= \frac{l}{u} \mathfrak{B} x (\frac{uu}{ll} Vxx - s)$, punctum vero λ, cuius effigies est in υ, super retina S circello, cuius radius erit $= \frac{l}{u} \mathfrak{B} z (s - \frac{uu}{ll} Vzz)$; nunc igitur id punctum λ inuestigemus, vnde iste circellus minimus euadat: quod fit si sit $s = \frac{2uu}{ll} Vzz$; eritque huius circuli radius $= \frac{u}{l} \mathfrak{B} V z^3$, qui simul confusionem exhiberet, si modo a puncto l, non maior circulus oriretur: sed substituto pro s valore inuento huius circuli radius erit, $\frac{u}{l} \mathfrak{B} V x (xx - 2zz)$, qui ergo illi $\frac{2u}{l} \mathfrak{B} V z^3$ aequalis statuatur, vnde nascitur haec aequatio:

$$x^3 - 3xzz = 2z^3, \text{ hincque } z = \tfrac{1}{2} x.$$

Quare circelli, quo minima confusio mensuratur, radius erit $= \frac{u}{4l} \mathfrak{B} V x^3$, quadruplo minor, quam si retina esset in V; indeque ipsa confusio sedecies minor. Etsi autem illa aequatio cubica tres habet radices, praeter $z = \frac{1}{2} x$ binae reliquae sunt aequales et $z = -x$, sicque $zz = xx$, vnde non minima sed quasi maxima confusio nasceretur.

Coroll. 1.

171. Cum igitur, vt oculus minimam confusionem sentiat, debeat esse $z = \frac{1}{2} x$, ideoque $s = \frac{1uu}{4ll} Vxx$,

patet retinam in eum locum S cogi debere, vt fit $VS = \frac{3}{4} Vv$ et $vS = \frac{1}{4} Vv$.

Coroll. 2.

172. Cum ergo oculus non folum praeditus fit facultate fefe parumper immutandi fed etiam hac facultate vti foleat ad confufionem euitandam, nullum eft dubium quin circellorum confufionem producentium radius fit $= \frac{11}{47} \mathfrak{B} V x^3$, ideoque quadruplo minor, quam problemate praecedente inueneramus dummodo imago fpectanda Ll propemodum in diftantia iufta reperiatur.

Coroll. 3.

173. Haec igitur expreffio $\frac{11}{47} \mathfrak{B} V x^3$ iuftam nobis exhibet menfuram confufionis, cum exprimat radium circellorum, quibus fingula obiecti puncta, quatenus id per lentes fpectatur, fuper retina repraefentantur. Si modo imago in diftantia iufta ab oculo reperiatur.

Scholion.

174. Si hi circelli euanefcerent, vifio plane effet diftincta hoc autem fieri nequit, nifi ipfum interuallum diffufionis Vxx euanefcat: vnde patet fi apertura primae lentis ad nihilum reduceretur, nullam confufionem fentiri debere. Verum vifio non ita eft delicata, vt prorfus nullam confufionem pati poffit fed dummodo confufio certum quendam terminum

num non ſuperet, quaſi eſſet nulla conſiderari poteſt. Iſte terminus, ſeu valor quem expreſſio $\frac{u}{4l}\mathfrak{B}Vx^3$ excedere non d:bet, ex exp.rientia potius peti debet, quam ex Theoria isque idcirco inſigni adhuc latitudine continetur: vnde fit vt pro diuerſo ſcopo modo maiore gradu confuſionis contenti eſſe ſoleamus, modo autem minorem gradum exigamus: quas circumſtantias cum ad praxin propius accedemus, accuratius ſumus examinaturi. Ceterum notandum eſt pro interuallo u, quo quaſi profunditas oculi exhibetur, vnum circiter pollicem aſſumi poſſe, quae quidem menſura fere erit arbitraria, cum deinceps limites confuſionis per experientiam conſtituemus.

Problema. 3.

175. Si oculus per vnicam lentem obiectum aſpiciat ita vt imago viſa ab oculo in diſtantia iuſta reperiatur, definire confuſionem, qua viſio afficietur.

Tab. I. Fig. 3.

Solutio.

Sit diſtantia obiecti $E\epsilon$ ante lentem $AE = a$, imaginis vero principalis $F\zeta$ poſt lentem $BF = \alpha$, lentisque craſſities $AB = v$, quantitas vero arbitraria qua cum binis diſtantiis determinatricibus a et α lentis facies determinatur, ſit $= k$, ponatur autem breuitatis gratia $\frac{k-v}{k+v} = i$. Tum autem poſita ratione refractionis $= n$, debet eſſe lentis P.P

radius faciei anterioris $= \frac{(n-1)a(k+v)}{k+v+2na}$

radius faciei poſterioris $= \frac{(n-1)\alpha(k-v)}{k-v-2n\alpha}$

ac ſi ſemidiameter aperturae faciei anterioris ſit $=x$, poſterioris vero $> ix$ ſpatium diffuſionis Ff erit $= P\alpha\alpha xx$ exiſtente

$$P = \frac{n}{2(n-1)^2}\left(\frac{1}{ii}\left(\frac{n}{a}+\frac{2}{k+v}\right)\left(\frac{1}{a}+\frac{1}{k+v}\right)^2 + ii\left(\frac{n}{\alpha}-\frac{2}{k-v}\right)\left(\frac{1}{\alpha}-\frac{1}{k-v}\right)^2\right)$$

radiorum vero in f concurrentium inclinatio ad axem $= i.\,\frac{x}{\alpha}$. His, quae §. 86 ſunt ſtabilita, praemiſſis, ſit diſtantia oculi poſt lentem $BO = O$, et quia imago $F\zeta$ ante oculum in diſtantia iuſta exiſtere aſſumitur, erit $O = \alpha + l$, ideoque $\alpha = O - l$, ſi locus oculi vt datus conſideretur. Hinc ergo habebimus $V = P\alpha\alpha$ et $\mathfrak{B} = \frac{i}{\alpha}$ ideoque $\mathfrak{B}V = i\alpha P$. Conſequenter radius circellorum in oculo confuſionem metientium, ſeu vt in poſterum loquemur menſura confuſionis erit

$$= \frac{n}{4l}.\, i\alpha x^3.\, P = \frac{in}{4l}(O-l)x^3 P.$$

Si lentis craſſities v euaneſcat, et pro determinatione lentis numerus arbitrarius λ loco k introducatur, erit ex §. 91 $P = \mu\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left(\lambda\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2+\frac{\nu}{a\alpha}\right)$, et $i = 1$ ex quo loco citato etiam ipſa lentis conſtructio eſt petenda.

Coroll. I.

176. Si igitur lens fuerit data, locus oculi poſt lentem ita definitur, vt debeat eſſe diſtantia $BO = O = \alpha + l$, exiſtente l diſtantia oculi iuſta: Sin autem locus oculi detur, pro lentis conſtructione diſtantia determinatrix α ita capi debet, vt ſit $\alpha = O - l$.

Coroll.

Coroll. 2.

177. Quare si oculus ita fuerit comparatus, vt exigat distantiam iustam $l = \infty$, fiet $\alpha = -\infty$, et mensura confusionis erit $= -\frac{1}{4} i u x^3$. P seu $= \frac{1}{4} i u x^3$. P, quia signum $-$ nihil mutat in magnitudine circellorum confusionem producentium.

Coroll. 3.

178. Etsi ergo hoc casu quo $\alpha = \infty$ spatium diffusionis Ff est infinitum, tamen inde confusio in visione orta est finita, quia hoc non obstante valor ipsius P manet finitus: erit enim

$$P = \frac{n}{2(n-1)^2}\left(\frac{1}{ii}\left(\frac{n}{a} + \frac{2}{k+v}\right)\left(\frac{1}{a} + \frac{2}{k+v}\right)^2 - \frac{8ii}{(k-v)^3}\right)$$

Radius autem faciei posterioris fit $= -\frac{(n-1)}{2n}(k-v)$.

Coroll. 4.

179. Quia est $i = \frac{k-v}{k+v}$, valor ipsius P etiam in genere ita exprimi potest, vt sit:

$$P = \frac{n}{2(n-1)^2}\left(\left(\frac{n}{a} + \frac{2}{k+v}\right)\left(\frac{1}{ia} + \frac{2}{k-v}\right)^2 + \left(\frac{n}{a} - \frac{2}{k-v}\right)\left(\frac{i}{a} - \frac{2}{k+v}\right)^2\right)$$

vbi est $\frac{n}{2(n-1)^2} = \frac{310}{121} = 2,561983$ ob $n = \frac{31}{20}$. Sicque etiam valores reliquarum litterarum Q, R, S etc. in §. 86 transformari poterunt.

Problema 4.

180. Si oculus per duas lentes obiectum $E\epsilon$ aspiciat, ita vt imago per eas repraesentata $G\gamma$ in distantia iusta $OG = l$ ob oculo sit remota, definire confusionem, qua visio afficietur.

Tab. I. Fig. 5.

T 3 Solutio

Solutio.

Sint vt hactenus pro lente PP distantiae determinatrices $AE = a$, $aF = \alpha$, crassities $Aa = v$, et distantia arbitraria $= k$, ita vt sit

radius faciei anterioris $= \frac{(n-1)a(k+v)}{k+v+2na}$

radius faciei posterioris $= \frac{(n-1)\alpha(k-v)}{k-v-2n\alpha}$

tum vero posito $\frac{k-v}{k+v} = i$ ponatur

$$P = \frac{n}{2(n-1)^2}\left(\left(\frac{n}{a}+\frac{2}{k+v}\right)\left(\frac{1}{ia}+\frac{2}{k-v}\right)^2+\left(\frac{n}{\alpha}-\frac{2}{k-v}\right)\left(\frac{i}{\alpha}-\frac{2}{k+v}\right)^2\right)$$

Deinde pro lente posteriori QQ sint distantiae determinatrices $BF = b$, $bG = \beta$, crassities lentis $Bb = v'$ et distantia arbitraria $= k'$ vt sit

radius faciei anterioris $= \frac{(n-1)b(k'+v')}{k'+v'+2nb}$

radius faciei posterioris $= \frac{(n-1)\beta(k'-v')}{k'-v'-2n\beta}$

tum vero posito $\frac{k'-v'}{k'+v'} = i'$ ponatur

$$Q = \frac{n}{2(n-1)^2}\left(\left(\frac{n}{b}+\frac{2}{k'+v'}\right)\left(\frac{1}{i'b}+\frac{2}{k'-v'}\right)^2+\left(\frac{n}{\beta}-\frac{2}{k'-v'}\right)\left(\frac{i'}{\beta}-\frac{2}{k'+v'}\right)^2\right)$$

Atque iam erit spatium diffusionis

$$Gg = \beta\beta xx\left(\frac{1}{i'i'}\cdot\frac{\alpha\alpha}{bb}P + ii\cdot\frac{bb}{\alpha\alpha}Q\right) = Vxx$$

et radiorum in g concurrentium inclinatio ad axem

$$ii'\cdot\frac{bx}{\alpha\beta} = \mathfrak{B}x.$$

Sit iam oculus in O existente $OG = l$, ac ponatur eius post lentem QQ distantia $bO = O$ erit $O = \beta + l$, ideoque

ideoque $\mathfrak{E} = O - l$; quibus positis cum sit mensura confusionis $= \frac{u}{4l} \mathfrak{B} x . V x x$ erit ea pro nostro casu:

$$\frac{ii'u}{4\,l} \cdot \frac{b\mathfrak{E}}{\alpha} x^3 \left(\frac{1}{i'i'} \cdot \frac{\alpha\alpha}{bb} P + ii \, \frac{bb}{\alpha\alpha} Q \right)$$

vel pro $\mathfrak{E}$ posito valore $O - l$ et signo mutato

$$\tfrac{1}{4} i i' u \left(1 - \frac{O}{l} \right) . \frac{b}{\alpha} x^3 \left(\frac{1}{i'i'} \cdot \frac{\alpha\alpha}{bb} P + ii . \frac{bb}{\alpha\alpha} Q \right).$$

Coroll. 1.

181. Si ergo pro oculo fuerit $l = \infty$, ideoque et $\mathfrak{E} = \infty$, etsi spatium diffusionis Gg sit infinitum tamen ob $\frac{O}{l} = 0$ confusio visionem afficiens nihilominus erit finita: neque enim P neque Q ob $\mathfrak{E} = \infty$ fit infinita.

Coroll. 2.

182. Si crassities lentium euanescat, vt sit $v = 0$ et $v' = 0$ erit $i = 1$ et $i' = 1$. ideoque hoc casu mensura confusionis

$$\tfrac{1}{4} u \left(1 - \frac{O}{l} \right) . \frac{b}{\alpha} x^3 \left(\frac{\alpha\alpha}{bb} P + \frac{bb}{\alpha\alpha} Q \right)$$

At si loco k et k' introducantur numeri λ et λ' erit

$$P = \mu \left(\frac{1}{a} + \frac{1}{\alpha} \right) \left(\lambda \left(\frac{1}{a} + \frac{1}{\alpha} \right)^2 + \frac{\nu}{a\alpha} \right) \text{ et}$$

$$Q = \mu \left(\frac{1}{b} + \frac{1}{\mathfrak{E}} \right) \left(\lambda' \left(\frac{1}{b} + \frac{1}{\mathfrak{E}} \right)^2 + \frac{\nu}{b\mathfrak{E}} \right)$$

existente

$$\mu = 0,938191 \text{ et } \nu = 0,232692$$

Coroll.

Coroll. 3.

183. Eodem autem hoc casu constructio binarum lentium ita se habebit:

radius faciei

Pro lente PP $\begin{cases} \text{anterioris} = \frac{a\alpha}{\varrho\alpha+\sigma a \pm \tau(a+\alpha)\surd(\lambda-1)} \\ \text{posterioris} = \frac{a\alpha}{\varrho a+\sigma\alpha \mp \tau(a+\alpha)\surd(\lambda-1)} \end{cases}$

Pro lente QQ $\begin{cases} \text{anterioris} = \frac{b\beta}{\varrho\beta+\sigma b \pm \tau(b+\beta)\surd(\lambda'-1)} \\ \text{posterioris} = \frac{b\beta}{\varrho b+\sigma\beta \mp \tau(b+\beta)\surd(\lambda'-1)} \end{cases}$

existente $\varrho=0,190781$; $\sigma=1,627401$ et $\tau=0,905133$.

Problema 5.

Tab. II. Fig. 6.

184. Si oculus per tres lentes PP, QQ et RR obiectum $E\varepsilon$ aspiciat, ita vt imago per eas repraesentata $H\theta$ in distantia iusta $OH=l$ ab oculo O sit remota, definire confusionem, qua visio afficietur.

Solutio.

Positis duabus prioribus lentibus PP et QQ vt in problemate praecedente, indeque determinatis valoribus P et Q cadat imago per has duas lentes repraesentata principalis in $G\eta$, post quam tertia lens RR ita collocata sit, vt sint eius distantiae determinatrices $CG=c$, $cH=\gamma$, crassities $Cc=v''$ et quantitas arbitraria $=k''$ vt sit:

radius faciei anterioris $=\frac{(n-1)c(k''+v'')}{k''+v''+2nc}$

radius faciei posterioris $=\frac{(n-1)\gamma(k''-v'')}{k''-v''-2n\gamma}$

Tum

Tum vero poſito $\frac{k''-v''}{k''+v''}=i''$ ponatur

$$R=\frac{n}{2(n-1)}\left(\left(\frac{n}{c}+\frac{2}{k''+v''}\right)\left(\frac{1}{i''c}+\frac{2}{k''-v''}\right)^2+\left(\frac{n}{\gamma}-\frac{2}{k''-v''}\right)\left(\frac{i''}{\gamma}-\frac{2}{k''+v''}\right)^2\right)$$

atque iam ſpatium diffuſionis erit

$$Hb=\gamma\gamma xx\left(\frac{1}{i'i'.i''i''}\frac{\alpha\alpha\mathfrak{C}\mathfrak{C}}{bbcc}P+\frac{ii}{i''i''}.\frac{bb\mathfrak{C}\mathfrak{C}}{\alpha\alpha cc}Q+ii.i'i'.\frac{bbcc}{\alpha\alpha\mathfrak{C}\mathfrak{C}}R\right)$$

quod eſt valor ipſius $\mathfrak{V}xx$. Radiorum vero in b concurrentium inclinatio ad axem eſt

$$ii'i''.\frac{bcx}{\alpha\mathfrak{C}\gamma}=\mathfrak{B}x$$

ſit iam oculus in O, ac ponatur eius diſtantia poſt lentem RR $=$ O erit $O=\gamma+l$, ideoque $\gamma=O-l$. Hinc menſura confuſionis in oculo ortae colligitur

$$\tfrac{1}{4}ii'i''u\left(1-\frac{O}{l}\right)\frac{bc}{\alpha\mathfrak{C}}x^3\left(\frac{1}{i'i'.i''i''}.\frac{\alpha\alpha\mathfrak{C}\mathfrak{C}}{bbcc}P+\frac{ii}{i''i''}\frac{bb\mathfrak{C}\mathfrak{C}}{\alpha\alpha cc}Q+ii.i'i'.\frac{bbcc}{\alpha\alpha\mathfrak{C}\mathfrak{C}}R\right)$$

Coroll. 1.

185. Hic iterum vt ante patet, ſi fuerit $l=\infty$ ideoque et $\gamma=\infty$, quo caſu diffuſionis ſpatium in infinitum extenditur, menſuram confuſionis terminis finitis contineri, quod etiam locum habet pro quouis lentium numero.

Coroll. 2.

186. Si lentium craſſities pro euaneſcente habeatur, ob $i=1$ $i'=1$, $i''=1$, menſura confuſionis ita ſimplicius exprimetur, vt ſit

$$=\tfrac{1}{4}u\left(1-\frac{O}{l}\right)\frac{bc}{\alpha\mathfrak{C}}x^3\left(\frac{\alpha\alpha\mathfrak{C}\mathfrak{C}}{bbcc}P+\frac{bb\mathfrak{C}\mathfrak{C}}{\alpha\alpha cc}Q+\frac{bbcc}{\alpha\alpha\mathfrak{C}\mathfrak{C}}R\right)$$

At hoc casu loco k, k', k'' introductis numeris $\lambda, \lambda', \lambda''$ erit

$$P = \mu\left(\tfrac{1}{a}+\tfrac{1}{\alpha}\right)\left(\lambda\left(\tfrac{1}{a}+\tfrac{1}{\alpha}\right)^2+\tfrac{\nu}{a\alpha}\right)$$

$$Q = \mu\left(\tfrac{1}{b}+\tfrac{1}{\mathfrak{b}}\right)\left(\lambda'\left(\tfrac{1}{b}+\tfrac{1}{\mathfrak{b}}\right)^2+\tfrac{\nu}{b\mathfrak{b}}\right)$$

$$R = \mu\left(\tfrac{1}{c}+\tfrac{1}{\gamma}\right)\left(\lambda''\left(\tfrac{1}{c}+\tfrac{1}{\gamma}\right)^2+\tfrac{\nu}{c\gamma}\right)$$

Coroll. 3.

187. Eodem vero casu constructio lentium per istos numeros $\lambda, \lambda', \lambda''$ ita erit dirigenda vt sit

Radius faciei

Pro lente PP $\begin{cases} \text{anterioris} = \dfrac{a\alpha}{\rho\alpha+\sigma a \pm \tau(a+\alpha)\sqrt{(\lambda-1)}} \\ \text{posterioris} = \dfrac{a\alpha}{\rho a+\sigma\alpha \mp \tau(a+\alpha)\sqrt{(\lambda-1)}} \end{cases}$

Pro lente QQ $\begin{cases} \text{anterioris} = \dfrac{b\mathfrak{b}}{\rho\mathfrak{b}+\sigma b \pm \tau(b+\mathfrak{b})\sqrt{(\lambda'-1)}} \\ \text{posterioris} = \dfrac{b\mathfrak{b}}{\rho b+\sigma\mathfrak{b} \mp \tau(b+\mathfrak{b})\sqrt{(\lambda'-1)}} \end{cases}$

Pro lente RR $\begin{cases} \text{anterioris} = \dfrac{c\gamma}{\rho\gamma+\sigma c \pm \tau(c+\gamma)\sqrt{(\lambda''-1)}} \\ \text{posterioris} = \dfrac{c\gamma}{\rho c+\sigma\gamma \mp \tau(c+\gamma)\sqrt{(\lambda''-1)}} \end{cases}$

Scholion.

188. Hinc satis manifestum est, quemadmodum hae formulae pro pluribus lentibus progrediantur: verum antequam eas exponam, conueniet alias quoque circumstantias, quae hinc facillime deducuntur, perpendi, scilicet magnitudinem obiecti visam, et copiam radiorum a singulis eius punctis in oculum transmissorum vt hae simul cum confusione visionis dein-

deinceps coniunctim pro quouis lentium numero exhiberi queant; quo pacto plures taediosas repetitiones euitabimus. Duae autem res ex hactenus allatis facile definiri possunt, quarum altera est quantitas, qua imago obiecti per lentes repraesentata ab oculo cernitur, quae quantitas aestimanda est ex angulo, sub quo imago videtur, vt is deinceps comparari possit, cum eo angulo sub quo ipsum obiectum in data distantia a nudo oculo spectaretur, vnde qua ratione magnitudo per lentes visa augeatur, intelligetur. Altera res in copia radiorum, a singulis obiecti punctis in oculum transmissorum versatur, qua claritas visione percepta continetur, a quolibet scilicet puncto conus seu cylindrus radiosus in oculum ingreditur, qui si pupillam penitus expleat, claritas ad summum gradum erit euecta, nisi forte maiori illustratione ipsi obiecto maius lumen concilietur. At si sectio illius coni aut cylindri, qua in oculum intrat, minor fuerit pupilla, in eadem ratione claritas decrescet; quod cum in omnibus instrumentis dioptricis, quibus magnitudinem visam vehementer augere propositum est, vsu venire soleat, plurimum intererit amplitudinem illius coni seu cylindri, qua in oculum penetrat, accurate determinasse.

Problema 6.

189 Definire quantitatem, sub qua quaeuis obiecti portio per lentes quotcunque ab oculo in distantia iusta ab imagine vltima remoto cernetur.

Solutio.

Sit z linea in obiecto concepta, quae quanta per lentes oculo fit apparitura, definiri oporteat. Ostensum autem est in praecedentibus, quotcunque fuerint lentes, imaginem principalem huius lineae iterum esse lineam, cuius longitudo ad z certam teneat rationem a distantiis determinatricibus lentium et numeris i, i', i'', i''' etc. pendentem (86). Sit ergo haec longitudo imaginis $= Mz$, quae cum ab oculo in distantia l remoto aspiciatur, apparebit sub angulo $= \frac{Mz}{l}$; vel cuius tangens potius fit $= \frac{Mz}{l}$; sed quia hic angulus rarissime vltra aliquot gradus assurgere solet, tangens tuto pro ipso arcu assumitur. Effigies autem quae ab hac linea Mz in oculo exprimitur, erit $= \frac{Muz}{l}$; hic enim cogitationes a confusione abstraho, qua vtique fit, vt effigies maior imprimatur, propterea quod singula puncta circellis exhibentur. Iam positis iisdem lentium determinationibus, quibus supra §. 86 sum vsus, pro vario lentium numero angulus, sub quo linea z in obiecto sumta cernetur ita se habebit:

	Angulus visionis	situ
Pro vnica lente	$1 \cdot \frac{\alpha}{a} \cdot \frac{z}{l}$	inuerso
Pro duabus lentibus	$\frac{1}{ii'} \cdot \frac{\alpha\beta}{ab} \cdot \frac{z}{l}$	erecto
Pro tribus lentibus	$\frac{1}{ii'i''} \cdot \frac{\alpha\beta\gamma}{abc} \cdot \frac{z}{l}$	inuerso
Pro quatuor lentibus	$\frac{1}{ii'i''i'''} \cdot \frac{\alpha\beta\gamma\delta}{abcd} \cdot \frac{z}{l}$	erecto
	etc.	

Scilicet

Scilicet si hae formulae fiat positiuae, oculus lineam z situ siue erecto siue inuerso videbit, prout est notatum sin autem fuerint negatiuae, situm indicatum in contrarium verti oportet.

Coroll. 1.

190. Si eadem obiecti linea z in distantia $=b$ ab oculo nudo cerneretur, ea appraritura esset sub angulo $=\frac{z}{b}$, vnde perspicitur, quanto ea vel maior vel minor per lentes videatur.

Coroll. 2.

191. Si distantia oculi a postrema lente ponatur $=O$, erit casu vnius lentis $\alpha=O-l$, ideoque $\frac{\alpha}{l}=-(1-\frac{O}{l})$ vnde angulus opticus lineae obiecti z respondens erit $=\frac{1}{l}(1-\frac{O}{l})\frac{z}{a}$ pro situ erecto, quia signum mutauimus.

Coroll. 3.

192. Simili modo casu duarum lentium ob $\beta=O-l$ erit iste angulus $=\frac{1}{ll'}(1-\frac{O}{l})\frac{\alpha z}{ab}$ pro situ inuerso.

Casu vero trium lentium ob $\gamma=O-l$ erit

iste angulus $=\frac{1}{ll'l''}(1-\frac{O}{l})\frac{\alpha\beta z}{abc}$ pro situ erecto

Casu quatuor lentium ob $\delta=O-l$ erit

iste angulus $\frac{1}{ll'l''l'''}(1-\frac{O}{l})\frac{\alpha\beta\gamma z}{abcd}$ pro situ inuerso, et ita porro pro pluribus lentibus.

Scholion.

193. Hinc etiam modus se offert confusionem ob lentium aperturam in oculo natam distinctius aestimandi. Scilicet cum singula obiecti puncta in oculo exprimantur circulis, quorum radius est $=\frac{u}{4l}\mathfrak{B}Vx^3$, verus autem circulus cuius radius $=z$, oculo in distantia l expositus in oculo referatur circulo cuius radius $=\frac{u}{l}z$ singula puncta illius obiecti per lentes spectati, aeque magna apparebunt, ac orbes circulares radii $z=\frac{1}{4}\mathfrak{B}Vx^3$, si in distantia ab oculo $=l$ spectarentur. Vel cum horum orbium semidiameter apparens sit $=\frac{z}{l}$, ob confusionem singula obiecti puncta instar circulorum videbuntur, quorum semidiameter apparens esset $=\frac{1}{4l}\mathfrak{B}Vx^3$. In expressionibus igitur ante pro confusione inuentis deleatur quantitas u, et habebitur semidiameter apparens circulorum confusionem exprimentium. Hinc iudicari poterit quam parua esse debeat confusio, vt non amplius sentiatur, scilicet si oculus non amplius percipere valeat spatium circulare, cuius semidiameter esset 1″, seu $\frac{1}{205}$ pars radii circiter, euidens est, si fuerit nostra expressio $\frac{1}{4l}\mathfrak{B}Vx^3=\frac{1}{205}$, confusionem fore imperceptibilem. Ac experientiam consulentes deprehendimus multo maiores angulos non amplius percipi posse, ita vt confusio non sit metuenda etiamsi expressio $\frac{1}{4l}\mathfrak{B}Vx^3$ notabiliter maior fuerit, quam $\frac{1}{205}$; ne autem hic temere quicquam statuamus, ponamus limitem, quem formula $\frac{1}{4l}\mathfrak{B}Vx^3$ excedere

non

non debeat, esse $= \frac{1}{4\varkappa^3}$, ita vt esse oporteat $\frac{1}{4}\mathfrak{B} V x^3 < \frac{1}{\varkappa^3}$. Postea igitur ad praxin descendentes poterimus pro $\varkappa$ numerum vel 40 vel minorem assumere prout experientia quouis casu postulauerit. Quod si ergo hoc modo confusionis rationem habeamus, profunditas oculi a non amplius in computum ingredietur.

Definitio 3.

194. Semidiameter confusionis, est semidiameter apparens circuli, qui ab oculo aeque magnus videtur, ac singula obiecti puncta ipsi ob confusionem apparent.

Corollarium.

195. Inueniemus igitur facile semidiametrum confusionis si formulas supra pro confusione repertas per profunditatem oculi a diuidamus quo pacto eae formulae ad numeros absolutos reducentur.

Definitio 4.

196. Multiplicatio per lentes producta ex ratione quantitatis, qua obiecta per lentes spectantur, ad quantitatem, qua eadem obiecta in data distantia ab oculo nudo cernerentur aestimatur. Exponens autem multiplicationis inuenitur, si magnitudo, qua linea quaecunque in obiecto concepta per lentes videtur, diuidatur per magnitudinem, qua eadem linea in data distantia ab oculo nudo spectata esset apparitura.

Coroll.

Coroll. 1.

197. Inuoluit ergo diiudicatio multiplicationis distantiam quandam fixam, in qua rem obiecta a nudo oculo aspici assumimus quae prout diuersa assumatur, multiplicatio alio atque alio modo exprimetur.

Coroll. 2.

198. Si haec distantia fixa, ex qua multiplicatio diiudicatur ponatur $= b$ et exponens multiplicationis $= m$, sit linea quaepiam in obiecto concepta $= z$, quae ergo nudo oculo in distantia b appareret sub angulo $= \frac{z}{b}$; eadem autem linea per lentes spectetur sub angulo $= \frac{Mz}{l}$ (88) ex quo erit exponens multiplicationis $m = \frac{Mb}{l}$.

Coroll. 3.

199. In ratione ergo $m : 1$ dimensiones lineares per lentes augeri sunt censendae; vnde superficies auctae apparebunt in ratione $mm : 1$, et ipsa corpora in ratione $m^3 : 1$. Cum exponente autem multiplicationis coniungi debet situs, quo obiecta apparent siue is sit erectus siue inuersus.

Scholion.

200. Distantia haec fixa b ad quam multiplicatio refertur non eodem modo perpetuo assumi solet, quippe quod etiam pro diuersitate obiectorum omnino fieri non posset. Nam si per lentes obiecta valde

valde remota veluti coeleſtia contuemur, quoniam ea nunquam in diſtantia modica ſpectare ſolemus, conuenit vtique magnitudinem per lentes viſam cum ea comparare, qua in ea ipſa a nobis diſtantia nudis oculis cernerentur: ideoque his caſibus diſtantia fixa b ipſi diſtantiae a, qua a lentibus obiecta ſunt remota, aequalis conſtitui ſolet. Scilicet ſi diſtantia a fuerit valde magna, qui eſt caſus Teleſcopiorum, ſtatuitur $b = a$, et magnitudo per haec inſtrumenta viſa, cum magnitudine per nudos oculos viſa in eadem diſtantia commodiſſime comparatur. Sic de Teleſcopiis dicitur, quoties diametri corporum coeleſtium multiplicentur, eoque caſu littera m exponentem huius rationis indicabit. Sin autem obiecta propiora contemplamur, qui eſt vſus Microſcopiorum ea plerumque ita prope ad inſtrumentum admouentur, vt in tam exigua diſtantia nudis oculis nunquam diſtincte cerni poſſent: neque ergo his caſibus ſtatui $b = a$ conueniret. Aliam ergo rationem ineundo pro b ſumi ſolet eiusmodi diſtantia modica, in qua obiecta commode ac diſtincte cernere liceat, quae etſi vtique pro diuerſa oculorum indole diuerſa ſumi deberet; tamen vt aliquid fixi ſtatuatur, pro b diſtantia 8 pollicum vtpote maximae oculorum parti conveniens accipi ſolet, ita vt his caſibus definiamus, quoties huiusmodi obiecta maiora *appareant*, quam ſi eadem nudis oculis in diſtantia 8 digitorum aſpicerentur. Interim tamen ſi multiplicatio ad hanc diſtantiam fuerit relata, non difficile erit eam ad quam-

libet aliam referre, ita vt haec hypothesis naturam horum instrumentorum afficere non sit censenda.

Problema 7.

Tab. II. Fig. 10.

201. Si obiectum per lentes quotcunque aspiciatur, definire amplitudinem coni seu cylindri luminosi, qui a singulis obiecti punctis in oculum transmittitur.

Solutio.

Reperiatur vt supra oculus in distantia iusta l post imaginem postremam per lentes repraesentatam quae etsi est per spatium aliquod Ll diffusa, hic tamen a confusione inde nata mentem abstrahimus, quoniam confusionem iam seorsim determinauimus. Ex puncto igitur l oculum versus diffunditur conus luminosus, cuius radii extremi ad axem inclinati sunt angulo OlT quem posuimus supra $= \mathfrak{B}x$. Huius igitur coni sectio circa ingressum in oculum consideretur, cuius semidiameter erit $= \mathfrak{B}lx$, vnde pro vario lentium numero hic semidiameter sequenti modo definietur.

Pro vnica lente . . . . . . $il\frac{x}{\alpha}$

Pro duabus lentibus . . . . . $ii'l\frac{bx}{\alpha\beta}$

Pro tribus lentibus . . . . $ii'i''l\frac{bcx}{\alpha\beta\gamma}$

Pro quatuor lentibus . . . $ii'i''i'''l\frac{bcdx}{\alpha\beta\gamma\delta}$

etc.

hicque perinde est siue hae formulae sint positiuae siue negatiuae; quia circulus siue radio positiuo siue negatiuo describatur eiusdem prodit magnitudinis.

Coroll.

Coroll. 1.

202. Si iste semidiameter $\mathfrak{B}/x$ maior fuerit semidiametro pupillae, tota pupillae apertura radiis impletur neque propterea visio clarior procurari poterit, nisi forte ipsum obiectum fortiori illustratione splendidius reddatur.

Coroll. 2.

203. Sin autem haec quantitas $\mathfrak{B}/x$ minor fuerit semidiametro pupillae claritatis mensura existet, quae eo maior erit quo maior fuerit ista quantitas: dum contra minuta hac quantitate claritas tam exigua euadere potest, vt non amplius sensui visus excitando sufficiat.

Definitio 5.

204. Gradus claritatis per lentes perceptae commodissime definietur semidiametro coni luminosi, qui a quouis obiecti puncto in oculum transmittitur.

Coroll. 1.

205. Gradus ergo claritatis definitur quantitate supra inuenta $\mathfrak{B}/x$, ita vt si gradum claritatis ponamus $= y$ habeamus $y = \mathfrak{B}/x$, quo cognito facillime iudicabimus, quonam gradu visio sit clara habenda.

Coroll. 2.

206. Scilicet si semidiametrum pupillae ponamus $= \omega$, quamdiu fuerit $y > \omega$, claritate plena

 fruemur,

fruemur, quae nullius augmenti eſt capax niſi forte ipſam pupillam magis dilatare valeamus.

Coroll. 3.

207. At ſi fuerit $y < \omega$, claritatem vtique minorem percipiemus: ac ſi claritatem plenam vnitate deſignemus, claritas ex caſu $y < \omega$ reſultans erit $= \frac{yy}{\omega\omega}$; propterea quod copia radiorum in oculum immiſſorum eſt vt quadratum ſemidiametri y.

Coroll. 4.

208. Quod ſi gradus claritatis y eousque decreſcat, vt copia radiorum nimis ſit parua, quam vt ſenſum viſus excitare poſſit, nihil ob ſummam caliginem percipi poterit, vnde manifeſtum eſt ad viſionem requiri, vt gradus claritatis certum quempiam limitem ſuperet.

Scholion.

209. Tam in Teleſcopiis quam Microſcopiis maxime neceſſe eſt; vt obiecta certo claritatis gradu exhibeantur ne repraeſentatio nimis fiat obſcura. Hic autem gradus plurimum a lumine proprio obiectorum pendet, quae quo fuerint illuſtriora minor claritatis gradus iis ſatis clare videndis ſufficit; ideoque ſtellas illuſtriores contemplantes minori gradu claritatis contenti eſſe poſſumus: terreſtria vero obiecta multo maiorem claritatis gradum poſtulant. Quo igitur haec ad

ad omnes casus accommodare valeamus gradum claritatis hic littera indefinita y contentum in computum sum ducturus. Quamobrem his de multiplicatione et claritate praemissis, haec duo elementa simul cum confusione pro quouis lentium numero exhibebo: ac primo quidem non neglecta lentium crassitie, tum vero eadem seorsim lentium crassitie neglecta exponi conueniet.

Problema 8.

210. Si oculus per quotcunque lentes PP, QQ, RR, SS etc. obiectum Ee aspiciat; ita vt imago postrema per eas repraesentata ante oculum in iusta distantia $= l$ reperiatur, determinare tam multiplicationem et claritatem, quam confusionem, qua visio perturbabitur.

Solutio

Quotcunque fuerint lentes, sint pro singulis distantiae determinatrices, vt et crassities cum quantitate arbitraria, vt sequitur.

Pro Lente	Dist: determinatr:	Crassities	quant: arb:
Prima PP	$EA = a$; $aF = \alpha$;	$Aa = v$;	k
Secunda QQ	$FB = b$; $bG = \mathfrak{b}$;	$Bb = v'$;	k'
Tertia RR	$GC = c$; $cH = \gamma$;	$Cc = v''$;	k''
Quarta SS	$HD = d$; $dI = \delta$;	$Dd = v'''$;	k'''

etc.

atque hinc posita ratione refractionis $\frac{31}{20} = n$ constructio lentium ita se habebit;

Pro Lente	Radius Faciei anterioris	Radius Faciei posterioris
Prima PP	$\frac{(n-1)a(k+v)}{k+v+2na}$	$\frac{(n-1)\alpha(k-v)}{k-v-2n\alpha}$
Secunda QQ	$\frac{(n-1)b(k'+v')}{k'+v'+2nb}$	$\frac{(n-1)\beta(k'-v')}{k'-v'-2n\beta}$
Tertia RR	$\frac{(n-1)c(k''+v'')}{k''+v''+2nc}$	$\frac{(n-1)\gamma(k''-v'')}{k''-v''-2n\gamma}$
Quarta SS	$\frac{(n-1)d(k'''+v''')}{k'''+v'''+2nd}$	$\frac{(n-1)\delta(k'''-v''')}{k'''-v'''-2n\delta}$

etc.

Tum posito breuitatis gratia:

$$\frac{k-v}{k+v} = i;\ \frac{k'-v'}{k'+v'} = i';\ \frac{k''-v''}{k''+v''} = i'';\ \frac{k'''-v'''}{k'''+v'''} = i'''\ \text{etc.}$$

si aperturae primae lentis in facie anteriori semidiameter fuerit $= x$, tam pro facie posteriori quam pro vtraque facie singularum lentum sequentium aperturae maiores esse debent vel saltem non minores, quam sequens tabula ostendit:

Pro Lente	Semidiameter aperturae in facie anteriori	posteriori
Prima PP	x	ix
Secunda QQ	$i.\frac{bx}{a}$	$ii'.\frac{bx}{a}$
Tertia RR	$ii'.\frac{bcx}{a\beta}$	$ii'i''.\frac{bcx}{a\beta}$
Quarta SS	$ii'i''.\frac{bcdx}{a\beta\gamma}$	$ii'i''i'''.\frac{bcdx}{a\beta\gamma}$

etc.

Denique

Denique ad abbreuiandum ponatur:

$$P = \frac{n}{2(n-1)^2}\left(\left(\frac{n}{a}+\frac{2}{k+v}\right)\left(\frac{1}{ia}+\frac{2}{k-v}\right)^2+\left(\frac{n}{\alpha}-\frac{2}{k-v}\right)\left(\frac{i}{\alpha}-\frac{2}{k+v}\right)^2\right)$$

$$Q = \frac{n}{2(n-1)^2}\left(\left(\frac{n}{b}+\frac{2}{k'+v'}\right)\left(\frac{1}{i'b}+\frac{2}{k'-v'}\right)^2+\left(\frac{n}{\beta}-\frac{2}{k'-v'}\right)\left(\frac{i'}{\beta}-\frac{2}{k'+v'}\right)^2\right)$$

$$R = \frac{n}{2(n-1)^2}\left(\left(\frac{n}{c}+\frac{2}{k''+v''}\right)\left(\frac{1}{i''c}+\frac{2}{k''-v''}\right)^2+\left(\frac{n}{\gamma}-\frac{2}{k''-v''}\right)\left(\frac{i''}{\gamma}-\frac{2}{k''+v''}\right)^2\right)$$

$$S = \frac{n}{2(n-1)^2}\left(\left(\frac{n}{d}+\frac{2}{k'''+v'''}\right)\left(\frac{1}{i'''d}+\frac{2}{k'''-v'''}\right)^2+\left(\frac{n}{\delta}-\frac{2}{k'''-v'''}\right)\left(\frac{i'''}{\delta}\cdot\frac{2}{k'''+v'''}\right)^2\right)$$

etc.

His positis ponamus oculum in distantia $= O$ ab vltima lente locari ita vt post imaginem vltimam reperiatur in distantia $= l$: magnitudinem autem visam comparari cum magnitudine qua idem obiectum nudo oculo in distantia fixa $= b$ cerneretur; ac ponatur exponens multiplicationis $= m$.

Deinde pro claritate sit gradus claritatis $= y$, ita vt y indicet semidiametrum coni luminosi in oculum intrantis.

Confusio autem aestimetur per semidiametrum confusionis supra (194) definitum.

Iam pro quouis lentium numero hae tres res ita se habebunt.

I. Pro vnica lente $O = \alpha + l$

1. Exponens multiplicationis $m = \frac{1}{i}\cdot\frac{\alpha b}{\alpha l}$ situ inuerso
2. Gradus claritatis $y = il.\ \frac{x}{\alpha}$
3. Semid: Confusionis $= \frac{1}{4} i \frac{\alpha}{l} x^3 . P$

II.

II. Pro duabus Lentibus $O = \beta + l$

1. Exp. mult: $m = \frac{1}{ii'} \cdot \frac{\alpha\beta b}{abl}$ situ erecto

2. Gradus claritatis $y = ii'l \cdot \frac{bx}{\alpha\beta}$

3. Semid. Confusionis $= \frac{1}{4} ii' \cdot \frac{\beta}{l} \cdot \frac{b}{\alpha} x^3 \left(\frac{1}{i'i'} \cdot \frac{\alpha\alpha}{bb} P + ii \cdot \frac{bb}{\alpha\alpha} Q \right)$.

III. Pro tribus Lentibus $O = \gamma + l$

1. Exp. mult: $m = \frac{1}{ii'i''} \cdot \frac{\alpha\beta\gamma b}{abcl}$ situ inverso:

2. Gradus claritatis $y = ii'i''l \cdot \frac{bcx}{\alpha\beta\gamma}$

3. Semidiameter confusionis;

$$\tfrac{1}{4} ii'i'' \cdot \frac{\gamma}{l} \cdot \frac{bc}{\alpha\beta} x^3 \left(\frac{1}{i'i' \cdot i''i''} \cdot \frac{\alpha\alpha\beta\beta}{bbcc} P + \frac{ii}{i''i''} \cdot \frac{bb\beta\beta}{\alpha\alpha cc} Q + ii \cdot i'i' \cdot \frac{bbcc}{\alpha\alpha\beta\beta} R \right).$$

IV. Pro quatuor Lentibus $O = \delta + l$

1. Exp. multipl: $m = \frac{1}{ii'i''i'''} \cdot \frac{\alpha\beta\gamma\delta b}{abcdl}$ situ erecto

2. Gradus Claritatis $y = ii'i''i'''l \cdot \frac{bcdx}{\alpha\beta\gamma\delta}$

3. Semidiameter Confusionis:

$$\tfrac{1}{4} ii'i''i''' \cdot \frac{\delta}{l} \cdot \frac{bcd}{\alpha\beta\gamma} x^3 \left\{ \begin{array}{l} \frac{1}{i'i' \cdot i''i'' \cdot i'''i'''} \cdot \frac{\alpha\alpha\beta\beta\gamma\gamma}{bbccdd} P + \frac{ii}{i''i'' \cdot i'''i'''} \cdot \frac{bb\beta\beta\gamma\gamma}{\alpha\alpha ccdd} Q \\ + \frac{ii \cdot i'i'}{i'''i'''} \cdot \frac{bbcc\gamma\gamma}{\alpha\alpha\beta\beta dd} R + ii \cdot i'i' \cdot i''i'' \cdot \frac{bbccdd}{\alpha\alpha\beta\beta\gamma\gamma} S \end{array} \right\}$$

V.

V. Pro quinque Lentibus $O = \varepsilon + l$

1. Exp. multipl: $m = \frac{1}{i\,i'\,i''\,i'''\,i''''} \cdot \frac{\alpha\beta\gamma\delta\varepsilon b}{abcdel}$ situ inuerso

2. Gradus claritatis $y = i\,i'\,i''\,i'''\,i''''\,l \cdot \frac{bcdex}{\alpha\beta\gamma\delta\varepsilon}$

3. Semidiameter confusionis

$$\frac{1}{4} i\,i'\,i''\,i'''\,i'''' \cdot \frac{\varepsilon}{l} \cdot \frac{bcde}{\alpha\beta\gamma\delta} x^3 \begin{cases} + \frac{1}{i'i'.\, i''i''.\, i'''i'''.\, i''''i''''} \cdot \frac{\alpha\alpha\beta\beta\gamma\gamma\delta\delta}{bbccddee} P \\ + \frac{ii}{i''i''.\, i'''i'''.\, i''''i''''} \cdot \frac{bb\beta\beta\gamma\gamma\delta\delta}{\alpha\alpha ccddee} Q \\ + \frac{ii.\, i'i'}{i'''i'''.\, i''''i''''} \cdot \frac{bbcc\gamma\gamma\delta\delta}{\alpha\alpha\beta\beta ddee} R \\ + \frac{ii.\, i'i'.\, i''i''}{i''''i''''} \cdot \frac{bbccdd\delta\delta}{\alpha\alpha\beta\beta\gamma\gamma ee} S \\ + ii.\, i'i'.\, i''i''.\, i'''i''' \cdot \frac{bbccddee}{\alpha\alpha\beta\beta\gamma\gamma\delta\delta} T \end{cases}$$

atque hinc etiam progressus ad plures lentes est manifestus.

Coroll. 1.

211. His omnibus casibus euidens est fore generatim
$$my = \frac{bx}{a}.$$

Datis scilicet multiplicatione m cum claritate y statim definitur apertura primae lentis nempe $x = my \cdot \frac{a}{b}$. Quo maior scilicet tam multiplicatio quam claritas desideratur, eo maiorem esse oportet aperturam lentis primae.

Coroll. 2.

212. Quum autem x maius accipere non liceat, quam vt confusio infra certum limitem contineatur, dato x cum exponente multiplicationis m

defi-

definitur claritatis gradus $y = \frac{bx}{ma}$, vnde patet reliquis paribus, quo maior multiplicatio exigatur, eo minori claritate contentos nos esse oportere.

Coroll. 3.

213. Imprimis autem hic obseruandum est, has formulas aeque negotium conficere, quamcunque crassitiem lentes habuerint. *Euadent autem* tractabiliores, si lentium crassities negligatur, qui casus seorsim tractari meretur.

Problema 9.

214. Iisdem positis, quae in problemate praecedente, si lentium crassities vt euanescens consideretur, determinare tam multiplicationem et claritatem, quam confusionem, qua visio perturbabitur.

Solutio.

Haec tractatio a praecedenti in isto differt, quod lentium crassities v, v', v'' etc. euanescant, et loco quantitatum arbitrariarum k, k', k'' etc. numeri arbitrarii λ, λ', λ'' etc. in calculum introducantur. Ponatur ergo

Pro Lente		Dist. determinatr.	num. arb.
Prima	PP	$EA = a$; $aF = \alpha$	λ
Secunda	QQ	$FB = b$; $bG = \beta$	λ'
Tertia	RR	$GC = c$; $cH = \gamma$	λ''
Quarta	SS	$HD = d$; $dI = \delta$	λ'''
		etc.	

Hinc

Hinc si sit breuitatis gratia!

$\rho = 0,190781$; $\sigma = 1,627401$; et $\tau = 0,905133$
lentium constructio ita est instituenda

	Radius Faciei	
Pro Lente	anterioris	posterioris
Prima PP	$\frac{a\alpha}{\rho a + \sigma\alpha \pm \tau(a+\alpha)\sqrt{(\lambda-1)}}$	$\frac{a\alpha}{\rho\alpha + \sigma a \mp \tau(a+\alpha)\sqrt{(\lambda-1)}}$
Secunda QQ	$\frac{b\mathfrak{b}}{\rho\mathfrak{b} + \sigma b \pm \tau(b+\mathfrak{b})\sqrt{(\lambda'-1)}}$	$\frac{b\mathfrak{b}}{\rho b + \sigma\mathfrak{b} \mp \tau(b+\mathfrak{b})\sqrt{(\lambda'-1)}}$
Tertia RR	$\frac{c\gamma}{\rho\gamma + \sigma c \pm \tau(c+\gamma)\sqrt{(\lambda''-1)}}$	$\frac{c\gamma}{\rho c + \sigma\gamma \mp \tau(c+\gamma)\sqrt{(\lambda''-1)}}$
Quarta SS	$\frac{d\delta}{\rho\delta + \sigma d \pm \tau(d+\delta)\sqrt{(\lambda'''-1)}}$	$\frac{d\delta}{\rho d + \sigma\delta \mp \tau(d+\delta)\sqrt{(\lambda'''-1)}}$

Cum iam lentis primae PP semidiameter aperturae sit $= x$, et in qualibet lente vtriusque faciei eadem sit ratio, vt omnes radii per primam ingressi simul per reliquas transmittantur, apertura reliquarum sequentes limites superare debet:

Semid. aperturae
Lentis secundae QQ $> \frac{b}{\alpha}x$
Lentis tertiae RR $> \frac{bc}{\alpha\mathfrak{b}}x$
Lentis quartae SS $> \frac{bcd}{\alpha\mathfrak{b}\gamma}x$
etc.

Tum vero posito breuitatis ergo $\mu = 0,938191$ et $\nu = 0,232692$ statuatur:

$$P = \mu\left(\tfrac{1}{a}+\tfrac{1}{\alpha}\right)\left(\lambda\left(\tfrac{1}{a}+\tfrac{1}{\alpha}\right)^2+\tfrac{\nu}{a\alpha}\right)$$
$$Q = \mu\left(\tfrac{1}{b}+\tfrac{1}{\mathfrak{b}}\right)\left(\lambda'\left(\tfrac{1}{b}+\tfrac{1}{\mathfrak{b}}\right)^2+\tfrac{\nu}{b\mathfrak{b}}\right)$$
$$R = \mu\left(\tfrac{1}{c}+\tfrac{1}{\gamma}\right)\left(\lambda''\left(\tfrac{1}{c}+\tfrac{1}{\gamma}\right)^2+\tfrac{\nu}{c\gamma}\right)$$
$$S = \mu\left(\tfrac{1}{d}+\tfrac{1}{\delta}\right)\left(\lambda'''\left(\tfrac{1}{d}+\tfrac{1}{\delta}\right)^2+\tfrac{\nu}{d\delta}\right)$$

etc.

 Sit

Sit iam O diſtantia oculi poſt vltimam lentem, ita vt ab imagine poſtrema diſtet interuallo $= l$, comparetur magnitudo viſa cum ea, qua idem obiectum in diſtantia fixa b nudo oculo cerneretur, ſitque exponens multiplicationis $= m$; et gradus claritatis $= y$, quibus poſitis erit pro quouis lentium numero vt ſequitur.

I. Pro vnica lente $O = \alpha + l$

1. Exponens multiplicationis $m = \frac{\alpha b}{a l}$ ſitu inuerſo
2. Gradus claritatis $y = l. \frac{x}{\alpha}$ hinc $my = \frac{bx}{a}$
3. Semid: confuſionis $= \frac{\alpha}{4l}. x^3$ P

II. Pro duabus lentibus $O = \beta + l$

1. Exponens multiplicationis $m = \frac{\alpha\beta b}{a b l}$ ſitu erecto
2. Gradus claritatis $y = l. \frac{bx}{\alpha\beta}$, hinc $my = \frac{bx}{a}$
3. Semid: confuſionis $= \frac{\beta}{4l}. \frac{b}{\alpha} x^3 \left(\frac{\alpha\alpha}{bb} P + \frac{bb}{\alpha\alpha} Q\right)$

III. Pro tribus lentibus $O = \gamma + l$

1. Exponens multiplicationis $m = \frac{\alpha\beta\gamma b}{a b c l}$ ſitu inuerſo
2. Gradus claritatis $y = l. \frac{bcx}{\alpha\beta\gamma}$ hinc $my = \frac{bx}{a}$
3. Semidiameter confuſionis

$$\frac{\gamma}{4l}. \frac{bc}{\alpha\beta} x^3 \left(\frac{\alpha\alpha\beta\beta}{bbcc} P + \frac{bb\beta\beta}{\alpha\alpha cc} Q + \frac{bbcc}{\alpha\alpha\beta\beta} R\right).$$

IV. Pro quatuor lentibus $O = \delta + l$

1. Exponens multiplicationis $m = \frac{\alpha\beta\gamma\delta b}{a b c d l}$ ſitu erecto
2. Gradus claritatis $y = l \frac{bcdx}{\alpha\beta\gamma\delta}$ hinc $my = \frac{bx}{a}$
3. Semidiameter confuſionis:

$$\frac{\delta}{4l}. \frac{bcd}{\alpha\beta\gamma} x^3 \left(\frac{\alpha\alpha\beta\beta\gamma\gamma}{bbccdd} P + \frac{bb\beta\beta\gamma\gamma}{\alpha\alpha ccdd} Q + \frac{bbcc\gamma\gamma}{\alpha\alpha\beta\beta dd} R + \frac{bbccdd}{\alpha\alpha\beta\beta\gamma\gamma} S\right)$$

V.

V. Pro quinque lentibus $O = \varepsilon + l$

1. Exponens multipl. $m = \frac{\alpha\beta\gamma\delta\varepsilon b}{abcdel}$ situ inuerso

2. Gradus claritatis $y = l.\, \frac{bcdex}{\alpha\beta\gamma\delta\varepsilon}$ hinc $my = \frac{bx}{a}$

3. Semidiameter confusionis

$$\frac{\varepsilon}{el}.\ \frac{bcde}{\alpha\beta\gamma\delta}\, x^3 \left\{ \begin{array}{l} + \frac{\alpha\alpha\beta\beta\gamma\gamma\delta\delta}{bbccddee}\, P + \frac{bb\beta\beta\gamma\gamma\delta\delta}{\alpha\alpha ccddee}\, Q \\ + \frac{bbcc\gamma\gamma\delta\delta}{\alpha\alpha\beta\beta ddee}\, R \\ + \frac{bbccdd\delta\delta}{\alpha\alpha\beta\beta\gamma\gamma ee}\, S + \frac{bbccddee}{\alpha\alpha\beta\beta\gamma\gamma\delta\delta}\, T \end{array} \right.$$

vnde non difficile erit has formulas ad lentes etiam plures continuare.

Coroll. 1.

215. Lentes simplices adhibendo numeros $\lambda, \lambda', \lambda''$, etc. vnitate minores accipi non possunt. Verum hic nihil obstat quo minus loco lentium simplicium lentes duplicatae, triplicatae vel etiam quadruplicatae in vsum vocentur, quo pacto in his formulis numeri $\lambda, \lambda', \lambda''$, etc. non solum infra vnitatem diminui sed ad nihilum vsque perduci poterunt. Tum autem constructio lentium harum multiplicatarum ex superiori capite peti et ad distantias determinatrices hic positas accommodari debet.

Coroll. 2.

216. Veluti si pro lente prima PP debeat esse

$\lambda = 0,191827$, hanc lentem ita ex duabus componi oportet, vt fit

	Radius faciei	
Pro lente	anterioris	posterioris
Priori . .	$\frac{2a\alpha}{(2\varrho - \sigma)\alpha + \sigma a}$;	$\frac{2a\alpha}{(2\sigma - \varrho)\alpha + \varrho a}$
Posteriori	$\frac{2a\alpha}{\varrho\alpha + (2\sigma - \varrho)a}$;	$\frac{2a\alpha}{\sigma\alpha + (2\varrho - \sigma)a}$.

Coroll. 3.

217. Sin autem velimus, vt pro lente prima fit $\lambda = 0,042165$, ea erit triplicanda, hoc modo

	Radius faciei	
Pro lente	anterioris	posterioris
Priori	$\frac{3a\alpha}{(3\varrho - 2\sigma)\alpha + \sigma a}$;	$\frac{3a\alpha}{(3\sigma - 2\varrho)\alpha + \varrho a}$
Media	$\frac{3a\alpha}{(2\varrho - \sigma)\alpha + (2\sigma - \varrho)a}$;	$\frac{3a\alpha}{(2\sigma - \varrho)\alpha + (2\varrho - \sigma)a}$
Posteriori	$\frac{3a\alpha}{\varrho\alpha + (3\sigma - 2\varrho)a}$;	$\frac{3a\alpha}{\sigma\alpha + (3\varrho - 2\sigma)a}$

Coroll. 4.

218. At fi pro lente prima requiratur $\lambda = -0,010216$, quadruplicata erit vtendum ita construenda:

	Radius faciei	
Pro lente	anterioris	posterioris
Prima	$\frac{4a\alpha}{(4\varrho - 3\sigma)\alpha + \sigma a}$;	$\frac{4a\alpha}{(4\sigma - 3\varrho)\alpha + \varrho a}$
Secunda	$\frac{4a\alpha}{(3\varrho - 2\sigma)\alpha + (2\sigma - \varrho)a}$;	$\frac{4a\alpha}{(3\sigma - 2\varrho)\alpha + (2\varrho - \sigma)a}$
Tertia	$\frac{4a\alpha}{(2\varrho - \sigma)\alpha + (3\sigma - 2\varrho)a}$;	$\frac{4a\alpha}{(2\sigma - \varrho)\alpha + (3\varrho - 2\sigma)a}$
Quarta	$\frac{4a\alpha}{\varrho\alpha + (4\sigma - 3\varrho)a}$;	$\frac{4a\alpha}{\sigma\alpha + (4\varrho - 3\sigma)a}$

Coroll.

Coroll. 5.

219. Simili autem modo lens secunda QQ ex suis distantiis determinatricibus b et $\mathfrak{b}$ per multiplicationem erit construenda, numerus que ei respondens λ' debet esse vel 0,191827, vel 0,042165 vel −0,010216; quod idem de reliquis lentibus est intelligendum.

Scholion 1.

220. Alias lentium species hic nollem in praxi adhiberi, cum hae solae sine metu enormis erroris confici queant; tum vero etsi aliae ab his non admodum discrepantes fere aequo successu in praxin introduci possent, tamen quia discrimen non admodum est notabile, iis facile carere poterimus. Cum igitur sit $\varrho = 0,190781$ et $\sigma = 1,627401$, ideoque

$$\varrho = 0,190781; \quad \sigma = 1,627401$$

$$2\varrho - \sigma = -1,245839; \quad 2\sigma - \varrho = 3,064021$$

$$3\varrho - 2\sigma = -2,682459; \quad 3\sigma - 2\varrho = 4,500641$$

$$4\varrho - 3\sigma = -4,119079; \quad 4\sigma - 3\varrho = 5,937261$$

easdem constructiones in numeris euolutis exhiberi conueniet.

I. Si igitur pro lente PP debeat esse $\lambda = 1$, ea erit simplex hoc modo construenda.

Radius faciei anterioris posterioris

$$\frac{aa}{+0,190781a + 1,627401\,\alpha}; \quad \frac{aa}{+1,627401a + 0,190781\,\alpha}$$

II.

II. Si pro lente PP debeat esse $\lambda = 0,191827$ ea erit duplicata hoc modo construenda

Radius Faciei

Pro lente	anterioris	posterioris
Priori	$\frac{a\alpha}{-0,622515\alpha + 0,013700a}$;	$\frac{a\alpha}{+1,332010\alpha + 0,095350a}$
Posteriori	$\frac{a\alpha}{+0,095350\alpha + 1,332010a}$;	$\frac{a\alpha}{+0,013700\alpha - 0,622515a}$

III. Si pro lente PP debeat esse $\lambda = 0,042165$ ea erit triplicata hoc modo construenda

Radius Faciei

Pro lente	anterioris	posterioris
Priori	$\frac{a\alpha}{-0,894353\alpha + 0,342467a}$;	$\frac{a\alpha}{+1,500214\alpha + 0,063594a}$
Media	$\frac{a\alpha}{-0,415280\alpha + 1,021340a}$;	$\frac{a\alpha}{+1,021340\alpha - 0,415280a}$
Posteriori	$\frac{a\alpha}{+0,063594\alpha + 1,500214a}$;	$\frac{a\alpha}{+0,342467\alpha - 0,894353a}$

IV. Si pro lente PP debeat esse $\lambda = -0,010216$ ea erit quadruplicata hoc modo construenda

Radius Faciei

Pro lente	anterioris	posterioris
Prima	$\frac{a\alpha}{-1,025770\alpha + 0,406850a}$;	$\frac{a\alpha}{+1,484315\alpha + 0,047655a}$
Secunda	$\frac{a\alpha}{-0,670615\alpha + 0,766005a}$;	$\frac{a\alpha}{+1,125160\alpha - 0,311460a}$
Tertia	$\frac{a\alpha}{-0,311460\alpha + 1,125160a}$;	$\frac{a\alpha}{+0,766005\alpha - 0,670615a}$
Quarta	$\frac{a\alpha}{+0,047655\alpha + 1,484315a}$;	$\frac{a\alpha}{+0,406850\alpha - 1,025770a}$

Scholion

Scholion 2.

221. Interim tamen ſi lentes deſideremus, in quibus valor ipſius λ maior ſit, quam hic eſt aſſumtus, conſtructio earum leui additione ex his ipſis formulis concinnari poterit. In binis ſcilicet fractionibus, quibus radii facierum cuiusque lentis deſignantur, alter denominator augeri alter vero diminui debet eadem quantitate, quae quantitas ſemper $= \tau(a+\alpha)\sqrt{v}$ denotante v exceſſum valoris ipſius λ ſupra ante aſſumtum. Ita ſi eſſe debeat

I. $\lambda = 1+v$; II. $\lambda = 0,191827+v$; III. $\lambda = 0,042165+v$
vel IV. $\lambda = -0,010216+v$

denominatores fractionum in Scholio praecedente traditarum pro quauis lente ſimplici alternatim ſunt augendi et minuendi quantitate $0,905133\,(a+\alpha)\sqrt{v}$. Vnde ſi lens quadruplicata deſideretur, pro qua ſit praeciſe $\lambda = 0$, erit $v = 0,010216$ et $\tau\sqrt{v} = 0,091487$; hincque pro quauis lente ſimplici denominatorum alter augeri alter vero diminui debet hac quantitate

$$0,091487\alpha + 0,091487a$$

ex quo talis naſcitur conſtructio huiusmodi lentis quadruplicatae pro qua eſt $\lambda = 0$

Pro lente	Radius faciei anterioris	Radius faciei poſterioris
Prima	$\frac{a\alpha}{-1,121259\alpha+0,315363a}$;	$\frac{a\alpha}{+1,575102\alpha+0,139182a}$
Secunda	$\frac{a\alpha}{-0,762102\alpha+0,674518a}$;	$\frac{a\alpha}{+1,216647\alpha-0,215973a}$
Tertia	$\frac{a\alpha}{-0,402547\alpha+1,033673a}$;	$\frac{a\alpha}{+0,857458\alpha-0,579121a}$
Quarta	$\frac{a\alpha}{-0,043752\alpha+1,532828a}$;	$\frac{a\alpha}{+0,458537\alpha-0,538283a}$

Sed hoc caſu quaelibet lens adhuc alio modo conſtrui poteſt: ſic quarta ordine retrogrado expoſita permutatis a et α dabit alteram formam lentis primae.

Supplementum III.

Ad Problem. 8.

Si lentes ratione refractionis diſcrepent, vt ſit ratio refractionis pro prima lente $= n$, pro ſecunda $= n'$, pro tertia $= n''$; inde neque in multiplicatione m neque in gradu claritatis quicquam mutatur; at vero in ſemidiametro confuſionis valores litterarum P, Q, R ſequenti modo immutari debent.

$$P = \frac{n}{2(n-1)^2}\left\{\begin{array}{l}\left(\frac{n}{\alpha}+\frac{2}{k+\nu}\right)\left(\frac{1}{i\alpha}+\frac{2}{k-\nu}\right)^2\\+\left(\frac{n}{\alpha}-\frac{2}{k-\nu}\right)\left(\frac{i}{\alpha}-\frac{2}{k+\nu}\right)^2\end{array}\right.$$

$$Q = \frac{n'}{2(n'-1)^2}\left\{\begin{array}{l}\left(\frac{n'}{b}+\frac{2}{k'+\nu'}\right)\left(\frac{1}{i'b}+\frac{2}{k'-\nu'}\right)^2\\+\left(\frac{n'}{\beta}-\frac{2}{k'-\nu'}\right)\left(\frac{i'}{\beta}-\frac{2}{k'+\nu'}\right)^2\end{array}\right.$$

$$R = \frac{n''}{2(n''-1)^2}\left\{\begin{array}{l}\left(\frac{n''}{c}+\frac{2}{k''+\nu''}\right)\left(\frac{1}{i''c}+\frac{2}{k''-\nu''}\right)^2\\+\left(\frac{n''}{\gamma}-\frac{2}{k''-\nu''}\right)\left(\frac{i''}{\gamma}-\frac{2}{k''+\nu''}\right)^2\end{array}\right.$$

$$S = \frac{n'''}{2(n'''-1)^2}\left\{\begin{array}{l}\left(\frac{n'''}{d}+\frac{2}{k'''+\nu'''}\right)\left(\frac{1}{i'''d}+\frac{2}{k'''-\nu'''}\right)^2\\+\left(\frac{n'''}{\delta}-\frac{2}{k'''-\nu'''}\right)\left(\frac{i'''}{\delta}-\frac{2}{k'''+\nu'''}\right)^2\end{array}\right.$$

etc.

Ad Problem. 9.

Si lentes ratione refractionis diſcrepent, loco litterarum ϱ, σ, τ in radiis facierum ſcribi oportet pro

pro ſecunda lente ϱ', σ', τ'; pro tertia ϱ'', σ'', τ'', etc. tum vero expreſſiones pro multiplicatione et gradu claritatis nulla mutatione egent; pro confuſione autem notari oportet, fore

$$P = \mu\left(\tfrac{1}{a}+\tfrac{1}{\alpha}\right)\left(\lambda\left(\tfrac{1}{a}+\tfrac{1}{\alpha}\right)^2+\tfrac{\nu}{a\alpha}\right)$$

$$Q = \mu'\left(\tfrac{1}{b}+\tfrac{1}{\beta}\right)\left(\lambda'\left(\tfrac{1}{b}+\tfrac{1}{\beta}\right)^2+\tfrac{\nu'}{b\beta}\right)$$

$$R = \mu''\left(\tfrac{1}{c}+\tfrac{1}{\gamma}\right)\left(\lambda''\left(\tfrac{1}{c}+\tfrac{1}{\gamma}\right)^2+\tfrac{\nu''}{c\gamma}\right)$$

etc.

CAPVT V.

DE
CAMPO APPARENTE OCVLIQVE LOCO MAXIME IDONEO.

Problema I.

222.

Tab. II. Fig. 10.

Si ex obiecti puncto extra axem sumto ϵ radius quicunque ϵM per lentem PP transmittatur, definire eius concursum cum axe O.

Solutio.

Sit $F\zeta$ imago principalis per lentem proiecta, a diffusione enim hic mentem abstrahimus, ac ponamus pro lente eius distantias determinatrices $AE = a$, $aF = \alpha$, eius crassitiem $Aa = v$, et quantitatem arbitrariam $= k$, sitque breuitatis ergo $\frac{k-v}{k+v} = i$. His positis si puncti ϵ imago cadat in ζ, voceturque $E\epsilon = z$, erit $F\zeta = \frac{1}{i} \cdot \frac{\alpha z}{a}$ at pro puncto lentis M statuatur eius distantia ab axe $AM = x$; ac supra ostendimus si radius a puncto E per punctum M transmitteretur, eum ita per m ad punctum F progressurum esse, vt foret $am = ix$. Radius igitur ϵM per punctum N ad ζ feretur, ita vt, cum obliquitas radiorum

rum in facies lentis incidentium perpetuo valde parva statuatur, fit proxime angulus $EM\varepsilon$ ad NMm vt n ad 1 posito $n = \frac{31}{20}$. Hinc erit $E\varepsilon$ ad mN in ratione composita istorum angulorum, et distantiarum AE ad Aa, seu $E\varepsilon : mN = na : v$, vnde fit $mN = \frac{vz}{na}$, ideoque $aN = ix - \frac{vz}{na}$. Iam vero radius a puncto N recta ad ζ pergit, et propterea axem ita in O secabit, vt sit $aN + F\zeta : aF = aN : aO$ sicque

$$aO = \left(i\alpha x - \frac{\alpha vz}{na}\right) : \left(ix - \frac{vz}{na} + \frac{\alpha z}{ia}\right) = \frac{niia\alpha x - i\alpha vz}{niiax - ivz + n\alpha z}$$

et $FO = \frac{n\alpha\alpha z}{niiax - ivz + n\alpha z}$

Si x sit semidiameter aperturae lentis in facie anteriori, haec intersectio O respondet casui, quo radius ab ε per lentis terminum summum transeat: sin autem is per terminum imum transmittatur, sumto x negatiuo fiet

$$FO = \frac{n\alpha\alpha z}{-niiax - ivz + n\alpha z}$$

At si radius ex ε per centrum lentis A transeat, punctum interfectionis ita cadet in O vt sit

$$FO = \frac{n\alpha\alpha}{na - iv}$$

Coroll. 1.

223. Si igitur x denotet semidiametrum aperturae lentis in facie anteriori PMAP, vt omnes radii a puncto ε in hanc faciem incidentes per lentem transmittantur, necesse est, vt faciei posterioris

PNaP semidiameter sit maior quam $\pm ix - \frac{vz}{na}$, sumto x tam negatiuo quam positiuo. Vnde hic semidiameter minor esse nequit, quam $ix + \frac{vz}{na}$.

Coroll. 2.

224. Si apertura in facie anteriori euanescat, vt sit $x = 0$ radii a puncto ε, cuius ab axe distantia $E\varepsilon = z$, per lentem non transmittentur, nisi in facie posteriori semidiameter aperturae sit $= \frac{vz}{na}$ vel maior. Vnde patet quo maior fuerit lentis crassities v eo maiori apertura in facie posteriori esse opus.

Coroll. 3.

225. Vicissim ergo si detur lentis apertura in facie posteriori, cuius semidiameter sit $= \frac{vz}{na}$; inde simul in obiecto extremum punctum ε determinatur, a quo radius in centrum lentis A incidens per eam transmittatur.

Coroll. 4.

226. Hic autem radius per A immissus post transitum cum axe in O occurret, vt ob $x = 0$ sit interuallum $FO = \frac{naa}{na - iv}$. seu $aO = \frac{-iav}{na - iv}$. Nisi ergo oculus in hoc axis loco teneatur, radius transmissus non in oculum ingredietur; si quidem apertura pupillae vt infinite parua spectetur.

Coroll.

Coroll. 5.

227. At si semidiameter pupillae ponatur $= \omega$, oculus etiam in o positus illum radium excipiet, si fuerit $o\,\omega = \omega$. Cum autem casu $x = 0$ sit

$$aN\left(\frac{vz}{na}\right) : aO\left(\frac{i\alpha v}{n\alpha - iv}\right) = o\omega\,(\omega) : Oo \text{ erit}$$

$$Oo = \frac{nia\alpha\omega}{z(n\alpha - iv)}.$$

Quod interuallum cum aeque positiuum ac negatiuum accipi possit, pro loco oculi o habebimus

$$ao = -\frac{i\alpha vz \pm nia\alpha\omega}{z(n\alpha - iv)} = \frac{-i\alpha(vz \pm na\omega)}{z(n\alpha - iv)}$$

Scholion.

228. In hac tractatione, vbi campum apparentem et locum oculi idoneum inuestigamus, tam aperturam lentis obiectiuae in facie anteriori quam pupillae amplitudinem pro nihilo habebimus, vt quaestiones obtineamus determinatas. Quare horum elementorum, quae in instrumentis dioptricis ad visionem accommodatis maximi sunt momenti, sequentes definitiones constituemus.

Definitio I.

229. Campus apparens est spatium in obiecto, ex cuius singulis punctis radii in centrum lentis obiectiuae incidentes per reliquas lentes omnes transmittuntur. Quod spatium cum sit circulare, eius radius vocatur semidiameter campi apparentis.

Coroll.

Coroll. 1.

230. Si ergo $E\varepsilon = z$ fuerit semidiameter campi apparentis, erit ε punctum obiecti extremum, sed ab axe maxime remotum, ex quo adhuc radii in centrum A lentis obiectiuae incidentes per omnes lentes transmittuntur.

Coroll. 2.

231. Determinatur igitur magnitudo campi apparentis per aperturam sequentium facierum refringentium, ac fortasse per aperturam vnius, si scilicet radii a puncto quodam magis remoto quam ε venientes nullum transitum per eam inuenirent, etiamsi per reliquas facies transmitterentur.

Scholion.

232. Si radii ex solo puncto E, quod est centrum campi apparentis, considerentur, ii quidem qui in A incidunt, quoniam perpetuo secundum axem progrediuntur, per omnes reliquas facies refringentes, quantumuis parua fuerit earum apertura, certe transmittentur; hincque campus apparens nunquam penitus euanescere potest. At quo magis punctum ε ab axe distans accipitur, vt radii ab eo per omnes facies transmittantur, eo maior earum apertura requiritur, quae cum ab earum curuatura pendeat, neque certum limitem superare debeat, hinc vltimum punctum ε, vnde radii etiam nunc transmittuntur, ac propterea semidiameter campi apparentis determinatur.

natur. In sequentibus quidem propositionibus campum apparentem seu eius semidiametrum $E\epsilon = z$ vt datum assumam, et quanta esse debeat cuiusque faciei apertura, inuestigabo: hinc enim facile vicissim, si quaeque apertura fuerit cognita, campum apparentem ipsum definire licebit. Ceterum in hac definitione assumsi aperturam primae faciei esse euanescentem: ex quo facile intelligitur ea aucta etiam campum aliquantum extendi oportere; verum hoc augmentum postea in lucrum cedet, quod cum nunquam soleat esse notabile, eius rationem hic non habendam censui; quemadmodum etiam in sequente definitione aperturae pupillae rationem non habebo.

Definitio 2.

233 Locus oculi idoneus est id punctum in axe, quo radii ab extremitate campi apparentis per lentes transmissi axem interfecant. Oculus scilicet in hoc loco constitutus totum campum apparentem conspiciet.

Coroll. 1.

234. Hinc igitur idoneus locus oculo assignabitur, si illa interfectio radiorum extremorum cum axe post lentem vltimam cadat, sin autem haec interfectio ante lentem vltimam reperiatur, fieri nequit vt oculus in eo loco teneatur neque propterea totum campum contueri poterit.

Coroll. 2.

235. At si ista intersectio pone lentem vltimam cadat, oculus totum campum apparentem perspiciet, etiamsi pupilla maxime esset contracta: neque tamen ob maiorem pupillae amplitudinem maiorem campum percipere valet.

Coroll. 3.

236. Verum ob amplitudinem pupillae hoc commodi assequimur, vt oculus etiamsi extra locum idoneum constituatur, dummodo distantia non sit nimis magna, tamen totum campum apparentem conspicere possit: id quod egregie vsu veniet iis casibus, quibus locus idoneus oculi ante faciem refringentem extremam cadit. Tum enim fieri poterit, vt oculus huic faciei immediate applicatus tamen totum campum percipiat.

Scholion.

237. Quando hic de visione loquor, id ita in genere est interpretandum, vt a puncto viso radius in oculum ingrediatur, neque hic curo, vtrum visio sit distincta nec ne? in sequentibus enim docebitur, quomodo lentes disponi conueniat, vt oculus in loco idoneo positus etiam in iusta ab vltima imagine distantia reperiatur, quo visio distincta reddatur. Hic igitur sine vllo respectu ad visionem distinctam habito, eum oculo locum assigno, vbi ab omnibus punctis

ctis in campo apparente contentis radios recipiat, et quoniam singula momenta, quae ad visionem pertinent, seorsim expediri conuenit; hic etiam non ad spatium diffusionis respicio, quod quidem semper per se euanescit, si apertura faciei primae euanescens statuatur.

Problema 2.

238. Si obiectum per vnicam lentem aspiciatur, determinare tam campum apparentem, quam locum idoneum oculi.

Tab. III. Fig. 12.

Solutio.

Sint vt in problemate superiori distantiae determinatrices huius lentis, scilicet distantia obiecti ante lentem $AE = a$, et imaginis post lentem $aF = \alpha$, tum vero lentis crassities $Aa = v$, et distantia arbitraria constructionem lentis plene determinans $= k$. Deinde ponamus semidiametrum campi apparentis $E\varepsilon = z$, ita vt posita faciei anterioris PAP apertura infinite parua etiamnum a puncto ε radii per lentem transmittantur. Sit porro breuitatis gratia $\frac{k-v}{k+v} = i$, atque supra demonstrauimus campum apparentem ad ε vsque extendi, si pro facie posteriori PaP fuerit semidiameter aperturae $aN = \frac{vz}{na}$ existente $n = \frac{31}{20}$; perinde enim est, siue hic semidiameter affirmatiue accipiatur siue negatiue. Hinc ergo vicissim si semidiameter huius aperturae ponatur $= \alpha$, erit semidiameter campi apparentis $E\varepsilon = z = \frac{na}{v}\alpha$.

 Quod

Quod ad locum idoneum oculi attinet, qui fit in O, quoniam inuenimus $FO = \frac{n\alpha a}{n\alpha - iv}$, erit diſtantia $aO = \frac{-i\alpha v}{n\alpha - iv}$; ideoque negatiua, niſi ſit vel i numerus negatiuus, vel $iv > n\alpha$. Sin autem haec diſtantia aO fuerit poſitiua, oculus in O poſitus totum campum apparentem perſpiciet.

Coroll. 1.

239. Si craſſities lentis v euaneſcat, ob $z = \frac{n\alpha}{v} a$, campus apparens euadet infinitus, ſeu potius indeterminatus; diſtantia vero aO euaneſcet. Oculus igitur lenti immediate applicatus tantum ſpatium conſpiciet, quantum per propriam indolem complecti valebit.

Coroll. 2.

240. Sin autem ob lentis craſſitiem $Aa = v$ diſtantia $aO = \frac{-i\alpha v}{n\alpha - iv}$ prodeat poſitiua, oculus in O poſitus totum campum apparentem aſpicere valebit, ſeu in obiecto ſpatium circulare ſpectabit, cuius ſemidiameter $E\varepsilon = z = \frac{n\alpha}{v} a = \frac{r\,i\,a\,a}{s\,i\,v}$, quod ergo eo erit maius, quo tenuior fuerit lens.

Coroll. 3.

241. At ſi diſtantia aO reſultet negatiua, oculum in loco idoneo conſtitui non licet, quoniam is neceſſario poſt lentem teneri debet. Vbicunque autem is poſt lentem collocetur, non vniuerſum campum apparentem contuebitur, ſed tantum eius partem, et quidem

quidem eo minorem, quo magis post lentem remoueatur, propterea quod hoc modo magis a loco idoneo recedit.

Coroll. 4.

242. Hoc igitur casu conueniet oculum immediate ad faciem lentis posteriorem applicare, quo situ eatenus tantum radios accipiet, quatenus pupilla patet; sicque campus visus ab apertura pupillae pendebit; quae si esset nulla etiam campus apparens euanesceret.

Coroll. 5.

243. Hinc patet si pupilla excedit aperturam faciei PaP seu si sit $\omega > a$, denotante ω semidiametrum pupillae, quia tum oculus huic faciei applicatus omnes radios transmissos recipit, eum totum campum apparentem esse visurum. Sin autem sit $\omega < a$, partem tantum totius campi perspiciet, cuius semidiameter erit $= \frac{na}{v}\omega = \frac{3 r a \omega}{a v}$, scilicet non maiorem quam si apertura faciei PaP aequalis esset pupillae.

Scholion.

244. Interim tamen si hoc casu postremo apertura faciei lentis PaP, cui oculus est applicatus maior sit quam pupilla, nihil obstat quominus ea successiue totam aperturam peragret, sicque pedetentim totum campum apparentem conspicere

ſpicere poterit, etiamſi eum ſimul contueri non valeat. Ceterum notandum eſt, ſi etiam faciei anteriori apertura tribuatur, inde campum apparentem aliquantum augeri, ſed partes obiecti vlteriores, quia non per medium lentis radios transmittunt, obſcuriores apparebunt, vnde merito a campo apparente excluduntur. At ſi faciei anteriori tribuatur apertura, cuius ſemidiameter $= x$, vt facies poſterior omnes radios in illam incidentes transmittat, eius apertura tanto maior eſſe debet, ſecundum regulas ſupra traditas; ſcilicet eius ſemidiametrum eſſe oportet $= ix + a$ ſeu $= ix + \frac{vz}{na}$.

Problema 3.

245. Si inſtrumentum dioptricum duabus conſtet lentibus, definire campum apparentem, et locum oculi idoneum.

Solutio.

Tab. III. Fig. 13.

Sit pro his lentibus vt hactenus:

pro PP: $AE = a, aF = \alpha, Aa = v$; diſt. arb: $= k$, et $\frac{k-v}{k+v} = i$

pro QQ: $BF = b, bG = \beta, Bb = v'$; diſt. arb: $= k'$ et $\frac{k'-v'}{k'+v'} = i'$

ac poſito ſemidiametro campi apparentis $E\varepsilon = z$, in imaginibus erit $F\zeta = \frac{1}{i} \cdot \frac{\alpha}{a} z$ et $G\eta = \frac{1}{ii'} \cdot \frac{\alpha\beta}{ab} z$. Conſideretur iam radius a puncto ε in centrum lentis primae A incidens et per lentes transmiſſus qui vtique per extremitates imaginum ζ et η transibit. Iam

Iam ex problemate praecedente patet esse $aN = \frac{vz}{na}$, vnde in altera lente punctum M' ita definietur, vt sit

$$F\zeta - aN : aF = BM' - F\zeta : BF \text{ siue}$$

$$BM' = F\zeta + \frac{BF(F\zeta - aN)}{aF} = \frac{aB.F\zeta - BF.aN}{aF}$$

vnde fit

$$BM' = \frac{1}{i}.\frac{a+b}{a}z - \frac{bvz}{naa}.$$

Nunc punctum N' hinc perinde definietur, atque ex problemate primo ex puncto M determinabatur punctum N; erit quippe

$$bN' = i'.BM' - \frac{v'}{nb}.F\zeta \text{ ideoque}$$

$$bN' = \frac{i'}{i}.\frac{a+b}{a}z - \frac{i'bv}{naa}z - \frac{1}{i}.\frac{av'}{nab}z$$

Hinc autem punctum O, vbi est locus oculi idoneus facile assignabitur erit enim $bN' + G\eta : bG = bN' : bO$, indeque

$$bO = \frac{bG.bN'}{bN' + G\eta} = \frac{\frac{i'}{i}.\frac{a+b}{a} - \frac{i'bv}{naa} - \frac{1}{i}.\frac{av'}{nab}}{\frac{i'}{i}.\frac{a+b}{a} - \frac{i'bv}{naa} - \frac{1}{i}.\frac{av'}{nab} + \frac{1}{ii'}.\frac{a\varepsilon}{ab}}\varepsilon$$

Quod si ergo ponamus semidiametrum aperturae

pro lente PP $\begin{cases} \text{faciei anterioris} = \mathfrak{A} = x \\ \text{faciei posterioris} = \mathfrak{a} \end{cases}$

pro lente QQ $\begin{cases} \text{faciei anterioris} = \mathfrak{B} \\ \text{faciei posterioris} = \mathfrak{b} \end{cases}$

habe-

habebimus

$$\mathfrak{A}=0$$

$$\mathfrak{a}=\frac{v}{n\alpha}z$$

$$\mathfrak{B}=\left(\frac{1}{i}\cdot\frac{\alpha+b}{a}-\frac{b\,v}{n\,a\,\alpha}\right)z$$

$$\mathfrak{b}=\left(\frac{i'}{i}\cdot\frac{\alpha+b}{a}-\frac{i'bv}{n\,a\,\alpha}-\frac{1}{i}\cdot\frac{\alpha\,v'}{n\,a\,b}\right)z$$

ac si distantia oculi post lentem QQ ponatur $bO=O$ erit

$$O=\frac{\mathfrak{b}}{\mathfrak{b}+\frac{1}{ii'}\cdot\frac{\alpha\,v'}{a\,b}z}\cdot\mathfrak{E}$$

Coroll. 1.

246. Si ambarum lentium crassities euanescat erit $v=0$. $v'=0$, et $i=i'=1$; quo ergo casu nostrae formulae in sequentes abibunt:

$$\mathfrak{A}=0;\ \mathfrak{a}=0;\ \mathfrak{B}=\frac{\alpha+b}{a}z \text{ et } \mathfrak{b}=\frac{\alpha+b}{a}z$$

Coroll. 2.

247. Datis ergo vicissim aperturis lentium ex aequationum traditarum ea, pro qua quantitas z minimum valorem adipiscitur, definietur campus adparens.

Coroll. 3.

248. Si igitur crassities lentium euanescat, campus apparens ex apertura lentis posterioris facillime determinatur. Erit enim $z=\frac{\alpha\,\mathfrak{B}}{\alpha+b}$; id quod intelligendum est, si distantia $bO=O$ fuerit positiua oculusque in O collocetur. Coroll.

Coroll. 4.

249. Sin autem diſtantia $bO = O$ prodeat negatiua oculusque vltimae lenti immediate applicetur, tum eius apertura plus non praeſtat quam ſi amplitudini pupillae eſſet aequalis. Quare ſi $\mathfrak{b}$ maior fuerit ſemidiametro pupillae ω loco $\mathfrak{b}$ ſcribatur ω et ex vltima aequatione verus valor ipſius z elicietur niſi forte ex aliqua reliquarum aequationum adhuc minor valor pro z eſſet proditurus.

Problema 4.

250. Si inſtrumentum dioptricum ex tribus conſtet lentibus determinare cum campum apparentem, tum locum oculi idoneum.

Tab. III. Fig. 14.

Solutio.

Exiſtentibus imaginibus per has lentes ſucceſſiue repraeſentatis in $F\zeta$, $G\eta$ et $H\theta$, obiecto vero ipſo in $E\epsilon$, ponamus vt hactenus:

Pro Lente	Diſtantias	craſſitiem	diſt. arb: et	
Prima PP..	$AE = a$; $aF = \alpha$;	$Aa = v$;	k;	$\frac{k-v}{k+v} = i$
Secunda QQ..	$BF = b$; $bG = \beta$;	$Bb = v'$;	k';	$\frac{k'-v'}{k'+v'} = i'$
Tertia RR..	$CG = c$; $cH = \gamma$;	$Cc = v''$;	k'';	$\frac{k''-v''}{k''+v''} = i''$

Semidiametros vero aperturarum

Pro Lente

PP faciei $\begin{cases} \text{anterioris} = \mathfrak{A} = \mathfrak{o} \\ \text{poſterioris} = \mathfrak{a} \end{cases}$

$$\text{QQ faciei} \begin{cases} \text{anterioris} = \mathfrak{B} \\ \text{posterioris} = \mathfrak{b} \end{cases}$$

$$\text{RR faciei} \begin{cases} \text{anterioris} = \mathfrak{C} \\ \text{posterioris} = \mathfrak{c} \end{cases}$$

Tum vero ſit ſemidiameter campi apparentis $E\epsilon = z$, ſupraque oſtendimus fore:

$$F\zeta = \frac{1}{i}\frac{\alpha}{a}z;\ G\eta = \frac{1}{ii'}\cdot\frac{\alpha\mathfrak{C}}{ab}z;\ H\theta = \frac{1}{ii'i''}\cdot\frac{\alpha\mathfrak{C}\gamma}{abc}z.$$

His poſitis habebimus $aN = \mathfrak{a} = \frac{vz}{n\alpha}$; vnde ſi per F recta ipſi NM' parallela ducta intelligatur erit

$$F\zeta - aN : aF = BM' - F\zeta : BF \text{ ſiue } BM' = \frac{aB.F\zeta - BF.aN}{aF}$$

ideoque

$$BM' = \frac{\alpha+b}{\alpha}\cdot\frac{1}{i}\cdot\frac{\alpha}{a}z - \frac{bvz}{n a\alpha} \text{ ſeu}$$

$$\mathfrak{B} = \frac{1}{i}\cdot\frac{\alpha+b}{a}z - \frac{b.v}{na\alpha}z$$

Porro vero eſt ex problemate primo $bN' = i'.BM' - \frac{v}{nb}.F\zeta$,

hincque $\mathfrak{b} = \frac{i'}{i}\cdot\frac{\alpha+b}{a}z - \frac{i'bv}{na\alpha}z - \frac{1}{i}\cdot\frac{\alpha v}{nab}z$

ſimili modo per G ducta intelligatur recta ipſi $N'M''$ parallela, eritque $bG : bN' + G\eta = CG : CM'' - G\eta$ ſiue

$$CM'' = \mathfrak{C} = G\eta + \frac{c}{\mathfrak{C}}(\mathfrak{b} + G\eta) = \frac{\mathfrak{C}+c}{\mathfrak{C}}.G\eta + \frac{c}{\mathfrak{C}}\mathfrak{b},$$

vnde fit

$$\mathfrak{C} = \frac{1}{ii'}\cdot\frac{\alpha(\mathfrak{C}+c)}{ab}z + \frac{i'}{i}\cdot\frac{c(\alpha+b)}{a\mathfrak{C}}z - \frac{i'bcv}{na\alpha\mathfrak{C}}z - \frac{1}{i}\cdot\frac{\alpha cv}{nab\mathfrak{C}}z$$

Deinde

Deinde ex CM'' ita definitur cN'' per problema primum, vt fit

$$cN'' = i''.\, CM'' - \frac{v''}{nc}.\, G\eta \text{ feu } \mathfrak{c} = i''.\, \mathfrak{C} - \frac{1}{ii'}.\, \frac{\alpha\beta v''}{nabc}z$$

hincque

$$\mathfrak{c} = \frac{i''}{ii'}.\, \frac{\alpha(\beta+c)}{ab}z + \frac{i'i''}{i}.\, \frac{\alpha(a+b)}{ab}z - i'i''.\, \frac{bcv}{naa\beta}z - \frac{i''}{i}.\, \frac{a c v'}{nab\beta}z - \frac{1}{ii'}.\, \frac{\alpha\beta v''}{nabc}z$$

Isti ergo valores fequenti modo determinantur:

$$\mathfrak{A} = 0$$

$$\mathfrak{a} = \frac{v}{na}.\, E\epsilon$$

$$\mathfrak{B} = \frac{\alpha+b}{\alpha}.\, F\zeta - \frac{b}{a}.\, \mathfrak{a}$$

$$\mathfrak{b} = i'.\, \mathfrak{B} - \frac{v'}{nb}.\, F\zeta$$

$$\mathfrak{C} = \frac{\beta+c}{\beta}.\, G\eta + \frac{c}{\beta}.\, \mathfrak{b}$$

$$\mathfrak{c} = i''.\, \mathfrak{C} - \frac{v''}{nc}.\, G\eta$$

Cum iam punctum O praebeat locum oculi iustum, fi ponamus $cO = O$, erit $cN'' + H\theta : cH = cN'' : cO$ vnde reperitur

$$O = \frac{\gamma c}{c + H\theta}, \text{ vel } \frac{1}{O} = \frac{1}{\gamma} + \frac{1}{ii'i''}.\, \frac{\alpha\beta}{ab}.\, \frac{z}{cc}.$$

Coroll. I.

251. Ex his aequationibus fequitur fore

$$\frac{\mathfrak{B}}{b} + \frac{\mathfrak{a}}{a} = \frac{\alpha+b}{ab}.\, F\zeta = \frac{1}{i}\left(\frac{\alpha}{b} + 1\right)\frac{z}{a} \text{ et}$$

$$\frac{\mathfrak{C}}{c} - \frac{\mathfrak{b}}{\beta} = \frac{\beta+c}{\beta c}.\, G\eta = \frac{1}{ii'}\left(\frac{\alpha\beta}{bc} + \frac{\alpha}{b}\right)\frac{z}{a}.$$

hincque porro:

$$\frac{i\mathfrak{a}}{a} + \frac{i\mathfrak{B}}{b} + \frac{ii'\mathfrak{b}}{\beta} - \frac{ii'\mathfrak{C}}{c} = \left(1 - \frac{\alpha\beta}{bc}\right)\frac{z}{a}.$$

Coroll. 2.

252. Si ergo craſſities lentium euaneſcat, cum ſit $a=0$, $b=\mathfrak{B}$ et $c=\mathfrak{C}$, definitio campi apparentis reducitur ad has duas aequationes.

I. $\mathfrak{B}\cdot\frac{1}{b}=\left(1+\frac{\alpha}{b}\right)\frac{z}{a}$

II. $\mathfrak{B}\left(\frac{1}{b}+\frac{1}{c}\right)-\mathfrak{C}\cdot\frac{1}{c}=\left(1-\frac{\alpha\zeta}{bc}\right)\frac{z}{a}$

vnde minor valor ipſius z praebet ſemidiametrum campi apparentis.

Coroll. 3.

253. Deinde ſi diſtantia $cO=O$ prodeat negatiua, vt oculum cogamur lenti vltimae immediate applicare, pro c ſcribi oportet ſemidiametrum pupillae ω, et ex vltima aequatione definietur ſemidiameter campi z, niſi forte ex alia aequatione adhuc minor valor pro z prodeat.

Problema 5.

254. Si inſtrumentum Dioptricum ex quatuor lentibus ſuper eodem axe diſpoſitis conſtet, determinare cum campum apparentem, tum locum oculi idoneum.

Solutio.

Tab. III. Fig. 15. Exiſtente obiecto $E\varepsilon=z$, ſint imagines per lentes ſucceſſiue repraeſentatae $F\zeta$, $G\eta$, $H\theta$, et $I\iota$, ponamusque

musque pro lentium fingularum determinatione vt hactenus:

Pro lente	distantias		crassitiem	dist. arb.	et
Prima PP	$AE = a$;	$aF = \alpha$;	$Aa = v$;	.. k;	$\frac{k-v}{k+v} = i$
Secunda QQ	$BF = b$;	$bG = \beta$;	$Bb = v'$;	.. k',	$\frac{k'-v'}{k'+v'} = i'$
Tertia RR	$CG = c$;	$cH = \gamma$;	$Cc = v''$;	.. k'';	$\frac{k''-v''}{k''+v''} = i''$
Quarta SS	$DH = d$;	$dI = \delta$;	$Dd = v'''$;	.. k''';	$\frac{k'''-v'''}{k'''+v'''} = i'''$

Semidiametri vero aperturarum fint:

Pro Lente

Prima PP faciei $\begin{cases} \text{anterioris} = \mathfrak{A} = 0 \\ \text{posterioris} = \mathfrak{a} \end{cases}$

Secunda QQ faciei $\begin{cases} \text{anterioris} = \mathfrak{B} \\ \text{posterioris} = \mathfrak{b} \end{cases}$

Tertia RR faciei $\begin{cases} \text{anterioris} = \mathfrak{C} \\ \text{posterioris} = \mathfrak{c} \end{cases}$

Quarta SS faciei $\begin{cases} \text{anterioris} = \mathfrak{D} \\ \text{posterioris} = \mathfrak{d} \end{cases}$

Si iam $F\varepsilon = z$ exhibeat femidiametrum campi apparentis erit, vt iam fupra oftendimus:

$$F\zeta = \frac{1}{i} \cdot \frac{\alpha}{a} z;\ G\eta = \frac{1}{ii'} \cdot \frac{\alpha\beta}{ab} z;\ H\theta = \frac{1}{ii'i''} \cdot \frac{\alpha\beta\gamma}{abc} z$$

$$\text{et } I\iota = \frac{1}{ii'i''i'''} \cdot \frac{\alpha\beta\gamma\delta}{abcd} z$$

Quod ſi ratiocinium nunc vt ante inſtituamus, obtinebimus ſequentes aequationes:

$\mathfrak{A}=0;$	$\mathfrak{a}=\frac{v}{n\alpha}\cdot E\epsilon$
$\mathfrak{B}=(1+\frac{b}{\alpha})F\zeta-\frac{b}{\alpha}\mathfrak{a}:$	$\mathfrak{b}=i'\mathfrak{B}-\frac{v'}{nb}\cdot F\zeta$
$\mathfrak{C}=(1+\frac{c}{\beta})G\eta+\frac{c}{\beta}\cdot\mathfrak{b}:$	$\mathfrak{c}=i''\mathfrak{C}-\frac{v''}{nc}\cdot G\eta$
$\mathfrak{D}=(1+\frac{d}{\gamma})H\theta+\frac{d}{\gamma}\cdot\mathfrak{c}:$	$\mathfrak{d}=i'''\mathfrak{D}-\frac{v'''}{nd}\cdot H\theta$

Ex quarum ordine priori conſequimur:

$$\frac{\mathfrak{B}}{b}+\frac{\mathfrak{a}}{\alpha}=(\frac{1}{\alpha}+\frac{1}{b})F\zeta=\frac{1}{i}(1+\frac{\alpha}{b})\frac{z}{a}$$

$$\frac{\mathfrak{C}}{c}-\frac{\mathfrak{b}}{\beta}=(\frac{1}{\beta}+\frac{1}{c})G\eta=\frac{1}{ii'}(\frac{\alpha}{b}+\frac{\alpha\beta}{bc})\frac{z}{a}$$

$$\frac{\mathfrak{D}}{d}-\frac{\mathfrak{c}}{\gamma}=(\frac{1}{\gamma}+\frac{1}{d})H\theta=\frac{1}{ii'i''}(\frac{\alpha\beta}{bc}+\frac{\alpha\beta\gamma}{bcd})\frac{z}{a}$$

ex ordine vero poſteriori

$$\mathfrak{a}=\frac{vz}{na}$$

$$\mathfrak{b}=i'\mathfrak{B}-\frac{1}{i}\cdot\frac{\alpha}{b}\cdot\frac{v'z}{na}$$

$$\mathfrak{c}=i''\mathfrak{C}-\frac{1}{ii'}\cdot\frac{\alpha\beta}{bc}\cdot\frac{v''z}{na}$$

$$\mathfrak{d}=i'''\mathfrak{D}-\frac{1}{ii'i''}\cdot\frac{\alpha\beta\gamma}{bcd}\cdot\frac{v'''z}{na}$$

Si denique pro loco oculi idoneo ponamus $dO=O$ erit

$$O=\frac{\delta\mathfrak{d}}{\mathfrak{d}+iz}\text{ ſeu }\frac{1}{O}=\frac{1}{\delta}+\frac{1}{ii'i''i'''}\cdot\frac{\alpha\beta\gamma}{bcd}\cdot\frac{z}{a\delta}.$$

Coroll.

Coroll. 1.

255. Ex aequationibus prioribus deducimus sequentes:

$$\frac{i\mathfrak{a}}{\alpha}+\frac{i\mathfrak{B}}{b}=\left(1+\frac{\alpha}{b}\right)\frac{z}{a}$$

$$\frac{i\mathfrak{a}}{\alpha}+i\frac{\mathfrak{B}}{b}+\frac{ii'\mathfrak{b}}{\beta}-\frac{ii'\mathfrak{C}}{c}=\left(1-\frac{\alpha\beta}{bc}\right)\frac{z}{a}$$

$$\frac{i\mathfrak{a}}{\alpha}+\frac{i\mathfrak{B}}{b}+\frac{ii'\mathfrak{b}}{\beta}-\frac{ii'\mathfrak{C}}{c}-\frac{ii'i''\mathfrak{c}}{\gamma}+\frac{ii'i''\mathfrak{D}}{d}=\left(1+\frac{\alpha\beta\gamma}{bcd}\right)\frac{z}{a}.$$

quae quomodo ad plures lentes sint continuandae, facile perspicitur.

Coroll. 2.

256. Si lentium crassities euanescat, fiet $\mathfrak{a}=0$; $\mathfrak{b}=\mathfrak{B}$; $\mathfrak{c}=\mathfrak{C}$ et $\mathfrak{d}=\mathfrak{D}$, porro $i=i'=i''=i'''=1$, vnde hae aequationes in sequentes formas abibunt:

$$\mathfrak{B}.\frac{1}{b}=\left(1+\frac{\alpha}{b}\right)\frac{z}{a}$$

$$\mathfrak{B}\left(\frac{1}{b}+\frac{1}{\beta}\right)-\mathfrak{C}\frac{1}{c}=\left(1-\frac{\alpha\beta}{bc}\right)\frac{z}{a}$$

$$\mathfrak{B}\left(\frac{1}{b}+\frac{1}{\beta}\right)-\mathfrak{C}\left(\frac{1}{c}+\frac{1}{\gamma}\right)+\mathfrak{D}\frac{1}{d}=\left(1+\frac{\alpha\beta\gamma}{bcd}\right)\frac{z}{a}$$

ex quibus tribus aequationibus, vti in genere, valor ipsius z, qui prodierit minimus verum semidiametrum campi apparentis praebebit.

Coroll. 3.

257. Totus iste campus apparens reuera spectabitur ab oculo in puncto O constituto, dummodo distantia $dO=O$ fuerit positiua. Sed si ea sit negatiua, oculusque lenti vltimo SS immediate applicetur, pona-

ponatur $\mathfrak{b} = \omega$, ſcilicet ſemidiametro pupillae, et ex vltima aequatione elicietur ſemidiameter ſpatii in obiecto reuera conſpicui.

Scholion 1.

258. Hinc igitur perſpicitur, quomodo campus apparens a ſingularum lentium apertura pendeat; ſimulque patet quanta eſſe debeat cuiusque lentis apertura, vt campus apparens datae magnitudinis obtineatur. Si enim quantitas z cum quantitatibus ad lentium determinationem pertinentibus pro data aſſumatur, per noſtras formulas ſucceſſiue ſemidiametri aperturarum pro ſingulis lentibus definiuntur: vbi quidem deinceps eſt diſpiciendum, num lentes tantae aperturae ſint capaces. Hinc ſcilicet campo apparenti limites praefiniuntur, quos transgredi non liceat; vnde ſequitur campum apparentem maiorem aſſumi non poſſe, quam vt aperturae inde pro ſingulis lentibus oriundae admitti queant. His autem definitis perinde eſt ſiue cuique lenti ea ipſa, quae fuerit inuenta apertura tribuatur, ſiue maior, dum ne ſit minor quandoquidem hic aperturam lentis obiectiuae euaneſcentem aſſumſimus. Verum ſi inſuper claritatis ratio habeatur, neceſſe eſt vt vera apertura cuiusque lentis eam, quam hic aſſignauimus, aliquantum ſuperet, et quidem ea quantitate, quam ſupra pro limitibus ob claritatem requiſitis exhibuimus, niſi enim hoc augmentum acceſſerit, extremitas in campo apparente minore lumine praedita erit quam

quam medium. Tum autem campus apparens latius patebit oramque obſcuriorem complectetur; quamobrem ſi circa extremitates minori lumine contenti eſſe velimus, ne opus quidem eſt, vt lentibus maior apertura, quam quidem per formulas noſtras definitur, tribuatur; ſuperfluumque foret aperturas vltra hos limites augere, ita vt hinc cuique lenti conueniens apertura conſtituatur.

Scholion 2.

259. Etſi pro caſu, quo lentium craſſities negligitur formulae noſtrae non multo ſimpliciores euadunt, tamen in iis percommode vſu venit, vt litterarum $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$, coefficientes, ſcilicet $\frac{1}{b}+\frac{1}{\beta}$; $\frac{1}{c}+\frac{1}{\gamma}$; $\frac{1}{d}+\frac{1}{\delta}$; ipſam diſtantiam focalem, inuoluant cuiuſque lentis; in praxi autem apertura ſatis tuto ex diſtantia focali colligi ſolet. Nam ſi lentis QQ diſtantia focalis ponatur $=q$, erit $\frac{1}{b}+\frac{1}{\beta}=\frac{1}{q}$, ſicque $\mathfrak{B}(\frac{1}{b}+\frac{1}{\beta})=\frac{\mathfrak{B}}{q}$; ac ne arcus nimis magni in apertura comprehendantur, neceſſe eſt vt ſit $\mathfrak{B} < \frac{1}{4}q$; et pro varia lentis forma valor fractionis $\frac{\mathfrak{B}}{q}$ vſque ad $\frac{1}{4}$, vel $\frac{1}{5}$ diminui debet. Quare ſi ponamus

$$\mathfrak{B}(\tfrac{1}{b}+\tfrac{1}{\beta})=\pi;\ \mathfrak{C}(\tfrac{1}{c}+\tfrac{1}{\gamma})=\pi';\ \mathfrak{D}(\tfrac{1}{d}+\tfrac{1}{\delta})=\pi''$$

hae litterae π, π', π'' eiusmodi denotabunt fractiones, quarum valor vt plurimum erit vel $\frac{1}{3}$ vel $\frac{1}{4}$ vel $\frac{1}{5}$, id ſcilicet tantum cauendum eſt, ne his litteris nimis magnus valor tribuatur. Quo obſeruato cum arbitrio noſtro relinquantur, imprimis conueniet has ipſas lit-

teras in calculum introduci, ex iisque reliquas determinari; earum enim beneficio campus apparens facillime definitur. Quin etiam ipse campus apparens ſtatim quoque in calculum induci poterit, quippe cuius determinatio deinceps per formulam ſimpliciſſimam expedietur. Hunc in finem vt tota inueſtigatio ad meros numeros redigatur, ponam $\frac{z}{a} = \Phi$ ita vt Φ ſit angulus, ſub quo ſemidiameter campi apparentis ab oculo ad lentem obiectiuam collocato ſpectaretur. Videamus ergo quomodo per hos numeros π, π', π'', etc. et Φ reliquae quantitates definiantur.

Definitio 3.

260. Ratio aperturae cuiuſque lentis mihi vocabitur quotus, qui oritur, ſi ſemidiameter aperturae diuidatur per diſtantiam focalem lentis; eius craſſitie pro nihilo habita.

Coroll. 1.

261. Ita ſi b et $\mathfrak{b}$ ſint diſtantiae determinatrices lentis, et $\mathfrak{B}$ ſemidiameter aperturae eiuſdem, quia diſtantia focalis eſt $= \frac{b\mathfrak{b}}{b+\mathfrak{b}}$, ratio aperturae erit $= \mathfrak{B}(\frac{1}{b} + \frac{1}{\mathfrak{b}})$

Coroll. 2.

262. Ratio igitur aperturae cuiuſuis lentis eſt fractio minor quam $\frac{1}{2}$, quandoquidem hanc legem ſanciuimus vt neutrius faciei arcus 60 gradibus maior in apertura contineatur.

Coroll.

Coroll. 3.

263. Si ſcilicet ambae facies fuerint aeque curuae ratio aperturae per hanc legem vsque ad $\frac{1}{2}$ augeri poterit; ſin autem altera facies fuerit plana, ratio aperturae $\frac{1}{4}$ ſuperare vix poterit: ac ſi lens ſit meniscus, ea adhuc minor ſtatui debebit.

Coroll. 4.

264. Cum autem nihil ſit, quo apertura lentium accuratius definiatur, ea fere arbitrio noſtro relinquitur, et quouis caſu commodiſſime per experientiam determinatur, ſufficietque notaſſe, eam fractioni ſiue $\frac{1}{3}$ ſiue $\frac{1}{4}$ ſiue etiam $\frac{1}{5}$ pro forma lentis aequalem ſtatui debere.

Scholion.

265. Vt ſcilicet quouis caſu ratio aperturae recte definiatur, radios vtriusque faciei lentis contemplari oportet, qui ſi fuerint f et g, erit diſtantia focalis $= \frac{fg}{(n-1)(f+g)} = \frac{10}{11}.\frac{fg}{f+g}$. Iam ſemidiameter aperturae minor eſſe debet quam $\frac{1}{4}f$ vel quam $\frac{1}{4}g$, prout vel f vel g fuerit minor. Sit $g < f$, et cum ſemidiameter aperturae minor eſſe debeat quam $\frac{1}{4}g$, ratio aperturae minor accipienda eſt quam $\frac{11}{40}(1+\frac{g}{f})$. Vnde patet, ſi lens ſit vtrinque aeque conuexa, ſeu $g=f$, rationem aperturae capi debere infra $\frac{11}{20}$; ſin autem ſit altera facies plana ſeu $f=\infty$, illum limitem eſſe $\frac{11}{40}$, qui adhuc minor fiet, ſi lens ſit

men'fcus, feu $\frac{\mathfrak{E}}{f}$ numerus negatiuus. Ceterum fi ratio aperturae fit $= \pi$, eique hoc modo idoneus valor tribuatur, perinde eft fiue is negatiue fiue pofitiue accipiatur: femper autem conducet rationi aperturae minorem valorem tribui, quam fecundum hanc regulam; partim vt obliquitas radiorum incidentium diminuatur, partim vero potiffimum, vt ob claritatem aperturas lentium adhuc vltra augere liceat.

Problema 6.

266. Si inftrumentum dioptricum ex quotcunque lentibus, quarum craffitiem vt nullam fpectare liceat, fit compofitum, dataque fit ratio aperturae pro fingulis lentibus vna cum campo apparente, definire diftantias determinatrices fingularum lentium.

Solutio.

Sit diftantia obiecti ante lentem primam $AE = a$, imaginisque per eam repraefentatae $aF = \alpha$, ac pro fequentibus lentibus ponatur.

Pro Lente	Dift: determinatrices	ratio aperturae
Secunda . .	$BF = b$; $bG = \mathfrak{E}$;	π
Tertia . .	$CG = c$; $cH = \gamma$;	π'
Quarta . .	$DH = d$; $dI = \delta$;	π''
Quinta . .	$EI = e$; $eK = \varepsilon$;	π'''

etc.

Hinc

Hinc ergo ſi ſemidiametri aperturarum harum lentium vt ante indicentur literis $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$, $\mathfrak{E}$, etc. erit $\pi = \mathfrak{B}(\frac{1}{b}+\frac{1}{\beta})$; $\pi' = \mathfrak{C}(\frac{1}{c}+\frac{1}{\gamma})$; $\pi'' = \mathfrak{D}(\frac{1}{d}+\frac{1}{\delta})$; $\pi''' = \mathfrak{E}(\frac{1}{e}+\frac{1}{\varepsilon})$ etc. Tum vero ſi ſit ſemidiameter campi apparentis $= z$ ponatur etiam $\frac{z}{a} = \Phi$. Cum igitur hinc ſit:

$$\mathfrak{B} = \frac{\pi b \beta}{b+\beta};\ \mathfrak{C} = \frac{\pi' c \gamma}{c+\gamma};\ \mathfrak{D} = \frac{\pi'' d \delta}{d+\delta};\ \mathfrak{E} = \frac{\pi''' e \varepsilon}{e+\varepsilon} \text{ etc.}$$

habebimus ex §. 256 ſequentes aequationes:

$$\frac{\pi \beta}{b+\beta} = (1+\frac{\alpha}{b})\Phi$$

$$\pi - \frac{\pi' \gamma}{c+\gamma} = (1-\frac{\alpha\beta}{bc})\Phi$$

$$\pi - \pi' + \frac{\pi'' \delta}{d+\delta} = (1+\frac{\alpha\beta\gamma}{bcd})\Phi$$

$$\pi - \pi' + \pi'' - \frac{\pi''' \varepsilon}{e+\varepsilon} = (1-\frac{\alpha\beta\gamma\delta}{bcde})\Phi$$

etc.

Quo iam faciliùs hinc per π, π', π'', π''' etc. et Φ diſtantiae determinatrices lentium definiri queant, ponatur $\alpha = \mathrm{A}a$; $\beta = \mathrm{B}b$; $\gamma = \mathrm{C}c$; $\delta = \mathrm{D}d$; $\varepsilon = \mathrm{E}e$ etc. Ita vt litterae A, B, C, D, E etc. denotent numeros abſolutos, ac noſtrae aequationes induent has formas:

$$\frac{\mathrm{B}\pi}{\mathrm{B}+1} = (1+\frac{\mathrm{A}a}{b})\Phi$$

$$\pi - \frac{\mathrm{C}\pi'}{\mathrm{C}+1} = (1-\frac{\mathrm{AB}a}{c})\Phi$$

$$\pi - \pi' + \frac{\mathrm{D}\pi''}{\mathrm{D}+1} = (1+\frac{\mathrm{ABC}a}{d})\Phi$$

$$\pi - \pi' + \pi'' - \frac{\mathrm{E}\pi'''}{\mathrm{E}+1} = (1-\frac{\mathrm{ABCD}a}{e})\Phi$$

etc.

vnde eliciuntur ſequentes determinationes:

$$b = \frac{A(B+1)a\Phi}{B\pi - (B+1)\Phi}$$

$$c = \frac{AB(C+1)a\Phi}{C\pi' - (C+1)(\pi-\Phi)}$$

$$d = \frac{ABC(D+1)a\Phi}{D\pi'' - (D+1)(\pi'-\pi+\Phi)}$$

$$e = \frac{ABCD(E+1)a\Phi}{E\pi''' - (E+1)(\pi''-\pi'+\pi-\Phi)}$$

etc.

Datis ergo praeter numeros $\Phi, \pi, \pi', \pi'', \pi'''$ etc. numeris A, B, C, D, E etc. cum diſtantia obiecti $AE = a$, per has formulas diſtantiae b, c, d, e etc. determinantur indeque inſuper alterae $\alpha, \beta, \gamma, \delta, \varepsilon$ etc hoc modo:

$$\alpha = Aa$$

$$\beta = \frac{AB(B+1)a\Phi}{B\pi - (B+1)\Phi}$$

$$\gamma = \frac{ABC(C+1)a\Phi}{C\pi' - (C+1)(\pi-\Phi)}$$

$$\delta = \frac{ABCD(D+1)a\Phi}{D\pi'' - (D+1)(\pi'-\pi+\Phi)}$$

$$\varepsilon = \frac{ABCDE(E+1)a\Phi}{E\pi''' - (E+1)(\pi''-\pi'+\pi-\Phi)}$$

etc.

Hinc naneiscimur diſtantias focales lentium

Primae $PP = \frac{Aa}{A+1}$

Secundae $QQ = \frac{ABa\Phi}{B\pi-(B+1)\Phi}$

Tertiae $RR = \frac{ABCa\Phi}{C\pi'-(C+1)(\pi-\Phi)}$

Quartae $SS = \frac{ABCDa\Phi}{D\pi''-(D+1)(\pi'-\pi+\Phi)}$

Quintae $TT = \frac{ABCDEa\Phi}{E\pi'''-(E+1)(\pi''-\pi'+\pi-\Phi)}$

etc.

Coroll.

Coroll. 1.

267. Ex angulo Φ cum diſtantia obiecti ante lentem primam $AE = a$, ita definitur ſemidiameter campi apparentis z, vt ſit $z = a\Phi$: neque tamen campus apparens pro lubitu aſſumi poteſt, ſed is per multiplicationem determinabitur, vt mox videbimus.

Coroll. 2.

268. Cum omnes numeri hic in calculum introducti aeque negatiue ac poſitiue accipi queant, obſeruandum eſt eos perpetuo ita aſſumi debere, vt internalla lentium quae ſunt $\alpha+b$; $\beta+c$; $\gamma+d$; $\delta+e$; etc. omnia prodeant poſitiua.

Coroll. 3.

269. Quod ad aperturam cuiusque lentis attinet eius ſemidiameter habebitur ſi eius diſtantia focalis multiplicetur per rationem aperturae littera π inſignitam.

Scholion.

270. Quo formulas hic inuentas ſimpliciores reddamus quoniam litteris $\mathfrak{A}$, $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$; etc. non amplius indigebimus, ponamus ad abbreuiandum;

$$\frac{A}{A+1} = \mathfrak{A};\ \frac{B}{B+1} = \mathfrak{B};\ \frac{C}{C+1} = \mathfrak{C};\ \frac{D}{D+1} = \mathfrak{D};\ \frac{E}{E+1} = \mathfrak{E} \text{ etc.}$$

vt ſit

$$A = \frac{\mathfrak{A}}{1-\mathfrak{A}};\ B = \frac{\mathfrak{B}}{1-\mathfrak{B}};\ C = \frac{\mathfrak{C}}{1-\mathfrak{C}};\ D = \frac{\mathfrak{D}}{1-\mathfrak{D}};\ E = \frac{\mathfrak{E}}{1-\mathfrak{E}} \text{ etc.}$$

at-

atque habebimus:

$$\alpha = \mathrm{A}a; \qquad b = \frac{\mathrm{A}a\Phi}{\mathfrak{B}\pi - \Phi}$$

$$\beta = \frac{\mathrm{AB}a\Phi}{\mathfrak{B}\pi - \Phi} \qquad c = \frac{\mathrm{AB}a\Phi}{\mathfrak{C}\pi' - \pi + \Phi}$$

$$\gamma = \frac{\mathrm{ABC}a\Phi}{\mathfrak{C}\pi' - \pi + \Phi} \qquad d = \frac{\mathrm{ABC}a\Phi}{\mathfrak{D}\pi'' - \pi' + \pi - \Phi}$$

$$\delta = \frac{\mathrm{ABCD}a\Phi}{\mathfrak{D}\pi'' - \pi' + \pi - \Phi} \qquad e = \frac{\mathrm{ABCD}a\Phi}{\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi}$$

$$\varepsilon = \frac{\mathrm{ABCDE}a\Phi}{\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi} \qquad \text{etc.}$$

Hincque porro definientur distantiae focales lentium:

Primae $PP = \mathfrak{A}a$

Secundae $QQ = \frac{\mathrm{A}\mathfrak{B}a\Phi}{\mathfrak{B}\pi - \Phi}$

Tertiae $RR = \frac{\mathrm{AB}\mathfrak{C}a\Phi}{\mathfrak{C}\pi' - \pi + \Phi}$

Quartae $SS = \frac{\mathrm{ABC}\mathfrak{D}a\Phi}{\mathfrak{D}\pi'' - \pi' + \pi - \Phi}$

Quintae $TT = \frac{\mathrm{ABCD}\mathfrak{E}a\Phi}{\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi}$

etc.

et lentium interualla:

$$\text{I et II} = \frac{\mathrm{A}\mathfrak{B}a\pi}{\mathfrak{B}\pi - \Phi}$$

$$\text{II et III} = \frac{\mathrm{AB}a\Phi(\mathfrak{C}\pi' - (1-\mathfrak{B})\pi)}{(\mathfrak{B}\pi - \Phi)(\mathfrak{C}\pi' - \pi + \Phi)}$$

$$\text{III et IV} = \frac{\mathrm{ABC}a\Phi(\mathfrak{D}\pi'' - (1-\mathfrak{C})\pi')}{(\mathfrak{C}\pi' - \pi + \Phi)(\mathfrak{D}\pi'' - \pi' + \pi - \Phi)}$$

$$\text{IV et V} = \frac{\mathrm{ABCD}a\Phi(\mathfrak{E}\pi''' - (1-\mathfrak{D})\pi'')}{(\mathfrak{D}\pi'' - \pi' + \pi - \Phi)(\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi)}$$

Quae interualla debent esse positiua.

Problema 7.

271. Positis iisdem, quae in problemate praecedente sunt assumta, definire locum idoneum oculi, vnde totus campus apparens conspici queat.

So-

Solutio.

Maneant omnes denominationes vt ante, et quia apertura lentis PP vt nulla ſpectatur, pro reliquis lentibus ex data aperturae ratione, ſemidiameter aperturae cuiusque ita ſe habebit

Lentis	ſemidiameter aperturae
Secundae QQ . .	$\frac{A\mathfrak{B}\alpha\pi}{\mathfrak{B}\pi-\Phi}\cdot\Phi$
Tertiae RR . .	$\frac{AB\mathfrak{C}\alpha\pi'}{\mathfrak{C}\pi'-\pi'+\Phi}\Phi$
Quartae SS . .	$\frac{ABC\mathfrak{D}\alpha\pi''}{\mathfrak{D}\pi''-\pi'+\pi-\Phi}\Phi$
Quintae TT . .	$\frac{ABCD\mathfrak{E}\alpha\pi'''}{\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi}\Phi$

vbi litterae $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$, $\mathfrak{E}$, etc. valores in praecedente Scholio aſſignatos obtinent.

Deinde magnitudines ſingularum imaginum conſiderari conuenit, quae ob $E\epsilon = z = \alpha\Phi$ et

$$\alpha = Aa, \; \mathfrak{b} = Bb; \; \gamma = Cc; \; \delta = Dd \text{ etc.}$$

erunt

$$F\zeta = \alpha\Phi = Aa\Phi$$

$$G\eta = ABa\Phi$$

$$H\theta = ABCa\Phi$$

$$I\iota = ABCDa\Phi$$

etc.

Iam pro quolibet lentium numero locus oculi idoneus ſeorſim definiri debet; denotante ergo O diſtantiam oculi poſt vltimam lentem

I. Pro vnica lente

Quia craſſities lentis vt nulla ſpectatur, euidens eſt pro loco oculi idoneo fore $O = o$

II. Pro duabus Lentibus

Tab. III. Fig. 13.

Cum hic ſit $bN' + G\eta : bG = bN' : bO$ erit $bO = O = \frac{bN'}{bN' + G\eta}\mathfrak{G}$. Sed eſt $bN' = \frac{A\mathfrak{B}\pi}{\mathfrak{B}\pi - \Phi}a\Phi$ et $G\eta = ABa\Phi$ vnde fit

$$bN' + G\eta = Aa\Phi . \frac{(B+1)\mathfrak{B}\pi - B\Phi}{\mathfrak{B}\pi - \Phi} = \frac{ABa\Phi(\pi - \Phi)}{\mathfrak{B}\pi - \Phi}$$

ob $(B+1)\mathfrak{B} = B$. Erit ergo $\frac{bN'}{bN' + G\eta} = \frac{\mathfrak{B}\pi}{B(\pi - \Phi)}$, quae fractio per $\mathfrak{G} = \frac{ABa\Phi}{(\mathfrak{B}\pi - \Phi)}$ multiplicata dat locum oculi idoneum

$$O = \frac{A\mathfrak{B}a\pi\Phi}{(\pi - \Phi)(\mathfrak{B}\pi - \Phi)}$$

III. Pro tribus Lentibus

Fig. 14.

Cum hic ſit $cN'' + H\theta : cH = cN'' : cO$, erit $cO = O = \frac{cN''}{cN'' + H\theta}.\gamma$. Sed eſt

$$cN'' = \frac{AB\mathfrak{C}a\pi'\Phi}{\mathfrak{C}\pi' - \pi + \Phi} \text{ et } H\theta = ABCa\Phi$$

hincque ob $(C+1)\mathfrak{C} = C$ fiet

$$cN'' + H\theta = \frac{ABCa\Phi(\pi' - \pi + \Phi)}{\mathfrak{C}\pi' - \pi + \Phi}$$

et $\frac{cN''}{cN'' + H\theta} = \frac{\mathfrak{C}\pi'}{C(\pi' - \pi + \Phi)}$ Nunc igitur ob $\gamma = \frac{ABCa\Phi}{\mathfrak{C}\pi' - \pi + \Phi}$ habebimus diſtantiam oculi idoneam:

$$O = \frac{AB\mathfrak{C}a\pi'\Phi}{(\pi' - \pi + \Phi)(\mathfrak{C}\pi' - \pi + \Phi)}$$

IV.

IV. Pro quatuor Lentibus

Cum fit $dN''' + I\iota : dI = dN''' : dO$, erit $dO = O = \frac{dN'''}{dN''' + I\iota} \cdot \delta$. Sed eft

Tab. III. Fig. 15.

$$dN''' = \frac{ABC\mathfrak{D}a\pi''\Phi}{\mathfrak{D}\pi'' - \pi' + \pi - \Phi} \text{ et } I\iota = ABCDa\Phi$$

hincque ob $(D+1)\mathfrak{D} = D$ fiet

$$dN''' + I\iota = \frac{ABCDa\Phi(\pi'' - \pi' + \pi - \Phi)}{\mathfrak{D}\pi'' - \pi' + \pi - \Phi} \text{ et}$$

$$\frac{dN'''}{dN''' + I\iota} = \frac{\mathfrak{D}\pi''}{D(\pi'' - \pi' + \pi - \Phi)}. \text{ Ergo ob } \delta = \frac{ABCDa\Phi}{\mathfrak{D}\pi'' - \pi' + \pi - \Phi}$$

prodit diftantia oculi idonea

$$O = \frac{ABC\mathfrak{D}a\pi''\Phi}{(\pi'' - \pi' + \pi - \Phi)(\mathfrak{D}\pi'' - \pi' + \pi - \Phi)}.$$

V. Pro quinque Lentibus

Si ratiocinium fimili modo ad cafum quinque lentium extendatur, reperiemus diftantiam oculi idoneam

$$O = \frac{ABCD\mathfrak{E}a\pi'''\Phi}{(\pi''' - \pi'' + \pi' - \pi + \Phi)(\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi)}.$$

VI. Pro fex Lentibus

Eodemque modo progrediendo colligitur fore pro cafu fex lentium diftantiam oculi idoneam

$$O = \frac{ABCDE\mathfrak{F}a\pi''''\Phi}{(\pi'''' - \pi''' + \pi'' - \pi' + \pi - \Phi)(\mathfrak{F}\pi'''' - \pi''' + \pi'' - \pi' + \Phi)}$$

ficque vlterius, quousque libuerit progredi licet.

Coroll. 1.

272. Quouis ergo casu necesse est, vt distantia oculi idonea prodeat positiua: si enim fieret negatiua totus campus apparens nusquam conspici posset.

Coroll. 2.

273. Iis autem casibus, quibus distantia O fit negatiua oculum immediate vltimae lenti applicari conueniet; Tum vero oculus plus non cernet, quam si vltimae lentis apertura aequalis esset amplitudini pupillae.

Coroll. 3.

274. Hoc ergo casu statuatur semidiameter aperturae vltimae lentis $= \omega$ semidiametro pupillae, ex eaque aequatione eliciatur valor ipsius Φ, quo invento erit $a\Phi$ semidiameter campi apparentis, qui in obiecto reuera conspicietur.

Problema 8.

275. Positis iisdem atque in problematibus praecedentibus, eam conditionem in lentium dispositione definire, vt oculus in loco idoneo positus obiectum simul distincte videat.

Solutio.

Quia aperturam lentis obiectiuae euanescentem assumimus, in visione alia confusio locum habere nequit nisi quatenus oculus non in distantia iusta ab

vltima imagine, quam intuetur exiſtit, quae ergo tolletur, ſi lentes ita diſponantur, vt imago vltima ante oculum in O ſitum in diſtantia iuſta, quam littera l deſignauimus, verſetur. Cum igitur in figuris locus oculi ante imaginem vltimam cadat haec diſtantia negatiue ſumta ipſi l aequalis eſt ponenda; vnde pro quouis lentium numero ſequentes habebimus determinationes.

I. Pro vnica lente

Tab. III. Fig. 12.

Cum hic ſit $O = o$, et $OF = \alpha = Aa$, oportet eſſe $Aa = -l$, ideoque $A = -\frac{l}{a}$: et $\alpha = -l$, vnde indoles huius lentis determinatur, ita vt eius diſtantia focalis eſſe debeat $= \frac{al}{l-a}$.

II. Pro duabus lentibus

Fig. 13.

Ex inuenta diſtantia $bO = O$, erit $OG = \frac{O}{bN'}$. $G\eta$. Eſt vero $\frac{O}{bN'} = \frac{1}{\pi - \Phi}$; vnde fit $\frac{ABa\Phi}{\pi - \Phi} = -l$, hincque pro ſecunda lente $B = -\frac{(\pi - \Phi)l}{Aa\Phi}$. et $\mathfrak{B} = \frac{B}{B+1}$. Vel cum ſit $Aa\Phi = -\frac{(\pi - \Phi)l}{B}$, erit pro loco oculi

$$O = \frac{-\mathfrak{B}\pi}{B(\mathfrak{B}\pi - \Phi)} l$$

III. Pro tribus lentibus

Fig. 14.

Hic eſt $OH = \frac{O}{cN''}$. $H\theta = \frac{H\theta}{\pi' - \pi + \Phi}$, vnde obtinetur:

$$OH = \frac{ABCa\Phi}{\pi' - \pi + \Phi} = -l$$

ſicque pro vltima lente habebitur:

$$C = \frac{-(\pi' - \pi + \Phi)l}{ABa\Phi}$$

At fi pro determinatione primae lentis capiatur $Aa\Phi = \frac{-(\pi' - \pi + \Phi)l}{BC}$, erit diftantia oculi

$$O = \frac{-\mathfrak{C}\pi'}{C(\mathfrak{C}\pi' - \pi + \Phi)}\, l$$

IV. Pro quatuor lentibus

Tab. III. Fig. 15.

Cum fit $OI = \frac{O}{a\,[illegible]}$, $I\iota = \frac{Il}{\pi'' - \pi' + \pi - \Phi}$, habebitur

$$OI = \frac{ABCDa\Phi}{\pi'' - \pi' + \pi - \Phi} = -l$$

Vnde pro vltima lente

$$D = -\frac{(\pi'' - \pi' + \pi - \Phi)l}{ABCa\Phi}$$

Cum autem fit $ABCa\Phi = -\frac{(\pi'' - \pi' + \pi - \Phi)l}{D}$ erit in loco oculi hoc valore furrogando

$$O = \frac{-\mathfrak{D}\pi''}{D(\mathfrak{D}\pi'' - \pi' + \pi - \Phi)}\, l.$$

V. Pro quinque Lentibus

Simili modo pro quinque lentibus vltima ita comparata effe debet, vt fit

$$E = \frac{-(\pi''' - \pi'' + \pi' - \pi + \Phi)l}{ABCDa\Phi}$$

Prima autem inde definita fit

$$O = \frac{-\mathfrak{E}\pi'''}{E(\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi)}\, l.$$

VI. Pro fex Lentibus

Eodem modo patet. pro fex lentibus fore

$$F = \frac{-(\pi'''' - \pi''' + \pi'' - \pi' + \pi - \Phi)l}{ABCDEa\Phi}$$

atque

atque si hinc Aa definiatur:

$$O = \frac{-\mathfrak{Z}\pi''''}{F(\mathfrak{Z}\pi'''' - \pi''' + \pi'' - \pi' + \pi - \Phi)}\, l$$

quas formulas quousque libuerit, continuare licet.

Coroll. 1.

276. Si distantia oculi iusta l fuerit infinita erit vt sequitur:

I. Pro vna lente $A = \infty$ et $\mathfrak{A} = 1$

II. Pro duabus lentibus $B = \infty$ et $\mathfrak{B} = 1$

III. Pro tribus lentibus $C = \infty$ et $\mathfrak{C} = 1$

IV. Pro quatuor lentibus $D = \infty$ et $\mathfrak{D} = 1$

etc.

Coroll. 2.

277. Casu ergo, quo distantia oculi iusta l est infinita, distantia oculi post lentem vltimam erit pro quouis lentium numero:

I. Pro vnica lente $O = 0$

II. Pro duabus lentibus $O = \frac{Aa\pi\Phi}{(\pi - \Phi)^2}$

III. Pro tribus lentibus $O = \frac{ABb\pi'\Phi}{(\pi' - \pi + \Phi)^2}$

IV. Pro quatuor lentibus $O = \frac{ABCc\pi''\Phi}{(\pi'' - \pi' + \pi - \Phi)^2}$

etc.

Scholion.

278. Hactenus campum apparentem Φ vt datum consideraui, ex eoque tam lentium indolem quam

quam earum dispositionem determinaui, vt campus datae amplitudinis appareat, nihilque obstare deprehendimus, quominus huic conditioni satisfiat; cum numeri A, B, C, D, etc. penitus arbitrio relinquantur, aperturarum vero rationes π, π', π'', etc. infra $\frac{1}{3}$ vel $\frac{1}{4}$ accipi debeant. Verum hic multiplicationis ratio nondum in computum est ducta qua simul campus apparens ita adstringitur, vt certum limitem excedere nequeat. Quoniam igitur in omnibus instrumentis dioptricis multiplicatio imprimis proposita esse solet, quemadmodum per eam campus apparens definiatur in sequente problemate exponamus.

Problema 9.

279. Si instrumentum dioptricum ex quotcunque lentibus fuerit compositum, quarum quidem crassities vt nulla spectetur, simul vero multiplicationis ratio sit proposita, determinare campum apparentem.

Solutio.

Manentibus omnibus denominationibus, quibus hactenus sumus vsi, ita vt $a\Phi$ semidiametrum campi apparentis denotet, sit b distantia, ad quam multiplicationem referamus. Magnitudo igitur $a\Phi$ in distantia hac$=b$ nudo oculo cerneretur sub angulo, cuius tangens est $=\frac{a\Phi}{b}$. Quare si multiplicationis ratio statuatur $=m$, necesse est vt eadem magnitudo $a\Phi$ per lentes spectetur sub angulo, cuius tangens sit $=\frac{m a \Phi}{b}$.

Iam

Iam vero ex iis, quae in problematibus praecedentibus funt tradita ifte angulus facile affignatur, ficque obtinebitur angulus Φ, indeque femidiameter campi apparentis $a\Phi$. Cum autem haec multiplicatio non ad ipfos angulos fed eorum tangentes referatur euidens eft tantum partes obiecti minimas circa centrum E fitas in ratione propofita multiplicari, remotiores vero in ratione minore. Quo notato hanc multiplicationis rationem m pro quouis lentium numero contemplemur.

I. Pro vnica Lente

Tangens anguli, quo imago $F\zeta$ ab oculo in O conftituto confpicitur, eft $\frac{F\zeta}{OF} = \frac{F\zeta}{aF}$ ob $aO = o$, Erit ergo $\frac{ma\Phi}{b} = \Phi$, feu $ma = b$. Hoc ergo cafu campus apparens non determinatur fed multiplicationis ratio eft $m = \frac{b}{a}$. Verum vt vifio fit diftincta, per fuperius problema debet effe $A = -\frac{l}{a}$ et diftantia oculi poft lentem $O = o$. At obiectum fitu erecto cernetur:

Tab. III. Fig. 12.

II. Pro duabus Lentibus

Tangens anguli, quo imago $G\eta$ ab oculo in O conftituto cernetur, eft $= \frac{bN'}{bO} = \pi - \Phi = \frac{ma\Phi}{b}$; vnde fequitur femidiameter campi apparentis

Fig. 13.

$$\Phi = \frac{\pi b}{ma + b} \text{ pro fitu inuerfo}$$

Quo inuento, vt vifio fit diftincta oportet effe

$$B = \frac{-ml}{Ab}, \text{ et } \mathfrak{B} = \frac{-ml}{Ab - ml}$$

hincque prodit diſtantia oculi poſt lentem ocularem

$$O = \frac{ABl(ma + b)}{mmal + ABb}.$$

III. Pro tribus Lentibus

Tab. III. Fig. 14.

Tangens anguli, quo imago $H\theta$ ab oculo in O conſtituto cernitur eſt $\frac{c\lambda''}{c\upsilon} = \pi' - \pi + \Phi = \frac{ma\Phi}{b}$, vnde fit ſemidiameter campi apparentis:

$$\Phi = \frac{(\pi' - \pi)b}{ma - b} \text{ pro ſitu erecto}$$

Deinde vt viſio ſit diſtincta, oportet eſſe

$$C = \frac{-ml}{ABb} \text{ et } \mathfrak{C} = \frac{-ml}{ABb - ml}$$

Cum igitur ſit $\pi' - \pi + \Phi = \frac{ma(\pi' - \pi)}{ma - b}$ erit

$$\mathfrak{C}\pi' - \pi + \Phi = \pi' - \pi + \Phi - \frac{ABb\pi'}{ABb - ml} = \frac{ma(\pi' - \pi)}{ma - b} - \frac{ABb\pi'}{ABb - ml}$$

hincque pro loco oculi

$$O = \frac{-ABbl\pi'}{(ABb - ml)(\mathfrak{C}\pi' - \pi + \Phi)} = \frac{ABbl(ma - b)\pi'}{(mmal - ABbb)\pi' + ma(ABb - ml)\pi}.$$

IV. Pro quatuor lentibus

Fig. 15.

Tangens anguli, quo imago $I\iota$ ab oculo in O conſtituto cernitur eſt $= \frac{d\lambda'''}{d\upsilon} = \pi'' - \pi' + \pi - \Phi = \frac{ma\Phi}{b}$;

vnde elicitur:

$$\Phi = \frac{(\pi'' - \pi' + \pi)b}{ma + b} \text{ pro ſitu inuerſo}$$

Hinc porro pro viſione diſtincta eſſe debet

$$D = \frac{-ml}{ABCb}, \text{ et } \mathfrak{D} = \frac{-ml}{ABCb - ml}$$

vnde fit

$$\mathfrak{D}\pi'' - \pi' + \pi - \Phi = \frac{ma(\pi'' - \pi' + \pi)}{ma + b} - \frac{ABCb\pi''}{ABCb - ml}$$

et

et pro loco oculi

$$O = \frac{ABCbl(ma+b)\pi''}{(mmal + ABCDbb)\pi'' + ma(ABCb - ml)(\pi' - \pi)}.$$

V. Pro quinque lentibus

Eodem modo progrediendo pro campo apparente reperitur:

$$\Phi = \frac{(\pi''' - \pi'' + \pi' - \pi)b}{ma - b} \text{ pro ſitu erecto}$$

et vt viſio euadat diſtincta

$$E = \frac{-ml}{ABCDb} \text{ et } \mathfrak{E} = \frac{-ml}{ABCDb - ml}$$

vnde pro loco oculi idoneo concluditur

$$O = \frac{ABCDbl(ma-b)\pi'''}{(mmal - ABCDbb)\pi''' + ma(ABCDb - ml)(\pi'' - \pi' + \pi)}.$$

VI. Pro ſex lentibus

Hic campus apparens ita definitur, vt ſit:

$$\Phi = \frac{(\pi'''' - \pi''' + \pi'' - \pi' + \pi)b}{ma + b} \text{ pro ſitu inuerſo}$$

viſio vero diſtincta exigit

$$F = \frac{-ml}{ABCDEb}; \ \mathfrak{F} = \frac{-ml}{ABCDEb - ml}$$

vnde pro loco oculi idoneo

$$O = \frac{ABCDEbl(ma+b)\pi''''}{(mmal + ABCDEbb)\pi'''' + ma(ABCDEb - ml)(\pi''' - \pi'' + \pi' - \pi)}$$

ſicque progreſſio ad plures lentes eſt manifeſta.

Coroll. I.

280. Datis ergo rationibus aperturarum ſingularum lentium π, π', π'', etc. vna cum ratione multiplicationis m, diſtantia b, ad quam multiplicatio refer-

refertur, et diſtantia obiecti ante inſtrumentum a, determinatur campus apparens.

Coroll. 2.

281. Vt ergo campus apparens pro data multiplicatione maximus obtineatur litteris π, π', π'', etc. ita valores maximos tribui conueniet, vt alternatim ſint poſitiui et negatiui.

Coroll. 3.

282. Si igitur valores π, π', π'', etc. vsque ad $\frac{1}{2}$ augeri liceat, maximus valor ipſius Φ pro quouis lentium numero erit vt ſequitur:

Pro caſu duarum lentium $\Phi = \frac{b}{2(ma+b)}$

Pro caſu trium lentium $\Phi = \frac{2b}{2(ma-b)}$

Pro caſu quatuor lentium $\Phi = \frac{3b}{2(ma+b)}$

Pro caſu quinque lentium $\Phi = \frac{4b}{2(ma-b)}$

etc.

Coroll. 4.

283. Quo plures ergo lentes adhibentur eo magis campus apparens augeri poteſt ſimul vero patet, quo maior multiplicatio deſideretur, eo minorem fieri campum apparentem.

Coroll.

Coroll. 5.

284. Ratio multiplicationis m tam poſitiue quam negatiue capi poteſt. Si poſitiue accipitur pro lentium numero pari ſitum inuerſum, pro impari autem ſitum erectum declarat. Contrarium vero euenit, ſi m fuerit numerus negatiuus.

Scholion 1.

285. Hic autem imprimis notandum eſt, valorem ipſius Φ tum ſolum angulum $EA\epsilon$ praebere, quando fuerit tam exiguus, vt aliquot gradus non ſuperet; ſi enim valor ipſius Φ prodeat multo maior, tum tangentem huius anguli $EA\epsilon$ exprimit. Plerumque autem, ſi quidem multiplicatio ſit modica, iſte valor ipſius Φ tam paruus reperitur, vt ſine errore pro ipſo angulo $EA\epsilon$ accipi poſſit. Hic igitur ob aliam cauſam amplitudo campi apparentis reſtringitur, vt certum limitem ſuperare nequeat; cum enim angulus, quo radii in oculum incidentes in O ad axem inclinantur, nunquam poſſit eſſe rectus, neque fortaſſe vix 60° ſuperare queat, quandoquidem ne nudo quidem oculo ſpatium in coelo maius quam 120° conſpicere valeamus; ſi illum angulum maximum, quem oculus capere valeat, circiter 63° ſtatuamus, vt eius tangens ſit $= 2$ pro quouis lentium numero habebimus $\frac{ma\Phi}{b} = 2$, vnde ſit $\Phi = \frac{2b}{ma}$, et $a\Phi = \frac{2b}{m}$. Data ergo multiplicatione m et diſtantia b ad quam refertur, ſemidiameter ſpatii in obiecto

conspicui nunquam maior existere potest quam $\frac{2b}{m}$ quotcunque etiam adhibeantur lentes, eaeque ita disponantur vt maximum campum patefaciant. In Telescopiis ergo, vbi sumitur $b = a$, et semidiameter campi ex ipso angulo Φ aestimatur, eius tangens nunquam maior esse potest quam $\frac{2}{m}$: vnde sequentem tabellam adiungo, quae pro quauis multiplicatione semidiametrum campi apparentis maximi ostendit, quem nunquam superare liceat.

Multiplicatio m	Semidiam: campi app: maximi	Multiplicatio m	Semidiam: campi app: maximi
5	21°, 48′	60	1°, 54′, 33″
10	11, 18	70	1, 38, 13
15	7, 35	80	1, 25, 57
20	5, 42	90	1, 16, 24
25	4, 34	100	1, 8, 45
30	3, 49	150	0, 45, 50
35	3, 16½	200	0, 34, 22
40	2, 52	250	0, 27, 30
45	2, 33	300	0, 22, 55
50	2, 17½	400	0, 17, 11
		500	0, 13, 45

Quod si ergo numerum lentium multiplicando iam fere ad tantum campum apparentem pertigerimus, is vlterius nullo modo augeri poterit.

Scho-

Scholion 2.

286. Quod ad locum oculi idoneum attinet, eum ideo in O constituimus, vt omnes radios per lentes transmissos accipiat, etiamsi pupilla maxime esset constricta: ex quo patet ob amplitudinem pupillae oculum de hoc loco sine vllo detrimento aliquantillum remoueri posse, ita vt superfluum foret hunc locum nimis sollicite obseruare, nisi forte apertura vltimae lentis fuerit admodum magna. Sin ea autem pupillam non superet, eaque adeo sit minor, manifestum est oculum ei immediate applicatum aeque omnes radios excipere, et eundem campum contueri, ac si in loco idoneo esset constitutus. His igitur casibus, si forte distantia O pro loco oculi prodeat negatiua, nihil de campo apparente perit, dummodo oculus lenti vltimae immediate applicetur. His itaque, quae ad visionem per instrumenta dioptrica in genere pertinent, expeditis, superest, vt inuestigemus, quantum visio ob diuersam radiorum refrangibilitatem turbetur, et quemadmodum hanc perturbationem euitare queamus.

CAPVT

CAPVT VI.

DE CONFVSIONE A DIVERSA RADIORVM INDOLE ORIVNDA.

Problema I.

287.

Si a puncto dato E radii per lentem PP transmittantur definire variationem in loco imaginis F, quae a diuerfa radiorum refrangibilitate oritur.

Solutio.

Sit diftantia puncti E ante lentem $AE = a$, facierum autem lentis radius anterioris $= f$ pofterioris $= g$ et craffities $Aa = v$, quae quantitates funt conftantes. Pofita nunc refractionis ratione ex aere in vitrum $= n : 1$, ob diuerfam radiorum naturam numerus n erit variabilis, ideoque etiam locus imaginis F poft lentem expreffae, cuius diftantia fi ponatur $aF = \alpha$, erit ex fupra inuentis

$$\frac{n-1}{f} = \frac{1}{a} + \frac{2n}{k+v} \text{ et } \frac{n-1}{g} = \frac{1}{\alpha} - \frac{2n}{k-v}$$

vbi

vbi quantitas k etiam pro variabili eſt habenda, quia tantum a, f, g et v ſunt conſtantes. Quaeſtio ergo huc redit, vt ſi numerus n differentiali ſuo dn creſcere ſumatur, definiatur differentiale diſtantiae α. Quare differentientur ambae aequationes illae:

$$\frac{dn}{f} = \frac{2dn}{k+v} - \frac{2ndk}{(k+v)^2}$$

$$\frac{dn}{g} = \frac{-d\alpha}{\alpha\alpha} - \frac{2dn}{k-v} + \frac{2ndk}{(k-v)^2}.$$

indeque eliminato dk habebitur

$$\frac{dn(k+v)^2}{f} + \frac{dn(k-v)^2}{g} = 2dn(k+v) - 2dn(k-v) - \frac{d\alpha(k-v)^2}{\alpha\alpha}$$

Reſtituantur pro f et g valores initio poſiti, ac peruenietur ad hanc aequationem:

$$\frac{dn(k+v)^2}{a} + \frac{dn(k-v)^2}{\alpha} + 4vdn + \frac{(n-1)d\alpha}{\alpha\alpha}(k-v)^2 = 0$$

vnde reperitur:

$$\frac{d\alpha}{\alpha\alpha} = \frac{-dn}{(n-1)a}\left(\frac{k+v}{k-v}\right)^2 - \frac{dn}{(n-1)\alpha} - \frac{4vdn}{(n-1)(k-v)^2} \text{ ſeu}$$

$$d\alpha = \frac{-\alpha dn}{n-1}\left(1 + \frac{\alpha}{a}\left(\frac{k+v}{k-v}\right)^2 + \frac{4\alpha v}{(k-v)^2}\right)$$

Tum vero cum etiam k ſit quantitas variabilis erit

$$dk = \frac{-(k+v)dn}{n(n-1)}\left(1 + \frac{k+v}{2a}\right)$$

Cum ergo poſuerimus $\frac{k-v}{k+v} = i$, ob $di = \frac{2vdk}{(k+v)^2}$ erit

$$di = \frac{-vdn}{n(n-1)}\left(\frac{1}{a} + \frac{2}{k+v}\right).$$

Coroll. I.

288. Si $n : 1$ denotet rationem refractionis radiorum mediae naturae, vt ſit $n = \frac{31}{20} = 1,55$. erit pro

pro radiis rubris ſeu minime refractis $n = 1,54$, et violaceis $n = 1,56$. quorum valorum diſcrimen a medio, cum ſit $= \frac{1}{155}$, pro differentiali dn haberi poterit.

Coroll. 2.

289. Quare ſi α denotet diſtantiam imaginis a radiis mediis formatae, pro ea quae a rubris formatur, erit $dn = -\frac{1}{155}$ et $\frac{dn}{n-1} = \frac{-1}{55}$. Hinc diſtantia imaginis rubrae poſt lentem erit $= \alpha + \frac{\alpha}{55}\left(1 + \frac{\alpha}{a}\left(\frac{k+v}{k-v}\right)^2 + \frac{4\alpha v}{(k-v)^2}\right)$ Diſtantia autem imaginis violaceae poſt lentem erit

$$\alpha - \frac{\alpha}{55}\left(1 + \frac{\alpha}{a}\left(\frac{k+v}{k-v}\right)^2 + \frac{4\alpha v}{(k-v)^2}\right).$$

Coroll. 3.

290. Si craſſities lentis euaneſcat, vt ſit $v = 0$, ob variabilitatem numeri n erit

$$d\alpha = \frac{-\alpha dn}{n-1}\left(1 + \frac{\alpha}{a}\right) = \frac{-\alpha\alpha dn}{n-1}\left(\frac{1}{a} + \frac{1}{\alpha}\right)$$

Ac ſi diſtantia focalis lentis ponatur $= p$, cum ſit $\frac{1}{a} + \frac{1}{\alpha} = \frac{1}{p}$, erit $d\alpha = \frac{-\alpha\alpha dn}{(n-1)p} = \frac{-20\alpha\alpha dn}{11p}$.

Scholion.

291. Hoc ergo modo ob diuerſam radiorum naturam valor diſtantiae α immutatur; vnde ſi ab ea tanquam ab obiecto radii porro ad lentem ſecundam emittantur, fiet etiam reſpectu huius lentis diſtantia obiecti variabilis. Quam ob cauſam in loco imaginis ab ea formatae duplex variatio orietur: id quod deinceps etiam in lentibus ſequentibus multo magis

magis eueniet. Hanc igitur variationem, quae pro quauis lente in loco imaginis nafcitur, in fequente problemate determinemus.

Problema 2.

292. Si locus imaginis F, quae refpectu lentis QQ vicem obiecti gerit, ob diuerfam radiorum naturam ipfe fit variabilis, determinare variationem, quam ob eandem caufam imago fequens in G patietur.

Solutio.

Sit pro radiis mediae naturae, quibus refpondet numerus n diftantia obiecti F ante lentem $BF = b$; imaginisque inde proiectae diftantia poft lentem $bG = \beta$; dum autem n abit in $n + dn$, hae diftantiae ambae b et β capiant fua incrementa differentialia db et $d\beta$. Ad quae inuenienda fit lentis QQ radius faciei anterioris $= f$, pofterioris $= g$, et craffities $Bb = v$, eritque vt ante:

$$\frac{n-1}{f} = \frac{1}{b} + \frac{2n}{k+v} \text{ et } \frac{n-1}{g} = \frac{1}{\beta} - \frac{2n}{k-v}$$

vbi k cum b et β pro variabili eft habenda. Differentiatione ergo inftituta habebitur.

$$\frac{dn}{f} = \frac{-db}{bb} + \frac{2dn}{k+v} - \frac{2n\,dk}{(k+v)^2}$$
$$\frac{dn}{g} = -\frac{d\beta}{\beta\beta} - \frac{2dn}{k-v} + \frac{2n\,dk}{(k-v)^2}$$

vnde eliminato dk, fit

$$\frac{dn(k+v)^2}{f} + \frac{dn(k-v)^2}{g} = \frac{-db}{bb}(k+v)^2 - \frac{d\beta}{\beta\beta}(k-v)^2 + 4v\,dn$$

 quae

quae multiplicata per $n-1$, si pro f et g valores dati substituantur prodit

$$\frac{dn(k+v)^2}{b}+2ndn(k+v)+\frac{dn(k-v)^2}{\mathfrak{g}}-2ndn(k-v)$$
$$=-\frac{(n-1)db}{bb}(k+v)^2-\frac{(n-1)d\mathfrak{g}}{\mathfrak{g}\mathfrak{g}}(k-v)^2+4(n-1)vdn$$

seu

$$\frac{dn(k+v)^2}{(n-1)b}+\frac{dn(k-v)^2}{(n-1)\mathfrak{g}}+\frac{4vdn}{n-1}+\frac{db}{bb}(k+v)^2+\frac{d\mathfrak{g}}{\mathfrak{g}\mathfrak{g}}(k-v)^2=0$$

Atque hinc elicitur:

$$d\mathfrak{g}=\frac{-\mathfrak{g}\mathfrak{g}db}{bb}\left(\frac{k+v}{k-v}\right)^2-\frac{\mathfrak{g}dn}{n-1}\left(1+\frac{\mathfrak{g}}{b}\left(\frac{k+v}{k-v}\right)^2+\frac{4\mathfrak{g}v}{(k-v)^2}\right)$$

Pro variabilitate autem ipsius k reperietur

$$\frac{dk}{(k+v)^2}=-\frac{db}{2nbb}-\frac{dn}{2n(n-1)b}-\frac{dn}{n(n-1)(k+v)}$$

Quare si ponatur $\frac{k-v}{k+v}=i$, ob $di=\frac{2vdk}{(k+v)^2}$ erit

$$di=\frac{-vdb}{nbb}-\frac{vdn}{n(n-1)b}-\frac{2vdn}{n(n-1)(k+v)}$$

ac si loco k numerus i introducatur, erit

$$d\mathfrak{g}=\frac{-\mathfrak{g}\mathfrak{g}db}{iibb}-\frac{\mathfrak{g}dn}{n-1}\left(1+\frac{\mathfrak{g}}{iib}+\frac{(1-i)^2\mathfrak{g}}{iiv}\right)\text{ et}$$

$$di=\frac{-vdb}{nbb}-\frac{vdn}{n(n-1)}\left(\frac{1}{b}+\frac{1-i}{v}\right).$$

Coroll. I.

293. Inuenta aequatio differentialis etiam hac forma repraesentari potest vt sit

$$\frac{id\mathfrak{g}}{\mathfrak{g}\mathfrak{g}}+\frac{db}{ibb}=\frac{-dn}{n-1}\left(\frac{i}{\mathfrak{g}}+\frac{1}{ib}+\frac{(1-i)^2}{iv}\right)$$

siue

fiue reftituendo k

$$\frac{d\mathfrak{b}}{\mathfrak{b}\mathfrak{b}}\left(\frac{k-v}{k+v}\right)+\frac{db}{bb}\left(\frac{k+v}{k-v}\right)=\frac{-in}{n-1}\left(\frac{1}{\mathfrak{b}}\left(\frac{k-v}{k+v}\right)+\frac{1}{b}\left(\frac{k+v}{k-v}\right)+\frac{4v}{kk-vv}\right)$$

vbi haec obferuanda eft analogia, vt quemadmodum ad b refertur $\frac{k+v}{k-v}$, ita ad $\mathfrak{b}$ referatur $\frac{k-v}{k+v}$.

Coroll. 2.

294. Si lentis huius craffities euanefcat, fit $v=0$, et $i=1$ vbi figura lentis non amplius in computum ingreditur fed fola diftantia focalis, vnde variatio in loco imaginis G ita erit comparata, vt fit

$$\frac{d\mathfrak{b}}{\mathfrak{b}\mathfrak{b}}+\frac{db}{bb}=\frac{-in}{n-1}\left(\frac{1}{\mathfrak{b}}+\frac{1}{b}\right) \text{ ideoque}$$

$$d\mathfrak{b}=\frac{-\mathfrak{b}\mathfrak{b}}{bb}db-\frac{\mathfrak{b}\mathfrak{b}dn}{n-1}\left(\frac{1}{\mathfrak{b}}+\frac{1}{b}\right).$$

Coroll. 3.

295. Si cafum a radiis mediae naturae, ad quos formulae hactenus traditae funt accommodatae, ad radios rubros transferre velimus, poni oportet $dn=-\frac{1}{100}$, fin autem ad radios violaceos $dn=+\frac{1}{100}$.

Problema 3.

296. Si radii ab obiecto E per lentes quotcunque transmittantur, determinare variationem in locis fingularum imaginum, quae a diuerfa radiorum refrangibilitate proficifcitur.

 Solutio.

Solutio.

Retineantur omnes denominationes, quibus in fuperioribus capitibus fumus vfi, ac fint $a, \alpha, b, \mathfrak{b}, c, \gamma$ etc. diftantiae determinatrices lentium pro radiis mediae naturae. Variata ergo ratione refractionis etiam hae diftantiae variabuntur, quarum variationes differentialibus indicemus. Cum autem diftantiae inter binas lentes maneant conftantes, illae variationes ita erunt comparatae vt fit

$$d\alpha + db = 0, \; d\mathfrak{b} + dc = 0, \; d\gamma + d.d = 0 \text{ etc.}$$

Cum iam diftantia obiecti $AE = a$ fit inuariabilis erit ex problemate primo fi ponatur $\frac{k-v}{k+v} = i$

$$d\alpha = -db = \frac{-\alpha\alpha dn}{i(n-1)}\left(\frac{i}{\alpha} + \frac{1}{ia} + \frac{(1-i)^2}{iv}\right) = \frac{-\alpha\alpha dn}{n-1} \cdot \frac{k+v}{k-v}\left(\frac{k-v}{\alpha(k+v)} + \frac{k+v}{a(k-v)} + \frac{4v}{kk-vv}\right)$$

$$\text{et } di = \frac{-v\,dn}{n(n-1)}\left(\frac{1}{a} + \frac{1-i}{v}\right)$$

Deinde pro fecunda lente, ad quam referuntur diftantiae determinatrices b, et $\mathfrak{b}$ cum arbitraria k' et craffitie v', vnde fecimus $\frac{k'-v'}{k'+v'} = i'$, habebimus

$$d\mathfrak{b} = -dc = \frac{-\mathfrak{b}\mathfrak{b}\,db}{i'i'bb} - \frac{\mathfrak{b}\mathfrak{b}\,dn}{i'(n-1)}\left(\frac{i'}{\mathfrak{b}} + \frac{1}{i'b} + \frac{(1-i')^2}{i'v'}\right)$$

$$\text{et } di' = \frac{-v'\,db}{nbb} - \frac{v'\,dn}{n(n-1)}\left(\frac{1}{b} + \frac{1-i'}{v'}\right)$$

Simili modo pro tertia lente, ad quam referuntur diftantiae determinatrices c et γ cum arbitraria k'' et craffitie v'' pofito $\frac{k''-v''}{k''+v''} = i''$ adipifcemur:

$$d\gamma = -dd = \frac{-\gamma\gamma\,dc}{i''i''cc} - \frac{\gamma\gamma\,dn}{i''(n-1)}\left(\frac{i''}{\gamma} + \frac{1}{i''c} + \frac{(1-i'')^2}{i''v''}\right)$$

$$\text{et } di'' = \frac{-v''\,dc}{ncc} - \frac{v''\,dn}{n(n-1)}\left(\frac{1}{c} + \frac{1-i''}{v''}\right)$$

atque

atque vlterius progrediendo obtinebimus ſequentes formulas:

$$d\delta = -de = \frac{-\delta\delta\, dd}{i''' i''' dd} - \frac{\delta\delta\, dn}{i'''(n-1)}\left(\frac{i'''}{\delta} + \frac{1}{i''' d} + \frac{(1-i''')^2}{i''' v'''}\right)$$

$$\text{et } di''' = \frac{-v'''\, dd}{n\, dd} - \frac{v'''\, dn}{n(n-1)}\left(\frac{1}{d} + \frac{1-i'''}{v'''}\right)$$

vnde haec differentialia facile ad quotcunque lentes extenduntur. Atque ſi hic ſucceſſiue valores differentialium dd, dc, db, iam ante definiti ſubſtituantur omnia haec differentialia tam diſtantiarum determinatricium, quam numerorum i, i', i'', i''' etc. per differentiale dn exprimentur.

Si ratio refractionis pro ſingulis lentibus ſit diuerſa pro iisque ordine exprimatur numeris, n, n', n'' etc. perſpicuum eſt, differentialia hic inuenta ſequenti modo expreſſum iri:

$$\text{I. } d\alpha = -db = \frac{-\alpha\alpha\, dn}{i(n-1)}\left(\frac{i}{\alpha} + \frac{1}{ia} + \frac{(1-i)^2}{iv}\right)$$

$$di = \frac{-v\, dn}{n(n-1)}\left(\frac{1}{a} + \frac{1-i}{v}\right)$$

$$\text{II. } d\mathfrak{G} = -dc = \frac{-\mathfrak{G}\mathfrak{G}\, db}{i'.\, i'.\, bb} - \frac{\mathfrak{G}\mathfrak{G}\, dn'}{i'(n'-1)}\left(\frac{i'}{\mathfrak{G}} + \frac{1}{i'b} + \frac{(1-i')^2}{i' v'}\right)$$

$$di' = \frac{-v'\, db}{n'\, bb} - \frac{v'\, dn'}{n'(n'-1)}\left(\frac{1}{b} + \frac{1-i'}{v'}\right)$$

$$\text{III. } d\gamma = -dc = \frac{-\gamma\gamma\, dc}{i''.\, i''.\, cc} - \frac{\gamma\gamma\, dn''}{i''(n''-1)}\left(\frac{i''}{\gamma} + \frac{1}{i'' c} + \frac{(1-i'')^2}{i''.\, v''}\right)$$

$$di'' = \frac{-v''\, dc}{n''\, cc} - \frac{v''\, dn''}{n''(n''-1)}\left(\frac{1}{c} + \frac{1-i''}{v''}\right)$$

etc.

ex quibus formulis etiam mutationes ſingularum imaginum ac proinde etiam tandem vltimae imaginis facile

facile definiri poterunt, feu potius loco imaginum angulos, fub quibus eae oculo ad iuftam diftantiam l pofitae fint adpariturae, confideremus.

Pro vna lente $\frac{1}{i}\cdot\frac{\alpha}{a}\cdot\frac{z}{l}$

Pro duabus lentibus $\frac{1}{ii'}\cdot\frac{\alpha\beta}{ab}\cdot\frac{z}{l}$

Pro tribus lentibus $\frac{1}{ii'i''}\cdot\frac{\alpha\beta\gamma}{abc}\cdot\frac{z}{l}$

etc.

quatenus fcilicet praeter diftantias $a, \alpha; b, \beta; c, \gamma$ etc. etiam litterae i, i', i'', i''' etc. funt variabiles.

Coroll. 1.

297. Hinc igitur per differentiationem definire licet, quanta mutatio in loco vltimae imaginis, quae obiectum vifionis conftituit, ob diuerfam radiorum refrangibilitatem oriri debeat.

Coroll. 2.

298. Deinde cum etiam magnitudinem cuiusque imaginis fupra per diftantias determinatrices et numeros i, i', i'' etc. definiuerimus, pro magnitudine imaginis habetur $\frac{1}{ii'i''i'''}\cdot\frac{\alpha\beta\gamma\delta}{abcd}\cdot\frac{z}{l}$ (189). fimili modo mutatio affignari poteft, quam magnitudo vltimae imaginis ob diuerfam refrangibilitatem radiorum patietur.

Coroll. 3.

299. Cognita autem vtraque mutatione, quam vltima imago tam refpectu loci, quam magnitudinis fubit,

ſubit, non difficulter colligetur, quanta confuſione ipſa viſio ob diuerſam radiorum refrangibilitatem perturbetur.

Coroll. 4.

300. Si craſſities lentium euaneſcat fiet

$$d\alpha = -db = \frac{-\alpha\alpha dn}{n-1}\left(\frac{1}{a}+\frac{1}{\alpha}\right)$$

$$d\beta = -dc = -\frac{\beta\beta db}{bb} - \frac{\beta\beta dn}{n-1}\left(\frac{1}{b}+\frac{1}{\beta}\right)$$

$$d\gamma = -dd = \frac{-\gamma\gamma dc}{cc} - \frac{\gamma\gamma dn}{n-1}\left(\frac{1}{c}+\frac{1}{\gamma}\right)$$

$$d\delta = -de = \frac{-\delta\delta dd}{dd} - \frac{\delta\delta dn}{n-1}\left(\frac{1}{d}+\frac{1}{\delta}\right)$$

etc.

numeri autem i, i', i'' etc. abeunt in vnitatem nullique mutationi amplius ſunt obnoxii.

Si ergo craſſities lentium euaneſcat, pro diuerſa refractione ſingularum lentium formulae ſuperiores abibunt in ſequentes:

$$\text{I. } d\alpha = -db = \frac{-\alpha\alpha dn}{n-1}\left(\frac{1}{a}+\frac{1}{\alpha}\right)$$

$$\text{II. } d\beta = -dc = -\frac{\beta\beta db}{bb} - \frac{\beta\beta dn'}{n'-1}\left(\frac{1}{\beta}+\frac{1}{b}\right)$$

$$\text{III. } d\gamma = -dd = \frac{-\gamma\gamma dc}{cc} - \frac{\gamma\gamma dn''}{n''-1}\left(\frac{1}{c}+\frac{1}{\gamma}\right)$$

etc.

Ceterum per ſe manifeſtum eſt, quando §. 288 dn vel $+\frac{1}{100}$ vel $-\frac{1}{155}$ ſignificare dicitur, id tantum de illa vitri ſpecie, pro qua eſt refractio radiorum me-

mediorum $n = \frac{31}{20}$ esse intelligendum; et pro aliis vitri speciebus differentialia dn', dn'', dn''', etc. haud mediocriter ab $\frac{1}{155}$ discrepare posse. Quanta autem futura sit haec diuersitas, optandum esset, vt ea potius experimentis, quam ex theoria quapiam definiretur.

Scholion

301. Cum igitur ob diuersam radiorum refrangibilitatem cuique imagini duplex alteratio inducatur, quarum altera eius magnitudinem, altera vero eius locum afficit, duplex inde confusio in visionem infertur. Si enim formulae in superioribus capitibus exhibitae ad radios mediae naturae restringantur, pro quibus est $n = \frac{31}{20}$, posito $dn = -\frac{1}{155}$, ex formulis hic traditis differentialibus locus et magnitudo imaginis a radiis rubris formatae definietur, posito autem $dn = +\frac{1}{155}$, locus et magnitudo imaginis violaceae declarabitur. *Tab. III. Fig. 15.* Scilicet si Ii fuerit imago vltima visioni obiecta, quae a radiis mediae naturae formatur, per formulas modo inuentas, prout vel $dn = -\frac{1}{155}$ vel $dn = +\frac{1}{155}$ ponatur, definietur tam imago rubra $R\rho$ quam violacea Vv; atque ex natura differentialium manifestum est cum interualla IR et IV inter se aequalia esse debere, tum etiam differentias $Ii - R\rho$ et $Vv - Ii$, ita vt oculo series innumerabilium imaginum inter extremas $R\rho$ et Vv sitarum simul cernenda offeratur, vnde eo maior confusio oriatur necesse est, quo maior fuerit differentia tam rationis loci quam magnitudinis. Quare haec confusio penitus tolleretur, si eiusmodi lentium dispositio definiri posset, vt tam interuallum RV, quam

quam differentia inter imagines $R\varrho$ et $V\upsilon$ ad nihilum redigeretur, quod vtrumque nisi simul praestari queat, confusionem perfecte tollere non licet Verumtamen etiamsi neutri harum conditionum satisfieri possit, tamen dabitur pro oculo eiusmodi locus O, vbi confusio minime sensibilis percipiatur, qui erit in concursu rectae $\upsilon\varrho$ productae cum axe: ibi enim omnes extremitates $\varrho\iota\upsilon$ communibus radiis cernentur, neque propterea extremitas obiecti colore tincta apparebit. Quare si simul punctum O conueniat cum loco oculi idoneo alia confusio non percipietur, nisi quae inde originem trahit, quod forte imagines extremae $R\varrho$ et $V\upsilon$ nimis a distantia iusta discrepent, siquidem media $I\iota$ ad distantiam iustam ob oculo fuerit remota. Neque tamen hinc ora obiecti coloribus iridis cincta apparebit, cui confusionis speciei maxime est occurrendum; ideoque ea quae adhuc adfuerit confusio facile tolerari poterit, quae vero etiam omnino tolleretur si modo interuallum RV vel in nihilum redigi vel saltem satis paruum reddi posset. Hinc ergo intelligimus vitium illud, quo obiecta coloribus iridis circumdata saepe repraesentantur, non tam necessario cum instrumentis dioptricis esse coniunctum, vt nullo pacto ab iis separari queat, quamobrem eo magis operae erit pretium, vt inuestigemus quomodo haec instrumenta ab isto vitio liberari possint. Quae tota inuestigatio huc redit, vt determinetur punctum O, vbi recta per terminos imaginum υ, ι, ϱ ducta cum axe concurrit hocque punctum cum loco oculi iam

supra definito conueniens reddatur, si quidem fieri potest: vnde perspicitur locum oculi O hac proprietate praeditum esse oportere, vt angulus, sub quo vltima imago cernitur ob variabilitatem numero n tributam nullam mutationem patiatur. Tum vero insuper videndum erit, num interualla IR et IV vel ad nihilum reduci, vel minima reddi queant.

Problema 4.

Tab. III. Fig. 12.

302. Proposita vnica lente definire locum oculi, vnde obiectum sine margine colorato cernatur.

Solutio.

Sit obiecti $E\varepsilon$ ante lentem distantia $EA = a$, imago vero per radios mediae naturae in $F\zeta$ repraesentetur, ponaturque $aF = \alpha$. Pro lente vero sit eius crassities $Aa = v$, et quantitas arbitraria $= k$, vnde capiatur $\frac{k-v}{k+v} = i$. Hinc posito $E\varepsilon = z$ erit $F\zeta = \frac{1}{i} \cdot \frac{\alpha}{a} z$ (86): quare si pro loco oculi statuatur distantia $aO = O$, quae est fixa, erit $OF = \alpha - O$, et anguli $FO\zeta$ tangens $= \frac{1}{i} \frac{\alpha}{a} \cdot \frac{z}{\alpha - O}$, quae formula ob diuersam radiorum refrangibilitatem nullam mutationem subire debet. Inde autem quantitates α et i tantum variantur, dum reliquae manent constantes. Quare istius formulae differentiale logarithmicum nihilo aequale positum praebet hanc aequationem

$$-\frac{di}{i} + \frac{d\alpha}{\alpha} - \frac{d\alpha}{\alpha - O} = 0 \text{ seu } \frac{-di}{i} - \frac{O\,d\alpha}{\alpha(\alpha - O)} = 0$$

vbi

vbi si valores supra inuenti substituantur, prodit

$$\frac{v\,dn}{in(n-1)}\left(\frac{1}{a}+\frac{1-i}{v}\right)+\frac{O\alpha\,dn}{i(n-1)(\alpha-O)}\left(\frac{i}{\alpha}+\frac{1}{i\alpha}+\frac{(1-i)^2}{iv}\right)=0$$

quae aequatio per $\frac{dn}{i(n-1)}$ diuisa praebet:

$$\frac{v}{n\alpha}+\frac{1-i}{n}+\frac{O\alpha}{\alpha-O}\left(\frac{i}{\alpha}+\frac{1}{i\alpha}+\frac{(1-i)^2}{iv}\right)=0$$

vnde locus oculi definiri poterit: qui si debeat conuenire cum supra inuento (238), vbi inuenimus

$O=\frac{-i\alpha v}{n\alpha-iv}$, erit

$\alpha-O=\frac{n\alpha\alpha}{n\alpha-iv}$ et $\frac{O\alpha}{\alpha-O}=\frac{-iv}{n}$, hincque nostra aequatio per n multiplicata abit in:

$$\frac{v}{\alpha}+1-i-\frac{iiv}{\alpha}-\frac{v}{\alpha}-(1-i)^2=0$$

seu $i-ii-\frac{iiv}{\alpha}=0$, ideoque $i=\frac{\alpha}{\alpha+v}=\frac{k-v}{k+v}$. Quamobrem quantitatem arbitrariam k ita definiri conueniet vt sit $k=2\alpha+v$: et cum sit $i=\frac{\alpha}{\alpha+v}$ pro loco oculi habebimus $O=\frac{-\alpha v}{n\alpha+(n-1)v}=\frac{-20\alpha v}{31\alpha+11v}$ ob $n=\frac{31}{20}$.

Quod si porro hinc variationem in loco imaginis desideremus definiri oportet differentiale $d\alpha$, quod fiet:

$$d\alpha=\frac{-\alpha\alpha\,dn}{i(n-1)}\left(\frac{i}{\alpha}+\frac{1}{i\alpha}+\frac{(1-i)^2}{iv}\right)$$

et pro i posito valore $\frac{\alpha}{\alpha+v}$

$$d\alpha=\frac{-(\alpha+v)dn}{n-1}\left(1+\frac{\alpha+v}{\alpha}\right)$$

qui valor si ad nihilum redigi posset, confusio omnis a diuersa radiorum refrangibilitate oriunda perfecte tolleretur.

Coroll. 1.

303. Pro lentis ergo constructione quantitas arbitraria k ita accipi debet, vt sit $k = 2\alpha + v$; atque tum oculus in eo loco constitutus, vbi totum campum apparentem percipiat, simul nullam confusionem a diuersa radiorum indole sentiet.

Coroll. 2.

304. Vt autem oculus simul imaginem in distantia iusta aspiciat, oportet sit $\alpha - O = -l$, ideoque $l = \frac{-11\alpha(\alpha+v)}{31\alpha+11v}$. Vnde colligitur: $\alpha = \frac{-l}{2} - \frac{-v}{2} - \sqrt{(\frac{1}{4}ll + \frac{9}{82}vl + \frac{1}{4}vv)}$, hincque
$O = \frac{1}{2}l - \frac{1}{2}v - \sqrt{(\frac{1}{4}ll + \frac{9}{82}vl + \frac{1}{4}vv)}$ et $k = -l - 2\sqrt{(\frac{1}{4}ll + \frac{9}{82}vl + \frac{1}{4}vv)}$

Coroll. 3.

305. Potest vero insuper effici, vt etiam $d\alpha$ euanescat, quod euenit si $a + \alpha + v = 0$, hoc est $a = \frac{1}{2}l - \frac{1}{2}v + \sqrt{(\frac{1}{4}ll + \frac{9}{82}vl + \frac{1}{4}vv)}$. Verum cum hoc casu ob $\alpha = -a - v$ imago in ipsum obiectum cadet, ita, vt radii nullam refractionem pati sint censendi, visio per lentem perinde erit comparata atque nudis oculis.

Coroll. 4.

306. Si crassities lentis v plane euanescat, tam ob campum apparentem, quam diuersam radiorum refrangibilitatem fit $O = 0$ hoc ergo casu oculus lenti immediate applicatus nullam confusionem ob diuersam radiorum naturam percipiet. Dum ergo fuerit $\alpha = -l$ visio erit distincta. Scho-

Scholion.

307. Hic scilicet penitus mentem abstrahimus a confusione iam supra determinata, quae a lentium apertura oritur, ideoque aperturam primae lentis vt euanescentem spectamus. Eam igitur hic tantum confusionis speciem contemplamur, quae a diuersa radiorum refrangibilitate originem ducit; quam plerumque tolli obseruauimus, si angulus ad O inuariabilis reddatur; tum enim ora obiecti satis bene terminata conspicietur, neque coloribus iridis cincta. Interim tamen adhuc aliqua confusio sentiri poterit inde oriunda, quod si imago media iustam ab oculo distantiam teneat, imaginum extremarum altera sit nimis propinqua alter animis remota, verum si earum interuallum non sit admodum magnum, confusio haec parum erit sensibilis. Ita hic inuenimus, quod experientia satis comprobatur, si obiecta per vnicam lentem spectemus, ea margine colorato destituta apparere dummodo oculus immediate applicetur quod si quando secus euenire videatur, causa aperturae lentis sine dubio erit tribuenda, cui conditioni rationes hic allegatae refragantur.

Problema 5.

308. Si instrumentum dioptricum duabus instructum sit lentibus, definire locum oculi, vnde obiectum sine margine colorato videatur.

Tab. III. Fig. 13.

Solutio

Solutio.

Posita obiecti distantia $AE = a$, sint pro radiis mediae naturae reliquae distantiae determinatrices $aF = \alpha$, $BF = b$ et $bG = \mathfrak{c}$ crassities vero lentium $Aa = v$, $Bb = v'$, et distantiae arbitrariae k et k', ponaturque $\frac{k-v}{k+v} = i$ et $\frac{k'-v'}{k'+v'} = i'$. His positis si magnitudo obiecti $E\varepsilon$ vocetur $= z$, erit imago $G\eta = \frac{1}{ii'}\frac{\alpha\mathfrak{c}}{ab}z$, vnde si oculi distantia ponatur $bO = O$,

ob $GO = \mathfrak{c} - O$, erit anguli $GO\eta$ tangens $\frac{1}{ii'}\cdot\frac{\alpha\mathfrak{c}}{ab}\cdot\frac{z}{\mathfrak{c}-O}$ cuius differentiale logarithmicum nihilo aequatum praebet,

$$-\frac{di}{i}-\frac{di'}{i'}+\frac{d\alpha}{\alpha}-\frac{db}{b}+\frac{d\mathfrak{c}}{\mathfrak{c}}-\frac{d\mathfrak{c}}{\mathfrak{c}-O}=0$$

quae valoribus supra (296) inuentis substitutis abit in:

$$\frac{v\,dn}{in(1-i)}\left(\frac{1}{a}+\frac{1-i}{v}\right)+\frac{v'\,db}{n'i'bb}+\frac{v'\,dn'}{i'n'(n'-1)}\left(\frac{1}{b}+\frac{1-i'}{v'}\right)$$
$$-\frac{db}{\alpha}-\frac{db}{b}+\frac{O}{\mathfrak{c}-O}\left(\frac{\mathfrak{c}\,db}{i'i'bb}+\frac{\mathfrak{c}\,dn'}{i'(n'-1)}\left(\frac{i'}{\mathfrak{c}}+\frac{1}{i'b}+\frac{(1-i')^2}{i'v'}\right)\right)=0$$

Verum conditio campi exigebat $O = \frac{\mathfrak{c}\mathfrak{c}}{\mathfrak{c}+\frac{1}{ii'}\cdot\frac{\alpha\mathfrak{c}}{ab}z}$, existente

$$\mathfrak{c} = \left(\frac{i'}{i}\cdot\frac{\alpha+b}{a}-\frac{i'bv}{n'a\alpha}-\frac{1}{i}\cdot\frac{\alpha v'}{n'ab}\right)z, \quad (245), \text{ vnde fit}$$

$$\frac{O}{\mathfrak{c}-O} = \frac{\mathfrak{c}\mathfrak{c}}{\frac{1}{ii'}\cdot\frac{\alpha\mathfrak{c}\mathfrak{c}}{ab}z} = \frac{ii'ab}{\alpha\mathfrak{c}}\left(\frac{i'}{i}\cdot\frac{\alpha+b}{a}-\frac{i'bv}{na\alpha}-\frac{1}{i}\cdot\frac{\alpha v'}{n'ab}\right)$$

Nunc

Nunc vero eſt $db = \frac{\alpha\alpha dn}{i(n-1)}(\frac{i}{\alpha}+\frac{1}{i\alpha}+\frac{(1-i)^2}{iv})$, quem valorem antequam ſubſtituamus, transformemus aequationem noſtram in hanc formam

$$\frac{dn}{n-1}\left(\frac{v}{ina}+\frac{1-i}{in}+\frac{v'}{i'nb}+\frac{1-i'}{i'n}+\frac{\beta O}{i'(\beta-O)}\left(\frac{i'}{\beta}+\frac{1}{i'b}+\frac{(1-i')^2}{i'v'}\right)\right)$$

$$+db\left(\frac{v'}{ni'bb}-\frac{1}{a}-\frac{1}{b}+\frac{\beta O}{i'i'bb(\beta-O)}\right)=0$$

vbi poſterius membrum abit in $-\frac{iv}{n\alpha\alpha}db$, tum vero erit

$$0=\frac{dn}{n-1}\left\{\begin{array}{l}1+\frac{b}{\alpha}+\frac{i'i'b}{\beta}(1+\frac{b}{\alpha})+\frac{(1-i')^2b(\alpha+b)}{\alpha v'}+\frac{2-i-i'}{n}\\ -\frac{iv}{n\alpha}-\frac{i'v'}{n\beta}-\frac{ibv}{n\alpha\alpha}-\frac{ii'i'bbv}{n\alpha\alpha\beta}-\frac{i(1-i')^2bbv}{n\alpha\alpha v'}\end{array}\right\}$$

diſtinguendo n' ab n erit

$$\frac{dn}{n-1}\left(\frac{iv}{\alpha\alpha}-\frac{(1-i)}{n}\right)=\frac{dn'}{n'-1}\left\{\begin{array}{l}1+\frac{b}{\alpha}+\frac{i'i'b}{\beta}(1+\frac{b}{\alpha})-\frac{ii'i'bbv}{n\alpha\alpha\beta}-\frac{i'v'}{n'\beta}+\frac{1-i'}{n'}\\ -\frac{ibv}{n\alpha\alpha}+\frac{b(1-i')^2}{v'}(1+\frac{b}{\alpha})-\frac{i(1-i')^2bbv}{n\alpha\alpha v'}\end{array}\right\}$$

cuius aequationis complicatio obſtat, quominus quicquam commode inde concludi poſſit.

Coroll. I.

309. Si ambae lentes craſſitie careant, vt ſit $v=0$, $v'=0$, et $i=i'=1$, aequatio differentialis prima eſt

$$\frac{d\alpha}{\alpha}-\frac{db}{b}-\frac{O\,d\beta}{\beta(\beta-O)}=0$$

tum vero:

$$d\alpha=-db=\frac{-\alpha\alpha dn}{n-1}(\frac{1}{\alpha}+\frac{1}{a})\text{ et}$$

$$d\beta=\frac{-\beta\beta db}{bb}-\frac{\beta\beta dn}{n-1}(\frac{1}{\beta}+\frac{1}{b})=\frac{-dn}{n-1}\left(\frac{\alpha\alpha\beta\beta}{bb}(\frac{1}{\alpha}+\frac{1}{a})+\beta\beta(\frac{1}{\beta}+\frac{1}{b})\right)$$

 qui-

quibus valoribus ſubſtitutis et per $\frac{dn}{n-1}$ diuiſione facta fit

$$-\alpha\alpha\left(\frac{1}{\alpha}+\frac{1}{a}\right)\left(\frac{1}{\alpha}+\frac{1}{b}\right)+\frac{O\mathfrak{b}}{\mathfrak{b}-O}\left(\frac{\alpha\alpha}{bb}\left(\frac{1}{\alpha}+\frac{1}{a}\right)+\frac{1}{\mathfrak{b}}+\frac{1}{b}\right)=0$$

Vnde ſi oculus lenti poſteriori immediate applicaretur vt eſſet

$O=0$, deberet eſſe $\left(\frac{1}{\alpha}+\frac{1}{a}\right)\left(\frac{1}{\alpha}+\frac{1}{b}\right)=0$.

Coroll. 2.

310. Verum in eadem hypotheſi, vt locus oculi congruat cum eo, quem viſio campi exigit, debet eſſe $O=\frac{b\mathfrak{b}(\alpha+b)}{b(\alpha+b)+\alpha\mathfrak{b}}$, vnde fit $\mathfrak{b}-O=\frac{\alpha\mathfrak{b}\mathfrak{b}}{b(\alpha+b)+\alpha\mathfrak{b}}$, ideoque $\frac{O}{\mathfrak{b}-O}=\frac{b(\alpha+b)}{\alpha\mathfrak{b}}=\frac{bb}{\mathfrak{b}}\left(\frac{1}{\alpha}+\frac{1}{b}\right)$

quo valore ſubſtituto noſtra aequatio erit

$$0=\left(\frac{1}{\alpha}+\frac{1}{b}\right)\left(-\alpha\alpha\left(\frac{1}{\alpha}+\frac{1}{a}\right)+bb\left(\frac{\alpha\alpha}{bb}\left(\frac{1}{\alpha}+\frac{1}{a}\right)+\frac{1}{\mathfrak{b}}+\frac{1}{b}\right)\right)$$

quae reducitur ad hanc formam

$$0=bb\left(\frac{1}{\alpha}+\frac{1}{b}\right)\left(\frac{1}{\mathfrak{b}}+\frac{1}{b}\right)$$

diſtinguendo n' ab n, membra a dn pendentia ſe deſtruunt et oritur

$$0=\frac{dn'}{n'-1}bb\left(\frac{1}{\alpha}+\frac{1}{b}\right)\left(\frac{1}{\mathfrak{b}}+\frac{1}{b}\right)$$

quod ita oſtenditur:

Cum aequatio prima differentialis praebeat:

$$\frac{d\alpha}{\alpha}-\frac{db}{b}-\frac{O\,d\mathfrak{b}}{\mathfrak{b}(\mathfrak{b}-O)}=0 \text{ ſiue}$$

$$d\alpha\left(\frac{1}{\alpha}+\frac{1}{b}\right)-\frac{O\,d\mathfrak{b}}{\mathfrak{b}(\mathfrak{b}-O)}=0$$

Cum

Cum igitur ex conditione campi apparentis fit

$$\frac{O}{\beta-O}=\frac{bb}{\beta}\left(\frac{1}{\alpha}+\frac{1}{b}\right) \text{ erit}$$

$$d\alpha\left(\frac{1}{\alpha}+\frac{1}{b}\right)-\frac{bb}{\beta^2}\left(\frac{1}{\alpha}+\frac{1}{b}\right)d\beta=0$$

ideoque loco $d\beta$ fuum valorem fustituendo

$$d\alpha\left(\frac{1}{\alpha}+\frac{1}{b}\right)-\frac{bb}{\beta^2}\left(\frac{1}{\alpha}+\frac{1}{b}\right)\left(\frac{\beta\beta d\alpha}{OO}-\frac{\beta^2 dn'}{n'-1}\left(\frac{1}{b}+\frac{1}{\beta}\right)\right)=0$$

vbi membra, quae $d\alpha$ continent, manifesto fe destruunt, et tota quaestio ad hanc aequationem perducitur

$$\frac{dn'}{n'-1}\cdot bb\left(\frac{1}{\alpha}+\frac{1}{b}\right)\left(\frac{1}{\beta}+\frac{1}{b}\right)=0.$$

Coroll. 3.

311. Quod fi ergo huic conditioni fatisfieri possit, obiectum fine margine colorato apparebit: praeterea vero confusio penitus tolleretur, fi reddi liceret $d\beta=0$, quod fit per hanc aequationem

$$\frac{\alpha\alpha}{bb}\left(\frac{1}{\alpha}+\frac{1}{a}\right)+\frac{1}{\beta}+\frac{1}{b}=0 \text{ fiue}$$

$$+\frac{dn}{n-1}\cdot\frac{\alpha\alpha}{bb}\left(\frac{1}{\alpha}+\frac{1}{a}\right)$$

$$+\frac{dn'}{n'-1}\left(\frac{1}{\beta}+\frac{1}{b}\right)=0$$

$$\alpha\alpha\left(\frac{1}{\alpha}+\frac{1}{a}\right)+bb\left(\frac{1}{\beta}+\frac{1}{b}\right)=0.$$

Coroll. 4.

312. Priori autem aequationi fatisfieri nequit nifi fuerit vel $\alpha+b=0$ vel $\frac{1}{\beta}+\frac{1}{b}=0$. Illo cafu ambae lentes coniungerentur, vt vnicam conftituerent, hoc vero pofterioris diftantia focalis fieret infinita, qui

 cafus

casus iterum ad casum vnicae lentis rediret; foret enim $O = -\alpha - b$, ob $\beta = -b$, et oculus priori lenti immediate applicari deberet.

Scholion.

313. Si simili modo has inuestigationes ad plures lentes extendere vellemus, non neglecta earum crassitie in formulas plane inextricabiles delaberemur vnde vix quicquam concludi posset. Verum quia in omnibus fere instrumentis dioptricis, praecipue quae pluribus lentibus constant, iis tam exigua crassities tribui solet, vt sine notabili errore pro nihilo haberi possit, tam taediosae indagationi facile supersedere poterimus. Ad quod accedit, quod hic non de summo rigore geometrico agatur, sed contenti esse queamus, dummodo hanc confusionem satis prope cognouerimus: ex quo sufficiet in consideratione plurium lentium earum crassitiem prorsus neglexisse.

Supplementum IV.

Si ratio refractionis in singulis lentibus sit diuersa solutio sequenti modo absoluetur:

I. Prima aequatio differentialis prorsus se habebit, vt in problemate, ita, vt sit

$$d\alpha\left(\frac{1}{\alpha}+\frac{1}{b}\right)+d\beta\left(\frac{1}{\beta}+\frac{1}{c}\right)-\frac{O\,d\gamma}{\gamma(\gamma-O)}=0$$

et quia etiam, vt ante, est

$$\frac{O}{\gamma-O}=\frac{bbcc}{\alpha\beta\beta\gamma}\left(1+\frac{\alpha}{b}\right)+\frac{cc}{\beta\gamma}\left(1+\frac{\beta}{c}\right)$$

erit

erit nostra aequatio

$$\left(\frac{1}{\alpha}+\frac{1}{b}\right)\left(d\alpha-\frac{bbcc}{\beta^2\gamma^2}d\gamma\right)+\left(\frac{1}{\beta}+\frac{1}{c}\right)\left(d\beta-\frac{cc}{\gamma\gamma}d\gamma\right)=0.$$

II. Nunc autem ratio diuersae refractionis est habenda; vnde in superioribus additamentis inuenimus esse

$$d\alpha=-\frac{\alpha\alpha dn}{n-1}\left(\frac{1}{a}+\frac{1}{\alpha}\right)$$

$$d\beta=\frac{\beta\beta d\alpha}{bb}-\frac{\beta\beta dn'}{n'-1}\left(\frac{1}{b}+\frac{1}{\beta}\right)$$

$$d\gamma=\frac{\gamma\gamma d\beta}{cc}-\frac{\gamma\gamma dn''}{n''-1}\left(\frac{1}{c}+\frac{1}{\gamma}\right)$$

Hincque ergo fiet

$$d\alpha-\frac{bbcc}{\beta^2\gamma^2}d\gamma=d\alpha-\frac{bbd\beta}{\beta^2}+\frac{bbccdn''}{\beta^2(n''-1)}\left(\frac{1}{c}+\frac{1}{\gamma}\right)$$

$$=\frac{bb\,dn'}{n'-1}\left(\frac{1}{b}+\frac{1}{\beta}\right)+\frac{bbccdn''}{\beta^2(n''-1)}\left(\frac{1}{c}+\frac{1}{\gamma}\right)$$

Deinde

$$d\beta-\frac{cc}{\gamma\gamma}d\gamma=\frac{ccdn''}{n''-1}\left(\frac{1}{c}+\frac{1}{\gamma}\right)$$

III. Ex his ergo nostra aequatio differentialis abibit in hanc formam:

$$\left(\frac{1}{\alpha}+\frac{1}{b}\right)\left(\frac{bbdn'}{n'-1}\left[\frac{1}{b}+\frac{1}{\beta}\right]+\frac{bbc^2dn''}{\beta^2(n''-1)}\left[\frac{1}{c}+\frac{1}{\gamma}\right]\right)$$

$$+\left(\frac{1}{\beta}+\frac{1}{c}\right)\left(\frac{ccdn''}{n''-1}\left[\frac{1}{c}+\frac{1}{\gamma}\right]\right)$$

siue

$$\frac{dn'}{n'-1}.\,bb.\left(\frac{1}{\alpha}+\frac{1}{b}\right)\left(\frac{1}{\beta}+\frac{1}{b}\right)$$

$$+\frac{dn''}{n''-1}.\,cc\left(\frac{1}{c}+\frac{1}{\gamma}\right)\left(\frac{bb}{\beta\beta}\left(\frac{1}{\alpha}+\frac{1}{b}\right)+\frac{1}{\beta}+\frac{1}{c}\right)=0$$

IV. Pro casu autem illo singulari, quo oculus lenti vltimae immediate, debet adplicari, ob $O=0$, habebitur simpliciter haec aequatio

$$d\alpha\left(\frac{1}{\alpha}+\frac{1}{b}\right)+d\beta\left(\frac{1}{\beta}+\frac{1}{c}\right)=0$$

 siue

fiue, fubftituto valore $d\beta$

$$d\alpha\left(\tfrac{1}{\alpha}+\tfrac{1}{b}\right)+\tfrac{\beta\beta d\alpha}{bb}\left(\tfrac{1}{\beta}+\tfrac{1}{c}\right)$$
$$-\tfrac{\beta\beta dn'}{n'-1}\left(\tfrac{1}{b}+\tfrac{1}{\beta}\right)=0.$$
$$d\alpha\left(\tfrac{1}{\alpha}+\tfrac{1}{b}+\tfrac{\beta\beta}{bb}\left[\tfrac{1}{\beta}+\tfrac{1}{c}\right]\right)$$
$$-\tfrac{\beta^2 dn'}{n'-1}\left(\tfrac{1}{b}+\tfrac{1}{\beta}\right)=0,$$

feu tandem

$$+\tfrac{dn}{n-1}\alpha\alpha\left(\tfrac{1}{b}+\tfrac{1}{\alpha}\right)\left(\tfrac{1}{\alpha}+\tfrac{1}{b}+\tfrac{\beta^2}{b^2}\left[\tfrac{1}{\beta}+\tfrac{1}{c}\right]\right)$$
$$+\tfrac{dn'}{n'-1}\beta\beta\left(\tfrac{1}{b}+\tfrac{1}{\beta}\right)=0.$$

V. Hoc modo tantum margo coloratus tollitur; vt autem tota confufio tollatur, quod fit fi $d\gamma=0$, infuper fatisfieri debet huic aequationi

$$0=\tfrac{dn}{n-1}\cdot\tfrac{\alpha\alpha\beta\beta\gamma\gamma}{bbcc}\left(\tfrac{1}{a}+\tfrac{1}{\alpha}\right)$$
$$+\tfrac{dn'}{n'-1}\cdot\tfrac{\beta\beta\gamma\gamma}{cc}\left(\tfrac{1}{b}+\tfrac{1}{\beta}\right)$$
$$+\tfrac{dn''}{n''-1}\cdot\gamma\gamma\cdot\left(\tfrac{1}{c}+\tfrac{1}{\gamma}\right).$$

Problema 6.

Tab. III. Fig. 14.

314. Si inftrumentum dioptricum tribus conftet lentibus, quarum craffities euanefcat, eam definire difpofitionem, vt oculus in eo loco, quem campus poftulat, conftitutus obiectum fine margine colorato confpiciat.

Solutio.

Pofita ergo diftantia obiecti ante lentem obiectiuam $AE=a$, eiusque magnitudine $E\varepsilon=z$, vocentur

centur diſtantiae imaginum a radiis mediae naturae formatarum, vt hactenus

$$aF = \alpha,\ BF = b;\ bG = \beta,\ CG = c,\ \text{et}\ cH = \gamma.$$

eritque imago vltima $H\theta = \frac{\alpha\beta\gamma}{abc}z$, et poſita oculi poſt lentem vltimam diſtantia $cO = O$, erit $OH = \gamma - O$ et anguli $HO\theta$ tangens $= \frac{\alpha\beta\gamma}{abc}\cdot\frac{z}{\gamma - O}$, quae debet eſſe inuariabilis. Poſito ergo eius differentiali logarithmico $= 0$ habebimus hanc aequationem:

$$\frac{d\alpha}{\alpha} - \frac{db}{b} + \frac{d\beta}{\beta} - \frac{dc}{c} + \frac{d\gamma}{\gamma} - \frac{d\gamma}{\gamma - O} = 0$$

At et formulis ſupra (300) erutis habemus

$$d\alpha = -db = \frac{-dn}{n-1}\cdot\alpha\alpha\left(\frac{1}{a} + \frac{1}{\alpha}\right)$$

$$d\beta = -dc = -\frac{\beta\beta dn}{n-1}\left(\frac{\alpha\alpha}{bb}\left(\frac{1}{a} + \frac{1}{\alpha}\right) + \frac{1}{b} + \frac{1}{\beta}\right)$$

$$d\gamma = -\frac{\gamma\gamma dn}{n-1}\left(\frac{\alpha\alpha\beta\beta}{bbcc}\left(\frac{1}{a} + \frac{1}{\alpha}\right) + \frac{\beta\beta}{cc}\left(\frac{1}{b} + \frac{1}{\beta}\right) + \frac{1}{c} + \frac{1}{\gamma}\right)$$

vnde noſtra aequatio erit

$$d\alpha\left(\frac{1}{\alpha} + \frac{1}{b}\right) + d\beta\left(\frac{1}{\beta} + \frac{1}{c}\right) - \frac{O\,d\gamma}{\gamma(\gamma - O)} = 0$$

Sed ob campum apparentem ſupra 254 inuenimus:

$$\mathfrak{B} = \beta = b\left(1 + \frac{\alpha}{b}\right)\frac{z}{a}$$

$$\mathfrak{C} = c = \frac{bc}{\beta}\left(1 + \frac{\alpha}{b}\right)\frac{z}{a} + \frac{\alpha c}{b}\left(1 + \frac{\beta}{c}\right)\frac{z}{a}$$

hincque

$$O = \frac{\gamma c}{c + H\theta}\ \text{et}\ \frac{O}{\gamma - O} = \frac{c}{H\theta}\ \text{vnde fit ob}$$

$$H\theta = \frac{\alpha\beta\gamma}{bc}\cdot\frac{z}{a}$$

$$\frac{O}{\gamma - O} = \frac{bbcc}{\alpha\beta\beta\gamma}\left(1 + \frac{\alpha}{b}\right) + \frac{cc}{\beta\gamma}\left(1 + \frac{\beta}{c}\right)\ \text{ſeu}$$

$$\frac{O}{\gamma(\gamma - O)} = \frac{bbcc}{\beta\beta\gamma\gamma}\left(\frac{1}{\alpha} + \frac{1}{b}\right) + \frac{cc}{\gamma\gamma}\left(\frac{1}{\beta} + \frac{1}{c}\right)$$

Valo-

Valoribus iam his substitutis habebimus:

$$\left(\frac{1}{\alpha}+\frac{1}{b}\right)\left(d\alpha-\frac{bbcc}{\mathfrak{b}\mathfrak{b}\gamma\gamma}d\gamma\right)+\left(\frac{1}{\mathfrak{b}}+\frac{1}{c}\right)\left(d\mathfrak{b}-\frac{cc}{\gamma\gamma}d\gamma\right)=0$$

At est

$$d\alpha-\frac{bbcc}{\mathfrak{b}\mathfrak{b}\gamma\gamma}d\gamma=\frac{dn}{n-1}\left(bb\left(\frac{1}{b}+\frac{1}{\mathfrak{b}}\right)+\frac{bbcc}{\mathfrak{b}\mathfrak{b}}\left(\frac{1}{c}+\frac{1}{\gamma}\right)\right)$$

$$d\mathfrak{b}-\frac{cc}{\gamma\gamma}d\gamma=\frac{dn}{n-1}\left(cc\left(\frac{1}{c}+\frac{1}{\gamma}\right)\right)$$

Quare facta diuisione per $\frac{dn}{n-1}$ nanciscemur hanc aequationem:

$$bb\left(\frac{1}{\alpha}+\frac{1}{b}\right)\left(\frac{1}{b}+\frac{1}{\mathfrak{b}}+\frac{cc}{\mathfrak{b}\mathfrak{b}}\left(\frac{1}{c}+\frac{1}{\gamma}\right)\right)+cc\left(\frac{1}{\mathfrak{b}}+\frac{1}{c}\right)\left(\frac{1}{c}+\frac{1}{\gamma}\right)=0$$

Quodsi vero oculus lenti postremae immediate applicetur, seu sit $O=0$, conditio praescripta hanc postulat aequationem:

$$a\alpha\left(\frac{1}{\alpha}+\frac{1}{b}\right)\left(\frac{1}{a}+\frac{1}{\alpha}\right)+\mathfrak{b}\mathfrak{b}\left(\frac{1}{\mathfrak{b}}+\frac{1}{c}\right)\left(\frac{\alpha\alpha}{bb}\left(\frac{1}{a}+\frac{1}{\alpha}\right)+\frac{1}{b}+\frac{1}{\mathfrak{b}}\right)=0$$

Confusio vero a diuersa radiorum refrangibilitate oriunda perfecte tolletur si praeterea fuerit $d\gamma=0$ seu

$$\frac{a\alpha\mathfrak{b}\mathfrak{b}}{bbcc}\left(\frac{1}{a}+\frac{1}{\alpha}\right)+\frac{\mathfrak{b}\mathfrak{b}}{cc}\left(\frac{1}{b}+\frac{1}{\mathfrak{b}}\right)+\frac{1}{c}+\frac{1}{\gamma}=0.$$

Coroll. 1.

315. Si rationes aperturarum lentium in computum ducantur, eaque pro lente secunda ponatur $=\pi$, pro tertia $=\pi'$ erit $bN'=\frac{\pi b\mathfrak{b}}{b+\mathfrak{b}}$ et $CM''=cN''=\frac{\pi' c\gamma}{c+\gamma}$. Tum vero posito $\frac{\pi}{a}=\Phi$, erit $G\eta=\frac{\alpha\mathfrak{b}}{ab}a\Phi$, et $H\theta=\frac{\alpha\mathfrak{b}\gamma}{abc}\cdot a\Phi$. Hinc fiet $\frac{O}{\gamma-O}=\frac{cN''}{H\theta}=\frac{\pi' abcc}{\alpha\mathfrak{b}(c+\gamma)a\Phi}$ et $\frac{O}{\gamma(\gamma-O)}=\frac{\pi' abcc}{\alpha\mathfrak{b}\gamma(c+\gamma)}\cdot\frac{1}{a\Phi}$.

Coroll. 2.

316. Quodſi porro vt ſupra ponatur

$\alpha = Aa,\ \beta = Bb,\ \gamma = Cc$ erit $\frac{O}{\gamma(\gamma-1)} = \frac{\pi'}{ABC(C+1)a\Phi}$;

tum vero $bN' = \frac{\pi Bb}{B+1}$ et $CM'' = \frac{\pi' Cc}{C+1}$: ac $G\eta = ABa\Phi$;

Cum iam ſit

$bN' + G\eta : bG = CM'' - G\eta : CG$ erit $\frac{bN'+G\eta}{bG} = \frac{CM''-G\eta}{CG}$.

ideoque

$\frac{1}{bG} + \frac{1}{CG} = \frac{CM''}{CG.G\eta} - \frac{bN'}{bG.G\eta}$ hoc eſt $\frac{1}{\beta} + \frac{1}{c} = \frac{\pi' C}{AB(C+1)a\Phi} - \frac{\pi}{AB(B+1)a\Phi}$

ſimili vero modo eſt $\frac{1}{\alpha} + \frac{1}{b} = \frac{\pi B}{A(B+1)a\Phi}$.

Coroll. 3.

317. Per easdem ſubſtitutiones fit

$\frac{1}{a} + \frac{1}{\alpha} = \frac{A+1}{Aa}$; $\frac{1}{b} + \frac{1}{\beta} = \frac{B+1}{Bb}$; $\frac{1}{c} + \frac{1}{\gamma} = \frac{C+1}{Cc}$ etc.

hincque:

$\alpha\alpha\ (\frac{1}{a} + \frac{1}{\alpha}) = A(A+1)a$

$\beta\beta\ (\frac{1}{b} + \frac{1}{\beta}) = B(B+1)b$

$\gamma\gamma\ (\frac{1}{c} + \frac{1}{\gamma}) = C(C+1)c$ ſicque porro.

Coroll. 4.

318. His ergo nouis denominationibus introductis differentialia ex §. 300 ita exprimentur

$d\alpha = -db = -\frac{dn}{n-1}.\ A(A+1)a$

$d\beta = -dc = -BBdb - \frac{dn}{n-1}.\ B(B+1)b$

$d\gamma = -d.d = -CCdc - \frac{dn}{n-1}.\ C(C+1)c$

quae formulae commodius in calculum introducentur.

Quemadmodum hic nouae formae adhibentur in fequentibus vfurpandae; ita et pro cafu diuerfae refractionis fequentibus formulis in pofterum vti licebit:

$$\frac{O}{\gamma(\gamma-O)} = \frac{\pi'}{ABC(C+1)a\Phi}$$

$$\frac{1}{\alpha}+\frac{1}{b} = \frac{\pi B}{A(B+1)a\Phi}.$$

$$\frac{1}{\beta}+\frac{1}{c} = \frac{\pi' C}{AB(C+1)a\Phi} - \frac{\pi}{AB(B+1)a\Phi}$$

$$\frac{1}{\gamma}+\frac{1}{d} = \frac{\pi'' D}{ABC(D+1)a\Phi} - \frac{\pi'}{ABC(C+1)a\Phi}$$

etc.

tum vero formulae differentiales erunt

$$d\alpha = -db = \frac{-dn}{n-1}.\ A(A+1)a$$

$$d\beta = -dc = B^2 d\alpha\ \frac{-dn'}{n'-1}.\ B(B+1)b$$

$$d\gamma = -dd = C^2 d\beta\ \frac{-dn''}{n''-1} C(C+1)c$$

$$d\delta = -de = D^2 d\gamma \frac{-dn'''}{n'''-1}.\ D(D+1)d$$

etc.

Atque ex his formulis vt margo coloratus euanefcat, fatisfieri debet huic aequationi

$$0 = \frac{dn'}{n'-1}.\ \frac{\pi b}{Aa\Phi} + \frac{dn''}{n''-1}.\ \frac{\pi' c}{ABa\Phi}$$

Vt autem haec confufio penitus tollatur, fieri debet

$$d\gamma = -\frac{dn}{n-1}.\ A(A+1)B^2C^2a$$

$$-\frac{dn'}{n'-1}.\ B(B+1)C^2.b$$

$$-\frac{dn''}{n''-1}.\ C(C+1)c = 0.$$

fiue

ſiue

$$\frac{d\pi}{\pi-1} \cdot \frac{(A+1)a}{A}$$
$$+\frac{d\pi'}{\pi'-1} \cdot \frac{(B+1)b}{AAB} = 0$$
$$+\frac{d\pi''}{\pi''-1} \cdot \frac{(C+1)c}{A^2 B \cdot C}.$$

Problema 7.

319. Si inſtrumentum dioptricum quatuor conſtet lentibus quarum craſſities negligi queat, eam definire diſpoſitionem vt oculus in eo loco quem campus apparens poſtulat, conſtitutus obiectum ſine confuſione a diuerſa radiorum indole oriunda conſpiciat.

Tab. III. Fig. 15.

Solutio.

Poſita diſtantia obiecti ante inſtrumentum $AE = a$, eiusque magnitudine $E\varepsilon = z$, vocentur diſtantiae imaginum a radiis mediae naturae formatarum vt hactenus.

$aF = \alpha$, $BF = b$; $bG = \beta$, $CG = c$; $cH = \gamma$; $DG = d$; $dI = \delta$

Tum vero ponamus praeterea

$$\alpha = Aa,\ \beta = Bb;\ \gamma = Cc;\ \delta = Dd$$

atque introducantur rationes aperturarum pro ſingulis lentibus poſt primam, quae ſint π pro ſecunda QQ, π' pro tertia RR et π'' pro quarta SS, ſumto pro campo $\frac{z}{a} = \Phi$. His poſitis erunt imagines $F\zeta = Aa\Phi$; $G\eta = ABa\Phi$; $H\theta = ABCa\Phi$ et $I\iota = ABCDa\Phi = \frac{\alpha\beta\gamma\delta}{abcd}z$.

 Iam

Iam posita oculi distantia post instrumentum $dO = O$, vt sit $OI = \delta - O$ erit anguli IOi tangens $\frac{\alpha\beta\gamma\delta}{abcd} \cdot \frac{z}{\delta - O}$, quae cum ob diuersam radiorum refrangibilitatem immutata manere debeat, differentiata dabit hanc aequationem

$$\frac{d\alpha}{\alpha} - \frac{db}{b} + \frac{d\beta}{\beta} - \frac{dc}{c} + \frac{d\gamma}{\gamma} - \frac{dd}{d} - \frac{O\, d\delta}{\delta(\delta - O)} = 0$$

quae ob $db = -d\alpha$, $dc = -d\beta$ et $dd = -d\gamma$ abit in hanc

$$d\alpha\left(\frac{1}{\alpha} + \frac{1}{b}\right) + d\beta\left(\frac{1}{\beta} + \frac{1}{c}\right) + d\gamma\left(\frac{1}{\gamma} + \frac{1}{d}\right) - \frac{O\, d\delta}{\delta(\delta - O)} = 0$$

Verum modo ante notauimus fore (318)

$$d\alpha = \frac{-dn}{n-1} \cdot A(A+1)a$$

$$d\beta = BB\,d\alpha - \frac{dn}{n-1} \cdot B(B+1)b$$

$$d\gamma = CC\,d\beta - \frac{dn}{n-1} \cdot C(C+1)c$$

$$d\delta = DD\,d\gamma - \frac{dn}{n-1} \cdot D(D+1)d$$

vbi pro b, c, d, valores §. 266 assignati substitui debent. Porro vero iam animaduertimus esse (316)

$$\frac{1}{\alpha} + \frac{1}{b} = \frac{1}{Aa\Phi} \cdot \frac{\pi B}{B+1}$$

$$\frac{1}{\beta} + \frac{1}{c} = \frac{1}{ABa\Phi}\left(\frac{\pi' C}{C+1} - \frac{\pi}{B+1}\right)$$

$$\frac{1}{\gamma} + \frac{1}{d} = \frac{1}{ABCa\Phi}\left(\frac{\pi'' D}{D+1} - \frac{\pi'}{C+1}\right).$$

atque

$$\frac{O}{\delta(\delta - O)} = \frac{\pi''}{ABCD(D+1)a\Phi}.$$

Quod

Quodſi iam priores valores in poſterioribus ſucceſſiue ſubſtituantur, habebimus

$$d\alpha = \frac{-dn}{n-1} . A(A+1)a$$

$$d\beta = \frac{-dn}{n-1} . B(AB(A+1)a+(B+1)b$$

$$d\gamma = \frac{-dn}{n-1} . C(AB^2C(A+1)a+BC(B+1)b+(C+1)c)$$

$$d\delta = \frac{-dn}{n-1} . D(AB^2C^2D(A+1)a+BC^2D(B+1)b+CD(C+1)c+(D+1)d)$$

His igitur valoribus in aequatione differentiali ſubſtitutis, et diuiſione per $\frac{dn}{(n-1)\Phi}$ facta aequatio noſtra ſecundum ſingula membra diſtributa erit

$$-\frac{\pi B}{(B+1)} . (A+1)a$$

$$-\frac{1}{A}\left(\frac{\pi' C}{C+1} - \frac{\pi}{B+1}\right)(AB(A+1)a+(B+1)b)$$

$$-\frac{1}{AB}\left(\frac{\pi'' D}{D+1} - \frac{\pi'}{C+1}\right)(AB^2C(A+1)a+BC(B+1)b+(C+1)c)$$

$$+\frac{1}{ABC} . \frac{\pi''}{D+1}(AB^2C^2D(A+1)a+BC^2D(B+1)b+CD(C+1)c+(D+1)d)=0$$

Terna autem priora membra negatiua ſola collecta praebent

$$-\frac{BCD(A+1)\pi''}{D+1}a - \frac{CD(B+1)\pi''}{A(D+1)}b - \frac{D(C+1)\pi''}{AB(D+1)}c$$

$$+\frac{\pi}{A}b + \frac{\pi'}{AB}c$$

quibus ſi quartum addatur prodit

$$\frac{\pi}{A}b + \frac{\pi'}{AB}c + \frac{\pi''}{ABC}d$$

Iam duo hic caſus conſiderari oportet, alterum quo punctum O poſt lentem vltimam cadit, alterum

vero, quo ob diſtantiam O negatiuam oculus lenti vltimae immediate applicatur. Pro priori caſu, quo diſtantia $dO = O$ prodit poſitiua habetur iſta aequatio ſi quidem in aequatione modo inuenta pro b, c, d valores §. 266 inuenti ſubſtituantur

$$\frac{(B+1)\pi}{B\pi-(B+1)\Phi}+\frac{(C+1)\pi'}{C\pi'-(C+1)(\pi-\Phi)}+\frac{(D+1)\pi''}{D\pi''-(D+1)(\pi'-\pi+\Phi)}=0$$

Pro caſu poſteriori, quo diſtantia O euaneſcens aſſumitur facta multiplicatione per $\frac{D+1}{a\Phi}$.

$$\frac{BCD(A+1)\pi''}{\Phi}+\frac{CD(B+1)^2\pi''}{B\pi-(B+1)\Phi}+\frac{D(C+1)^2\pi''}{C\pi'-(C+1)(\pi-\Phi)}$$
$$=\frac{(B+1)(D+1)\pi}{B\pi-(B+1)\Phi}+\frac{(C+1)(D+1)\pi'}{C\pi'-(C+1)(\pi-\Phi)}.$$

Hoc modo efficitur, vt obiectum ſine margine colorato appareat at omnis confuſio tolletur ſi praeterea fuerit

$$AB^2C^2D(A+1)+\frac{ABC^2D(B+1)^2\Phi}{B\pi-(B+1)\Phi}+\frac{ABCD(C+1)^2\Phi}{C\pi'-(C+1)(\pi-\Phi)}+\frac{ABC(D+1)^2\Phi}{D\pi''-(D+1)(\pi'-\pi+\Phi)}=0$$

ſeu per ABC diuidendo:

$$BCD(A+1)+\frac{CD(B+1)^2\Phi}{B\pi-(B+1)\Phi}+\frac{D(C+1)^2\Phi}{C\pi'-(C+1)(\pi-\Phi)}+\frac{(D+1)^2\Phi}{D\pi''-(D+1)(\pi'-\pi+\Phi)}=0$$

$$BCD(A+1)+\frac{CD(B+1)b}{Aa}+\frac{D(C+1)c}{ABa}+\frac{(D+1)d}{ABCa}=0$$

Coroll. 1.

320. Si vtrique conditioni ſatisfieri poteſt, vt nec locus nec magnitudo imaginis vllam mutationem patiatur locus oculi non amplius in computum ingreditur ſed imago, vndecunque cernatur, ab omni confuſione prorſus libera erit.

Coroll.

Coroll. 2.

321. Vt ergo hunc fummum perfectionis gradum confequamur, binis his aequationibus fatisfieri oportet.

$$\frac{(B+1)\pi}{B\pi-(B+1)\Phi}+\frac{(C+1)\pi'}{C\pi'-(C+1)(\pi-\Phi)}+\frac{(D+1)\pi''}{D\pi''-(D+1)(\pi'-\pi+\Phi)}=0$$

et

$$A+1+\frac{(B+1)^2\Phi}{B(B\pi-(B+1)\Phi)}+\frac{(C+1)^2\Phi}{BC(C\pi'-(C+1)(\pi-\Phi))}+\frac{(D+1)^2\Phi}{BCD(D\pi''-(D+1)(\pi'-\pi+\Phi))}=0.$$

Coroll. 3.

322. Si porro vt fupra fecimus ponamus

$$\frac{A}{A+1}=\mathfrak{A},\ \frac{B}{B+1}=\mathfrak{B},\ \frac{C}{C+1}=\mathfrak{C}\ \text{et}\ \frac{D}{D+1}=\mathfrak{D}$$

iftae aequationes fequenti modo fimplicius exprimentur:

$$+\frac{\pi}{\mathfrak{B}\pi-\Phi}+\frac{\pi}{\mathfrak{C}\pi'-\pi+\Phi}+\frac{\pi''}{\mathfrak{D}\pi''-\pi'+\pi-\Phi}=0\ \text{et}$$

$$\frac{1}{\mathfrak{A}}+\frac{\Phi}{\mathfrak{A}\mathfrak{B}(\mathfrak{B}\pi-\Phi)}+\frac{\Phi}{\mathfrak{A}\mathfrak{B}\mathfrak{C}(\mathfrak{C}\pi'-\pi+\Phi)}+\frac{\Phi}{\mathfrak{A}\mathfrak{B}\mathfrak{C}\mathfrak{D}(\mathfrak{D}\pi''-\pi'+\pi-\Phi)}=0$$

hic prima membra multiplicari debent per $\frac{d\,n}{n-1}$

fecunda per $\frac{d\,n'}{n'-1}$

tertia per $\frac{d\,n''}{n''-1}$

etc.

Coroll. 4.

323. Sin autem non liceat has ambas aequationes adimplere, curandum eft, vt faltem priori, qua magnitudo apparens a confufione liberatur, fatisfiat

fiat hoc enim modo obiectum fine margine colorato appareteit vbi quidem ad duos cafus refpici conuenit prout diftantia oculi dO prodierit pofitiua vel negatiua.

Scholion.

324. Introducendis ergo rationibus aperturarum quibus fupra iam commode fumus vfi ad campum apparentem definiendum aequationes etiam iftae confufionem a diuerfa radiorum refrangibilitate oriundam tollentes fatis fiunt fimplices, vt fine moleftia tractari queant; fi quidem craffities lentium negligatur. Hoc ergo modo problema generale, quicunque fuerit lentium numerus, expediri conueniet.

Supplementum V.

Si ratio refractionis in fingulis lentibus difcrepet, prodit primo quidem eadem aequatio differentialis

$$d\alpha\left(\frac{1}{a}+\frac{1}{b}\right)+d\beta\left(\frac{1}{b}+\frac{1}{c}\right)+d\gamma\left(\frac{1}{\gamma}+\frac{1}{d}\right)\frac{-Od\delta}{\delta(\delta-O)}=0$$

in qua ergo cafu $O=0$ vltimum membrum abiici debet.

I) Si autem O habeat valorem pofitiuum, erit, vt ante,

$$\frac{O}{\delta(\delta-O)}=\frac{\pi''}{ABCD(D-\phi-1)\alpha\Phi}$$

atque etiam valores $\frac{1}{\alpha}+\frac{1}{b}$; $\frac{1}{b}+\frac{1}{c}$; $\frac{1}{\gamma}+\frac{1}{d}$ manent iidem, vt ante.

At

At vero ob diuerſitatem refractionis habebimus

$$d\alpha = \frac{-dn}{n-1} . A(A+1)a$$

$$d\beta = BBd\alpha - \frac{dn'}{n'-1} . B(B+1)b$$

$$d\gamma = CCd\beta - \frac{dn''}{n''-1} C. (C+1)c.$$

$$d\delta = DDd\gamma - \frac{dn'''}{n'''-1} . D(D+1)d$$

Hinc ergo aequatio noſtra ſucceſſiue ita formetur:

$$\frac{-Od\delta}{\delta(\delta-O)} = \frac{-\pi'' d\delta}{ABCD(D+1)a\Phi}$$

$$= \frac{-\pi'' D d\gamma}{ABC(D+1)a\Phi} + \frac{dn'''}{n'''-1} . \frac{\pi''. d}{ABC.a\Phi}$$

Addatur $d\gamma(\frac{1}{\gamma}+\frac{1}{\delta}) = \frac{d\gamma.\pi'' D}{ABC(D+1)a\Phi} - \frac{\pi' d\gamma}{ABC(C+1)a\Phi}$

et prodibit $\frac{-\pi' d\gamma}{ABC(C+1)a\Phi} + \frac{dn'''}{n'''-1} . \frac{\pi''. d}{ABC.a\Phi}$

et pro $d\gamma$ ſubſtituto valore

$$\frac{-\pi' C d\beta}{AB(C+1)a\Phi} + \frac{dn''}{n''-1} . \frac{\pi' c}{ABa\Phi} + \frac{dn'''}{n'''-1} . \frac{\pi''. d}{ABC.a\Phi}$$

Iam addatur

$$d\beta(\frac{1}{\beta}+\frac{1}{c}) = \frac{\pi' C d\beta}{AB(C+1)a\Phi} - \frac{\pi d\beta}{AB(B+1)a\Phi}$$

proditque

$$\frac{-\pi d\beta}{AB(B+1)a\Phi} + \frac{dn''}{n''-1} . \frac{\pi' c}{AB.a\Phi} + \frac{dn'''}{n'''-1} . \frac{\pi'' d}{ABCa\Phi}$$

et ſubſtituto valore ipſius $d\beta$

$$\frac{-\pi B d\alpha}{A(B+1)a\Phi} + \frac{dn'}{n'-1} . \frac{\pi. b}{A.a\Phi} + \frac{dn''}{n''-1} . \frac{\pi' c}{ABa\Phi} + \frac{dn'''}{n'''-1} . \frac{\pi'' d}{ABC.a\Phi}$$

denique addatur $d\alpha(\frac{1}{\alpha}+\frac{1}{b}) = \frac{\pi B d\alpha}{A(B+1)a\Phi}$

ac aequatio quaeſita, qua margo coloratus euaneſcit, erit

$$\frac{dn'}{n'-1} . \frac{\pi b}{Aa\Phi} + \frac{dn''}{n''-1} . \frac{\pi' c}{ABa\Phi} + \frac{dn'''}{n'''-1} . \frac{\pi'' d}{ABCa\Phi} = 0$$

II) Sin autem O habeat valorem negatiuum: tunc fumi debet O=o et pro eodem fcopo aequatio ita formabitur:

Cum fit

$$d\gamma\left(\frac{1}{\gamma}+\frac{1}{a}\right)=\frac{\pi'' D\, d\gamma}{ABC(D+1)a\Phi}-\frac{\pi' d\gamma}{ABC(C+1)a\Phi}$$

fubftituto valore $d\gamma$ fiet.

$$CC\,d\beta\left(\frac{\pi'' D}{ABC(D+1)a\Phi}-\frac{\pi'}{ABC(C+1)a\Phi}\right)$$
$$-\frac{dn''}{n''-1}.\,C(C+1)c\left(\frac{\pi'' D}{ABC(D+1)a\Phi}-\frac{\pi'}{ABC(C+1)a\Phi}\right)$$

Addatur

$$d\beta\left(\frac{1}{\beta}+\frac{1}{c}\right)=\frac{\pi' C d\beta}{AB(C+1)a\Phi}-\frac{\pi\, d\beta}{AB(B+1)a\Phi}$$

eritque

$$d\beta\left(\frac{CD\pi''}{AB(D+1)a\Phi}-\frac{\pi}{AB(B+1)a\Phi}\right)-\frac{dn''}{n''-1}(C+1)c\left(\frac{\pi'' D}{AB(D+1)a\Phi}-\frac{\pi'}{AB(C+1)a\Phi}\right)$$

et fubftituto valore ipfius $d\beta$

$$d\alpha\left(\frac{BCD\pi''}{A(D+1)a\Phi}-\frac{B\pi}{A(B+1)a\Phi}\right)$$
$$-\frac{dn'}{n'-1}\left(\frac{(B+1)CDb\pi''}{A(D+1)a\Phi}-\frac{\pi b}{A.a\Phi}\right)-\frac{dn''}{n''-1}(C+1)c\left(\frac{\pi'' D}{AB(D+1)a\Phi}-\frac{\pi'}{AB(C+1)a\Phi}\right)$$

Addatur

$$d\alpha\left(\frac{1}{a}+\frac{1}{b}\right)=\frac{\pi B d\alpha}{A(B+1)a\Phi}$$

eritque

$$-\frac{dn}{n-1}.\left(\frac{(A+1)BCD\gamma\pi''}{(D+1)a\Phi}\right)$$
$$-\frac{dn'}{n'-1}\left(\frac{(B+1)CDb\pi''-(D+1)b\pi}{A(D+1)a\Phi}\right)$$
$$-\frac{dn''}{n''-1}\left(\frac{(C+1)Dc\pi''-(D+1)c\pi'}{AB(D+1)a\Phi}\right)$$

Vnde

Vnde pro casu $O = o$ aequatio, qua margo coloratus destruitur, erit

$$o = \frac{a\,dn}{n-1} \cdot (A+1) B C D \pi''$$
$$+ \frac{b\,dn'}{n'-1} \cdot \left(\frac{(B+1) C D \pi'' - (D+1)\pi}{A} \right)$$
$$+ \frac{c\,dn''}{n''-1} \left(\frac{(C+1) D \pi'' - (D+1)\pi'}{A\,B} \right)$$

III) Vt autem praeterea omnis confusio tollatur, insuper reddi oportet $d\delta = o$; vnde haec aequatio nascitur

$$o = \begin{cases} \frac{a\,dn}{n-1} \cdot A(A+1) B^2 C^2 D^2 \\ + \frac{b\,dn'}{n'-1} \cdot B(B+1) C^2 D^2 \\ + \frac{c\,dn''}{n''-1} \cdot C(C+1) D^2 \\ + \frac{d.\,dn'''}{n'''-1} \cdot D(D+1) \end{cases}$$

quae per $A^2 B^2 C^2 D^2$ diuisa dat

$$o = \begin{cases} \frac{a\,dn}{n-1} \cdot \frac{A+1}{A} \\ + \frac{b\,dn'}{n'-1} \cdot \frac{B+1}{A^2 B} \\ + \frac{c\,dn''}{n''-1} \cdot \frac{C+1}{A^2 B^2 C} \\ + \frac{d.\,dn'''}{n'''-1} \cdot \frac{D+1}{A^2 B^2 C^2 D} \end{cases}$$

Circa hanc autem aequationem inprimis notandum est, si omnes lentes pari facultate refringente sint praeditae, ei satisfieri haud posse; ex quo haec aequatio proprie pertinet ad casum, quo diuersae refractiones locum habent.

Pro-

Problema 8.

325. Si inftrumentum dioptricum ex quotcunque lentibus fit compofitum, quarum craffitiem negligere liceat, eam determinare difpofitionem, vt oculus in eo loco, quem campus apparens poftulat pofitus nullam confufionem fentiat.

Solutio.

Sit obiecti ante lentem primam diftantia $AE = a$, eiusque magnitudo $E\epsilon = z$, quae quidem confpici queat, et ftatuatur $\frac{z}{a} = \Phi$. Deinde fint diftantiae imaginum a radiis mediae naturae formatarum vt fupra:

$$AE = a;\ BF = b;\ CG = c;\ DH = d;\ EI = e$$
$$a\,F = \alpha;\ b\,G = \beta;\ c\,H = \gamma;\ d\,I = \delta;\ e\,K = \epsilon$$

etc.

ac vocemus breuitatis gratia

$$\alpha = Aa,\ \beta = Bb,\ \gamma = Cc,\ \delta = Dd,\ \epsilon = Ee \text{ etc.}$$

tum vero etiam

$$\frac{A}{A+1} = \mathfrak{A};\ \frac{B}{B+1} = \mathfrak{B};\ \frac{C}{C+1} = \mathfrak{C};\ \frac{D}{D+1} = \mathfrak{D},\ \frac{E}{E+1} = \mathfrak{E} \text{ etc.}$$

Iam apertura primae lentis PP vt euanefcente confiderata fit ratio aperturae pro reliquis lentibus

$$QQ = \pi;\ RR = \pi';\ SS = \pi'';\ TT = \pi''' \text{ etc.}$$

His

His poſitis ſupra vidimus (266) fore praeter $\alpha = \mathrm{A}a$

$$b = \frac{\mathrm{A}a\Phi}{\mathfrak{B}\pi - \Phi} \qquad \beta = \frac{\mathrm{AB}a\Phi}{\mathfrak{B}\pi - \Phi}$$

$$c = \frac{\mathrm{AB}a\Phi}{\mathfrak{C}\pi' - \pi + \Phi} \qquad \gamma = \frac{\mathrm{ABC}a\Phi}{\mathfrak{C}\pi' - \pi + \Phi}$$

$$d = \frac{\mathrm{ABC}a\Phi}{\mathfrak{D}\pi'' - \pi' + \pi - \Phi} \qquad \delta = \frac{\mathrm{ABCD}a\Phi}{\mathfrak{D}\pi'' - \pi' + \pi - \Phi}$$

$$e = \frac{\mathrm{ABCD}a\Phi}{\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi} \qquad \varepsilon = \frac{\mathrm{ABCDE}a\Phi}{\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi}$$

etc. etc.

Deinde ſingularum imaginum magnitudo ita ſe habebit

$$\mathrm{F}\zeta = \mathrm{A}a\Phi;\ \mathrm{G}\eta = \mathrm{AB}a\Phi;\ \mathrm{H}\theta = \mathrm{ABC}a\Phi;\ \mathrm{I}\iota = \mathrm{ABCD}a\Phi$$

etc.

et ipſi aperturarum ſemidiametri:

Lentis ſecundae $\mathrm{QQ} = \frac{\mathrm{A}\mathfrak{B}a\Phi}{\mathfrak{B}\pi - \Phi}\pi$

Lentis tertiae $\mathrm{RR} = \frac{\mathrm{AB}\mathfrak{C}a\Phi}{\mathfrak{C}\pi' - \pi + \Phi}\pi'$

Lentis quartae $\mathrm{SS} = \frac{\mathrm{ABC}\mathfrak{D}a\Phi}{\mathfrak{D}\pi'' - \pi' + \pi - \Phi}\pi''$

Lentis quintae $\mathrm{TT} = \frac{\mathrm{ABCD}\mathfrak{E}a\Phi}{\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi}\cdot\pi'''$

Mutata iam refractionis lege $n : 1$ infinite parum, diſtantiae $\alpha, b, \beta, c; \gamma, d, \delta$ etc. tales mutationes recipiunt.

$$d\alpha = -\frac{dn}{n-1}.\mathrm{A}a(\mathrm{A}+1)$$

$$d\beta = \frac{-dn}{n-1}.\mathrm{AB}a\left((\mathrm{A}+1)\mathrm{B} + \frac{(\mathrm{B}+1)\Phi}{\mathfrak{B}\pi - \Phi}\right)$$

$$d\gamma = \frac{-dn}{n-1}.\mathrm{ABC}a\left((\mathrm{A}+1)\mathrm{BC} + \frac{(\mathrm{B}+1)\mathrm{C}\Phi}{\mathfrak{B}\pi - \Phi} + \frac{(\mathrm{C}+1)\Phi}{\mathfrak{C}\pi' - \pi + \Phi}\right)$$

$$d\delta = \frac{-dn}{n-1}\cdot\mathrm{ABCD}a\left((\mathrm{A}+1)\mathrm{BCD} + \frac{(\mathrm{B}+1)\mathrm{CD}\Phi}{\mathfrak{B}\pi - \Phi} + \frac{(\mathrm{C}+1)\mathrm{D}\Phi}{\mathfrak{C}\pi' - \pi + \Phi} + \frac{(\mathrm{D}+1)\Phi}{\mathfrak{D}\pi'' - \pi' + \pi - \Phi}\right)$$

etc.

 Porro

Porro vero habetur:

$$\frac{1}{\alpha}+\frac{1}{b}=\frac{1}{Aa\Phi}\cdot\mathfrak{B}\pi$$

$$\frac{1}{\beta}+\frac{1}{c}=\frac{1}{ABa\Phi}\left(\mathfrak{C}\pi'-\frac{\mathfrak{B}\pi}{B}\right)$$

$$\frac{1}{\gamma}+\frac{1}{d}=\frac{1}{ABCa\Phi}\left(\mathfrak{D}\pi''-\frac{\mathfrak{C}\pi'}{C}\right)$$

$$\frac{1}{\delta}+\frac{1}{e}=\frac{1}{ABCDa\Phi}\left(\mathfrak{E}\pi'''-\frac{\mathfrak{D}\pi''}{D}\right)$$

etc.

His pofitis pro quolibet lentium numero feorfim formulas quaefito fatisfacientes expediamus: pofita diftantia oculi poft lentem vltimam $=O$.

I. Pro vnica Lente

Habetur haec aequatio differentialis $\frac{-O\,d\alpha}{\alpha(\alpha-O)}=0$, pro quo cafu tam magnitudo imaginis quam eius locus manebit inuariatus fi fuerit $d\alpha=0$, hoc eft $A(A+1)=0$, vnde deberet effe vel $A=0$ vel $A=-1$: quorum prius vifio non admittit pofterius autem lentem tollit. Tum vero ob campum effe debet $O=0$.

II. Pro duabus lentibus

Habetur haec aequatio differentialis, qua margo coloratus tollitur:

$$d\alpha\left(\frac{1}{\alpha}+\frac{1}{b}\right)-\frac{O\,d\mathfrak{C}}{\mathfrak{C}(\mathfrak{C}-O)}=0$$

at ob campum apparentem eft $\frac{O}{\mathfrak{C}(\mathfrak{C}-O)}=\frac{1}{BB}\left(\frac{1}{\alpha}+\frac{1}{b}\right)$, ita vt habeamus:

$$d\alpha\left(\frac{1}{\alpha}+\frac{1}{b}\right)-\frac{d\mathfrak{C}}{BB}\left(\frac{1}{\alpha}+\frac{1}{b}\right)=0$$

Verum

Verum eſt $d\mathfrak{E} = BBd\alpha - \frac{d\pi}{n-1} . ABa . \frac{(B+1)\Phi}{\mathfrak{B}\pi - \Phi}$

Quare ſi O habeat valorem poſitiuum erit ob

$$\mathfrak{B}(B+1) = B, \frac{\pi}{\mathfrak{B}\pi - \Phi} = 0$$

Sin autem valor O prodeat negatiuus, quo caſu capitur $O = 0$ erit $\frac{(A+1)\mathfrak{B}\pi}{\Phi} = 0.$

Omnis autem confuſio penitus tolletur, ſi inſuper fuerit

$$\frac{1}{\mathfrak{A}} + \frac{\Phi}{A\mathfrak{B}(\mathfrak{B}\pi - \Phi)} = 0$$

III. Pro tribus Lentibus

Si calculum eodem modo proſequamur, obiectum ſine margine colorato conſpicietur:

1. Si ex campo apparente diſtantia O prodeat poſitiua hanc aequationem adimplendo:

$$\frac{\pi}{\mathfrak{B}\pi - \Phi} + \frac{\pi'}{\mathfrak{C}\pi' - \pi + \Phi} = 0.$$

2. Si ob diſtantiam O prodeuntem negatiuam capiatur $O = 0$ huic aequationi erit ſatisfaciendum:

$$\frac{\pi'}{\mathfrak{A}\Phi} + \frac{\pi'}{A\mathfrak{B}(\mathfrak{B}\pi - \Phi)} = \frac{\pi}{AB\mathfrak{C}(\mathfrak{B}\pi - \Phi)}$$

Omnis autem confuſio penitus tolletur ſi fuerit inſuper

$$\frac{1}{\mathfrak{A}} + \frac{\Phi}{A\mathfrak{B}(\mathfrak{B}\pi - \Phi)} + \frac{\Phi}{AB\mathfrak{C}(\mathfrak{C}\pi' - \pi + \Phi)} = 0.$$

IV. Pro quatuor Lentibus

Vt obiectum ſine margine colorato ſpectetur:

1. Si

1. Si ex campo apparente diſtantia O prodeat poſitiua huic aequationi erit ſatisfaciendum:

$$\frac{\pi}{\mathfrak{B}\pi-\Phi}+\frac{\pi'}{\mathfrak{C}\pi'-\pi+\Phi}+\frac{\pi''}{\mathfrak{D}\pi''-\pi'+\pi-\Phi}=0$$

2. Sin autem capiatur $O=0$, huic

$$\frac{\pi''}{\mathfrak{A}\Phi}+\frac{\pi''}{A\mathfrak{B}(\mathfrak{B}\pi-\Phi)}+\frac{\pi''}{AB\mathfrak{C}(\mathfrak{C}\pi'-\pi+\Phi)}=\frac{\pi}{ABC\mathfrak{D}(\mathfrak{B}\pi-\Phi)}+\frac{\pi'}{ABC\mathfrak{D}(\mathfrak{C}\pi'-\pi+\Phi)}$$

Omnis vero confuſio penitus tolletur ſi fuerit praeterea

$$\frac{1}{\mathfrak{A}}+\frac{\Phi}{A\mathfrak{B}(\mathfrak{B}\pi-\Phi)}+\frac{\Phi}{AB\mathfrak{C}(\mathfrak{C}\pi'-\pi+\Phi)}+\frac{\Phi}{ABC\mathfrak{D}(\mathfrak{D}\pi''-\pi'+\pi-\Phi)}=0$$

V. Pro quinque Lentibus

Vt obiectum tantum ſine margine colorato conſpiciatur

1. Si ex campo apparente diſtantia O prodeat poſitiua, huic aequationi erit ſatisfaciendum:

$$\frac{\pi}{\mathfrak{B}\pi-\Phi}+\frac{\pi'}{\mathfrak{C}\pi'-\pi+\Phi}+\frac{\pi''}{\mathfrak{D}\pi''-\pi'+\pi-\Phi}+\frac{\pi'''}{\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi}=0$$

2. Sin autem capiatur $O=0$ huic

$$\frac{\pi'''}{\mathfrak{A}\Phi}+\frac{\pi'''}{A\mathfrak{B}(\mathfrak{B}\pi-\Phi)}+\frac{\pi'''}{AB\mathfrak{C}(\mathfrak{C}\pi'-\pi+\Phi)}+\frac{\pi'''}{ABC\mathfrak{D}(\mathfrak{D}\pi''-\pi'+\pi-\Phi)}.$$

$$\frac{1}{ABCD\mathfrak{E}}\left(\frac{\pi}{\mathfrak{B}\pi-\Phi}+\frac{\pi'}{\mathfrak{C}\pi'-\pi+\Phi}+\frac{\pi''}{\mathfrak{D}\pi''-\pi'+\pi-\Phi}\right)$$

Omnis autem confuſio penitus tolletur, ſi praeterea ſatisfiat huic aequationi:

$$\frac{1}{\mathfrak{A}}+\frac{\Phi}{A\mathfrak{B}(\mathfrak{B}\pi-\Phi)}+\frac{\Phi}{AB\mathfrak{C}(\mathfrak{C}\pi'-\pi+\Phi)}+\frac{\Phi}{ABC\mathfrak{D}(\mathfrak{D}\pi''-\pi'+\pi-\Phi)}+\frac{\Phi}{ABCD\mathfrak{E}(\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi)}=0$$

Atque hinc manifeſta eſt progreſſio ad maiorem lentium numerum.

Coroll.

Coroll. 1.

326. In casu ergo vnicae lentis licet quidem obiectum a margine colorato liberare neque vero confusionem penitus tollere. Casu autem duarum lentium ne margo quidem coloratus tolli potest, siquidem oculus in eo loco, quem campi apparentis conditio postulat, teneatur.

Coroll. 2.

327. Quodsi vero plures duabus habeantur lentes sufficiens quantitatum numerus adest, quarum determinatione non solum margo coloratus deleri, sed etiam forte omnis confusio penitus auferri posse videtur praecipue si lentium numerus ternarium superet.

Scholion.

328. Quod ergo incommodum a diuersa radiorum natura oriundum adeo graue vel summo Neutono est visum, vt instrumenta dioptrica nullo modo ab eo liberari posse sit arbitratus, id quidem saltem quod ad marginem coloratum attinet ad quem Neutonus inprimis spectabat, iam satis feliciter tolli posse certum est; ita vt saltem ob hanc causam non opus sit ad Telescopia Catoptrica confugere. Hoc autem vitio sublato si praeterea alterum confusionis fontem obstruamus lentes scilicet nullam confusionem parientes adhibendo, nullum est dubium quin instrumenta dioptrica ad summum perfectionis gradum euehi queant.

Quae igitur hactenus particulatim circa singulas horum instrumentorum affectiones proposuimus, ea colligi conueniet, vnde in capite sequente praecepta generalia pro omnium instrumentorum dioptricorum constructione tradere est visum.

Supplementum VI.

Ex iis, quae ante sunt adiecta, poterimus etiam problematis solutionem pro casu exhibere, quo singulae lentes peculiari refractione sunt praeditae, vbi quidem tantum postremae aequationes pro confusione vitanda mutationem quandam postulant; interim tamen etiam priores formulas, quibus locus oculi, quem campus apparens requirit, distinctius repraesentemus.

I. *Distantia Oculi post vltimam lentem* pro quouis lentium numero se habebit, vt sequitur.

Num. lentium	O id est, distantia oculi post lentem vltimam
I.	o
II.	$\frac{A\mathfrak{B}a\pi\Phi}{(\pi-\Phi)(\mathfrak{B}\pi-\Phi)}$ seu $\frac{\mathfrak{B}b\pi}{\pi-\Phi}$
III.	$\frac{AB.\mathfrak{C}a\pi'\Phi}{(\pi'-\pi+\Phi)(\mathfrak{C}\pi'-\pi+\Phi)}$ seu $\frac{\mathfrak{C}c\pi'}{\pi'-\pi+\Phi}$
IV.	$\frac{ABC\mathfrak{D}.a\pi''\Phi}{(\pi''-\pi'+\pi-\Phi)(\mathfrak{D}\pi''-\pi'+\pi-\Phi)}$ seu $\frac{\mathfrak{D}d\pi''}{\pi''-\pi'+\pi-\Phi}$
V.	$\frac{ABCD\mathfrak{E}.a\pi'''\Phi}{(\pi'''-\pi''+\pi'-\pi+\Phi)(\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi)}$ seu $\frac{\mathfrak{E}e\pi'''}{\pi'''-\pi''+\pi'-\pi+\Phi}$

etc.

II.

II. Si valor ipſius O ſit poſitiuus, ad marginem coloratum tollendum ſequentes aequationes ſunt adimplendae:

Num. lentium	
I.	$0 = 0$
II.	$0 = \frac{bdn'}{n'-1} \cdot \frac{\pi}{A\alpha\Phi}$
III.	$0 = \frac{bdn'}{n'-1} \cdot \frac{\pi}{A\alpha\Phi} + \frac{cdn''}{n''-1} \cdot \frac{\pi'}{AB\alpha\Phi}$
IV.	$0 = \frac{bdn'}{n'-1} \cdot \frac{\pi}{A\alpha\Phi} + \frac{cdn''}{n''-1} \cdot \frac{\pi'}{AB\alpha\Phi} + \frac{ddn'''}{n'''-1} \cdot \frac{\pi''}{ABC\alpha\Phi}$

etc.

III. Si valor ipſius O prodeat negatiuus, quo caſu capi debet $O = 0$, ad marginem coloratum tollendum ſequentes aequationes ſunt adimplendae.

Num. lentium	
I.	$0 = 0$
II.	$0 = \frac{adn}{n-1} \cdot (A+1) B \pi$
III.	$0 = \frac{adn}{n-1}(A+1) BC\pi' + \frac{bdn'}{n'-1} \cdot \frac{(B+1)C\pi' - (C+1)\pi}{A}$
IV.	$0 = \frac{adn}{n-1} \cdot (A+1) BCD\pi'' + \frac{bdn'}{n'-1}\left(\frac{(B+1)CD\pi'' - (D+1)\pi}{A}\right) + \frac{cdn''}{n''-1}\left(\frac{(C+1)D\pi'' - (D+1)\pi'}{AB}\right)$

etc.

 IV.

IV. Vt autem insuper omnis confusio huius generis tollatur, sequentes aequationes sunt adimplendae:

Num. lentium	
I.	$0 = \frac{a\,dn}{n-1} \cdot \frac{A+1}{A}$
II.	$0 = \frac{a\,dn}{n-1} \cdot \frac{A+1}{A} + \frac{b\,dn'}{n'-1} \cdot \frac{B+1}{A^2 B}$
III.	$0 = \frac{a\,dn}{n-1} \cdot \frac{A+1}{A} + \frac{b\,dn'}{n'-1} \cdot \frac{B+1}{A^2 B} + \frac{c\,dn''}{n''-1} \cdot \frac{C+1}{A^2 B^2 C}$
IV.	$0 = \frac{a\,dn}{n-1} \cdot \frac{A+1}{A} + \frac{b\,dn'}{n'-1} \cdot \frac{B+1}{A^2 B} + \frac{c\,dn''}{n''-1} \cdot \frac{C+1}{A^2 B^2 C} + \frac{d\,dn'''}{n'''-1} \cdot \frac{D+1}{A^2 B^2 C^2 D}$

etc.

Quarum formularum ordo hinc distinctius perspicitur, quam in problemate.

CAPVT

CAPVT VII.

DE CONSTRVCTIONE INSTRVMENTORVM DIOPTRICORVM IN GENERE.

Problema I.

329.

Si instrumentum dioptricum vnica constet lente crassitiei cuiuscunque, PP, definire omnia momenta ad visionem pertinentia.

Tab. III. Fig. 12.

Solutio.

Quod primo ad ipsius lentis structuram attinet ponatur obiecti $E\epsilon$ ante eam distantia $AE = a$, imaginisque $F\zeta$ post eam proiectae $aF = \alpha$; tum vero lentis crassities $Aa = v$ et quantitas arbitraria $= k$, vnde capiatur $\frac{k-v}{k+v} = i$. Hinc facies lentis ita erunt formatae, vt sit existente $n = \frac{31}{20}$ Conf. §. 68.

radius faciei anterioris $= \frac{(n-1)a(k+v)}{k+v+2na}$

radius faciei posterioris $= \frac{(n-1)\alpha(k-v)}{k-v-2n\alpha}$.

Ponatur porro semidiameter obiecti conspicui $E\epsilon = z$, et semidiameter aperturae in facie anteriori $AM = x$, in facie autem posteriori semidiameter aperturae non

ſit minor quam ix. Denique oculi poſt lentem diſtantia vocetur $aO = O$, cuius diſtantia iuſta ſit $= l$. His poſitis ſequentia momenta perpendi oportet.

1. Debet eſſe $O = \alpha + l$ vt diſtantia imaginis viſae naturae oculi conueniat.

2. Conſideranda venit multiplicatio, quae ex ratione definitur, quam diameter obiecti viſus tenet ad eiusdem diametrum viſum ſi nudo oculo in diſtantia data $= b$ ſpectaretur. Quodſi ergo hic exponens multiplicationis ponatur $= m$ erit

$m = \frac{1}{i} \cdot \frac{\alpha b}{a l}$ pro ſitu inuerſo

3. Gradus claritatis determinatur ſemidiametro coni luminoſi, qui a quouis obiecti puncto in oculum immittitur, qui ſi ponatur $= y$ erit

$y = i\, l. \frac{x}{\alpha} = \frac{b x}{m a}$

4. Confuſio inquinans viſionem menſuratur ſemidiametro apparente circuli, qui nudo oculo aeque magnus cernitur, ac ſingula obiecti puncta per lentem ſpectata. Hanc menſuram vocaui (194) ſemidiametrum confuſionis, ad quem definiendum ſi ponatur:

$$P = \frac{n}{2(n-1)^2}\left(\left(\frac{n}{\alpha} + \frac{2}{k+v}\right)\left(\frac{1}{i\alpha} + \frac{2}{k-v}\right)^2 + \left(\frac{n}{\alpha} - \frac{2}{k-v}\right)\left(\frac{i}{\alpha} - \frac{2}{k+v}\right)^2\right)$$

erit ſemidiameter confuſionis $= \frac{1}{4} i . \frac{\alpha}{l} x^3 P = \frac{1}{4} i^2 . \frac{m a}{b} x^3 . P.$

5. Vt oculus maximum campum apparentem percipiat, debet eſſe $O = \frac{-i\alpha v}{n\alpha - i v}$

6. Quae

6. Quae diſtantia ſi fuerit poſitiua, ſemidiameter campi ſeu obiecti conſpicui $E\varepsilon = z$, ita pendet ab apertura faciei poſterioris, vt poſito huius ſemidiametro $= a$ ſit $z = \frac{na}{v}.\ a$.

7. Sin autem diſtantia illa pro O aſſignata prodierit negatiua, oculum lenti immediate applicari conueniet, vt ſit $O = o$; tum vero pro a poſito ſemidiametro pupillae ω erit pro campo apparente $z = \frac{na}{v}\omega$.

8. Vt obiectum ſine margine colorato appareat, exiſtente diſtantia O ante inuenta poſitiua, debet eſſe $i = \frac{a}{a+v}$. ſeu $k = 2\,a + v$.

9. Sin autem ob illam diſtantiam negatiuam prodeuntem capiatur $O = o$, vt margo coloratus euitetur, oportet eſſe $\frac{v}{a} + 1 - i = o$, vnde ſit $k = -2\,a - v$.

10. Omnis denique confuſio a diuerſa radiorum refrangibilitate oriunda prorſus tolletur, ſi inſuper fieri poſſet $(a+v)(a+a+v) = o$

Coroll. 1.

330. Si exponens multiplicationis m cum gradu claritatis y proponatur, erit $my = \frac{bx}{a}$. quae proprietas ad lentium numerum quantumuis magnum patet. Cum ergo $\frac{b}{a}$ ſit quantitas data, erit x vt my; tum vero y vt $\frac{x}{m}$, ac m vt $\frac{x}{y}$

Coroll. 2.

331. Tam maior ergo multiplicatio quam maior claritatis gradus poſtulat maiorem aperturam. Verum

Verum aucto x confusio augetur in ratione triplicata, siquidem quantitas P maneat eadem: quare si x ex confusione etiamnum tolerabili determinetur, simul quantitas my determinatur.

Coroll. 3.

332. Vt igitur tam multiplicationem m quam claritatem y salua confusione maxime augere liceat, quantitatem arbitrariam k ita definiri conueniet, vt litterae P minimus valor concilietur.

Cum autem posuerimus $\frac{k-v}{k+v} = i$, valor ipsius P fiet minimus, huic aequationi satisfaciendo:

$$2i^4\left(\frac{n}{\alpha}+\frac{1}{v}\right)\left(\frac{1}{\alpha}+\frac{2}{v}\right)^2-\frac{2ni^3}{\alpha v}\left(\frac{1}{\alpha}+\frac{1}{v}\right)-\frac{i^3}{v}\left(\frac{1}{\alpha}+\frac{2}{v}\right)^2$$
$$+\frac{2ni}{av}\left(\frac{1}{a}+\frac{1}{v}\right)+\frac{i}{v}\left(\frac{1}{a}+\frac{2}{v}\right)^2-2\left(\frac{n}{a}+\frac{1}{v}\right)\left(\frac{1}{a}+\frac{1}{v}\right)^2=0$$

vti §. 45 inuenimus.

Coroll. 4.

333. Si loco i valor sustituatur suus $\frac{k-v}{k+v}$, aequatio transibit in hanc formam:

$$+\frac{2n}{a^3}(k+v)^4+\frac{(2n+1)}{aav}(k+v)^3(k+3v)+\frac{4(n+2)}{av}(k+v)^2$$
$$-\frac{2n}{\alpha^3}(k-v)^4-\frac{(2n+1)}{\alpha\alpha v}(k-v)^3(k-3v)+\frac{4(n+2)}{\alpha v}(k-v)^2+32k=0$$

quae per v multiplicata etiam ad casum accommodari potest quo est $v=0$: tum autem reperitur

$$k=\frac{4(n+2)a\alpha}{(2n+1)(a-\alpha)} \text{ et}$$

$$P=\frac{n}{8(n-1)^2(n+2)}\left(\frac{1}{a}+\frac{1}{\alpha}\right)\left((4n-1)\left(\frac{1}{a}+\frac{1}{\alpha}\right)^2+\frac{4(n-1)^2}{a\alpha}\right)$$

Scholion

Scholion

334. Hic non generatim omnem visionem, quae fit per vnicam lentem, considero, sed tantum eam, qua maximus campus apparens conspicitur; quamobrem locum oculi ita definiui, vt ipsi maximus campus adferatur. Sin autem minore campo velimus esse contenti oculus quoque alibi post lentem constitutus obiecta distincte cernere poterit, quemadmodum per lentes satis amplas, quae tabularum vitrearum nomine sunt notae, fieri solet. Hunc autem casum quoniam per se facile expeditur, atque in instrumentis dioptricis magnitudo campi potissimum spectatur, hic non attingo.

Problema 2.

335. Si instrumentum dioptricum duabus constet lentibus cuiuscunque crassitiei definire omnia momenta ad visionem pertinentia. Tab. III. Fig. 13.

Solutio.

Obiecto constituto in $E\epsilon$, eius imagines proiiciantur per istas lentes in $F\zeta$ et $G\eta$, ac ponantur quantitates vtramque lentem determinantes

$AE = a$, $aF = \alpha$, crassities $Aa = v$ et distantia arbitraria $= k$

$BF = b$, $bG = \beta$, crassities $Bb = v'$ et distantia arbitraria $= k'$

ponaturque breuitatis gratia $\frac{k-v}{k+v}=i$ et $\frac{k'-v'}{k'+v'}=i'$. Hinc existente $n=\frac{31}{20}$, constructio vtriusque lentis ita se habebit,

Radius faciei	anterioris	posterioris
Pro Lente PP . .	$\frac{(n-1)a(k+v)}{k+v+2na}$ . .	$\frac{(n-1)\alpha(k-v)}{k-v-2n\alpha}$
Pro Lente QQ . .	$\frac{(n-1)b(k'+v')}{k'+v'+2nb}$ . .	$\frac{(n-1)\beta(k'-v')}{k'-v'-2n\beta}$

Sit porro semidiameter aperturae lentis primae PP in facie anteriori $=x$, in facie autem posteriori maior sit quam ix. Tum vero lentis secundae QQ semidiameter aperturae in facie anteriori maior esse debet quam $i.\frac{bx}{\alpha}$, in posteriori vero maior quam $ii'.\frac{bx}{\alpha}$. Deinde sit semidiameter obiecti conspicui $E\varepsilon=z$, distantia oculi $bO=O$, eiusque distantia iusta $=l$. His positis ad sequentia momenta erit attendendum.

1. Vt oculus imaginem $G\eta$ in distantia iusta conspiciat, oportet esse $O=\beta+l$.

2. Posita distantia $=h$ ad quam multiplicatio referatur, sit exponens multiplicationis $=m$, ac supra inuenimus esse oportere $m=\frac{1}{ii'}.\frac{\alpha\beta h}{abl}$ pro situ erecto.

3. Denotante y gradum claritatis fiet

$$y=ii'l.\frac{bx}{\alpha\beta}\text{ ideoque }my=\frac{hx}{a}$$

4. Pro confusione posito brevitatis ergo

$$P=\frac{n}{2(n-1)^2}\left(\left(\frac{n}{a}+\frac{2}{k+v}\right)\left(\frac{1}{ia}+\frac{2}{k-v}\right)^2+\left(\frac{n}{\alpha}+\frac{2}{k-v}\right)\left(\frac{i}{\alpha}-\frac{2}{k+v}\right)^2\right)$$

$$Q=\frac{n}{2(n-1)^2}\left(\left(\frac{n}{b}+\frac{2}{k'+v'}\right)\left(\frac{1}{i'b}+\frac{2}{k'-v'}\right)^2+\left(\frac{n}{\beta}-\frac{2}{k'-v'}\right)\left(\frac{i'}{\beta}-\frac{2}{k'+v'}\right)^2\right)$$

erit-

eritque

femid. confufionis $= \frac{1}{4} i i' . \frac{\mathfrak{E}}{I} . \frac{b}{\alpha} x^3 \left(\frac{1}{i' i'} . \frac{\alpha \alpha}{b b} P + i i . \frac{b b}{\alpha \alpha} Q \right).$

5. Pro campo apparente definiendo, qui pendet ab apertura fingularum facierum lentium, fi ponamus

Semidiametrum aperturae

Pro lente PP $\begin{cases} \text{faciei anterioris} = \mathfrak{A} = x \\ \text{faciei pofterioris} = \mathfrak{a} \end{cases}$

Pro lente QQ $\begin{cases} \text{faciei anterioris} = \mathfrak{B} \\ \text{faciei pofterioris} = \mathfrak{b} \end{cases}$

habebimus fequentes aequationes

$$\mathfrak{a} = \frac{v}{n a} z$$

$$\mathfrak{B} = \left(\frac{1}{i} . \frac{\alpha + b}{a} - \frac{b v}{n a \alpha} \right) z$$

$$\mathfrak{b} = \left(\frac{i'}{i} . \frac{\alpha + b}{a} - \frac{i' b v}{n a \alpha} - \frac{1}{i} . \frac{\alpha v'}{n a b} \right) z$$

ex quibus tribus aequationibus valor ipfius z minimus praebet femidiametrum campi apparentis.

6. Quem campum vt oculus reuera perfpiciat, eius locus ita debet fumi, vt fit

$$O = \frac{\frac{i'}{i} \frac{\alpha + b}{a} - \frac{i' b v}{n a \alpha} - \frac{1}{i} \frac{\alpha v'}{n a b}}{\frac{i'}{i} . \frac{\alpha + b}{a} - \frac{i' b v}{n a \alpha} - \frac{1}{i} . \frac{\alpha v'}{n a b} + \frac{1}{\mu'} . \frac{\alpha \mathfrak{E}}{a b}} . \mathfrak{E}$$

fi quidem haec diftantia fuerit pofitiua.

7. Sin autem haec diftantia prodierit negatiua, oculum lenti QQ immediate applicari conuenit, vt fit $O = o$; tum vero pro campo apparente inue-

 niendo

niendo in aequationibus n°. 5 allatis loco $\mathfrak{C}$ scribatur semidiameter pupillae ω, atque ex iisdem valor ipsius z minimus erutus dabit semidiametrum campi apparentis.

8. Obiectum porro sine margine colorato cernetur, existente distantia O n° 6 inuenta positiua, si huic aequationi satisfiat:

$$0 = 1 + \frac{b}{\alpha} + \frac{i' \iota' b}{\mathfrak{C}}\left(1 + \frac{b}{\alpha}\right) + \frac{(1-i')^2 b(\alpha + b)}{\alpha v'} + \frac{2-i-i'}{n}$$
$$- \frac{iv}{n\alpha} - \frac{i'v'}{n\mathfrak{C}} - \frac{ibv}{n\alpha\alpha} - \frac{i\iota' i' bbv}{n\alpha\alpha\mathfrak{C}} - \frac{i(1-i')^2 bbv}{n\alpha\alpha v'}$$

9. Sin autem capiatur distantia O $= 0$, obiectum margine colorato carebit, si fuerit

$$0 = \frac{1-i}{in} + \frac{1-i'}{i'n} + \frac{v}{ina} + \frac{v'}{inb} + \alpha\left(1 + \frac{\alpha}{ia} + \frac{(1-i)^2\alpha}{iiv}\right)\left(\frac{v'}{n\iota' bb} - \frac{1}{\alpha} - \frac{1}{b}\right)$$

10. Omnis autem confusio penitus tolletur, quae quidem a diuersa radiorum refrangibilitate proficiscitur, si huic aequationi satisfacere licuerit $d\mathfrak{C} = 0$, seu huic

$$0 = \frac{\alpha\alpha}{i\iota' bb}\left(\frac{i}{\alpha} + \frac{1}{ia} + \frac{(1-i)^2}{iv}\right) + \frac{i'}{\mathfrak{C}} + \frac{1}{\iota' b} + \frac{(1-i')^2}{i' v'}.$$

Coroll. 1.

336. Quia ambarum lentium distantia aB necessario est positiua, oportet sit $\alpha + b > 0$. Tum vero aperturae ita debent esse comparatae vt sit $\mathfrak{a} > ix$; $\mathfrak{B} > i . \frac{bx}{\alpha}$ et $\mathfrak{C} > ii' . \frac{bx}{\alpha}$.

Coroll. 2.

337. Casu igitur, quo ob valorem ipsius O prodeuntem negatiuum distantia O $= 0$ assumitur, quia

quia tum $b = \omega$ ſtatuitur, etiam eſſe debet $\omega > ii'\frac{bx}{\alpha}$ ſeu $x < \frac{\alpha\omega}{n'b}$. Scilicet hoc caſu inutile eſſet primam aperturam maiorem ſumere, quia radii non in oculum ingrederentur.

Scholion.

338. Si ſimili modo inſtrumenta dioptrica pluribus lentibus inſtructa proſequi vellemus, in calculos tantopere intricatos delaberemur, vt ex iis nihil fere concludere liceret. Oritur autem haec calculi complicatio a craſſitie lentium, qua neglecta omnia fiunt multo concinniora. Quare ſi pluribus lentibus vti velimus, craſſitiem tam paruam aſſumamus, vt ſine errore pro nihilo haberi poſſit, id quod in praxi etiam ſedulo obſeruari ſolet. Praeterea etiam campi apparentis locique oculi determinatio non capit rigorem geometricum, parumque refert ſi in ea aliquantum aberretur. Similis quoque ratio eſt conditionum, quibus effectus a diuerſa radiorum refrangibilitate oriundus tollitur; ſufficiet enim iis proxime ſatisfeciſſe, cum perfecta huius confuſionis deſtructio ne ſperari quidem poſſit. Quocirca in ſequentibus, vbi inſtrumenta pluribus lentibus inſtructa euoluemus, craſſitiem lentium in calculo penitus praetermittamus: in quo negotio ne eadem toties repetere opus habeamus, problema generale praemittamus in quo omnia momenta generatim tantum exponamus eaque deinceps pro quouis lentium numero accurate deſcribamus.

 Problema.

Problema 5.

339. Si inſtrumentum dioptricum compoſitum ſit ex lentibus quotcunque quarum craſſitiem negligere liceat, elementa eius conſtructionem continentia exponere, ex quibus deinceps regulae dirigentes conſtructionem ipſam ſtabiliri poſſint.

Solutio.

Obiecto in E conſtituto ponantur diſtantiae determinatrices ſingularum lentium vna cum numeris arbitrariis ad ſingulas pertinentibus vt ſequitur.

Pro Lente prima $EA = a$; $aF = \alpha$; num: arb: $= \lambda$

Pro Lente ſecunda $FB = b$; $bG = \beta$; num: arb: $= \lambda'$

Pro Lente tertia $GC = c$; $cH = \gamma$; num: arb: $= \lambda''$

Pro Lente quarta $HD = d$; $dI = \delta$; num: arb: $= \lambda'''$

Pro Lente quinta $IE = e$; $eK = \varepsilon$; num: arb: $= \lambda''''$

Pro Lente ſexta $KF = f$; $fL = \zeta$; num: arb: $= \lambda'''''$

etc.

ex quibus elementis quomodo ſingulae lentes debeant formari ſupra eſt expoſitum.

Nunc autem porro ponatur breuitatis gratia:

$\alpha = Aa$; $\beta = Bb$; $\gamma = Cc$: $\delta = Dd$; $\varepsilon = Ee$; $\zeta = Ff$

etc.

tum

tum vero ftatuatur etiam

$$\frac{A}{A+1}=\mathfrak{A};\ \frac{B}{B+1}=\mathfrak{B};\ \frac{C}{C+1}=\mathfrak{C};\ \frac{D}{D+1}=\mathfrak{D};\ \frac{E}{E+1}=\mathfrak{E};\ \frac{F}{F+1}=\mathfrak{F}$$

etc.

ita vt fint diftantiae focales lentium:

$$\mathfrak{A}a;\ \mathfrak{B}b;\ \mathfrak{C}c;\ \mathfrak{D}d;\ \mathfrak{E}e;\ \mathfrak{F}f \quad \text{etc.}$$

Nunc fit femidiameter aperturae primae lentis obiectiuae $=x$, pro reliquis vero lentibus ratio aperturarum litteris π, π', π'', π''', π'''' etc. exponatur; ita vt femidiameter aperturae, cuiusque maior accipi debeat quam fecundum has rationes: fcilicet capi oportebit:

Semid: apert:

Lentis Secundae $>\pi\,\mathfrak{B}\,b$
Lentis tertiae $>\pi'\,\mathfrak{C}\,c$
Lentis quartae $>\pi''\,\mathfrak{D}\,d$
Lentis quintae $>\pi'''\,\mathfrak{E}\,e$
Lentis fextae $>\pi''''\,\mathfrak{F}\,f$ etc.

Quod fi iam femidiameter fpatii in obiecto confpicui vocetur $=z$, fiatque $\frac{z}{a}=\Phi$, vt fit $z=a\Phi$, atque hinc diftantiae determinatrices ita exprimentur, vt fit primo $\alpha=Aa$ tum vero:

$$b=\frac{Aa\Phi}{\mathfrak{B}\pi-\Phi}; \qquad \beta=\frac{ABa\Phi}{\mathfrak{B}\pi-\Phi}$$

$$c=\frac{ABa\Phi}{\mathfrak{C}\pi'-\pi+\Phi}; \qquad \gamma=\frac{ABCa\Phi}{\mathfrak{C}\pi'-\pi+\Phi}$$

$$d=\frac{ABCa\Phi}{\mathfrak{D}\pi''-\pi'+\pi-\Phi}; \qquad \delta=\frac{ABCDa\Phi}{\mathfrak{D}\pi''-\pi'+\pi-\Phi}$$

$$e=\frac{ABCDa\Phi}{\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi}; \quad \varepsilon=\frac{ABCDEa\Phi}{\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi}$$

$$f=\frac{ABCDEa\Phi}{\mathfrak{F}\pi''''-\pi'''+\pi''-\pi'+\pi-\Phi}; \quad \zeta=\frac{ABCDEFa\Phi}{\mathfrak{F}\pi''''-\pi'''+\pi''-\pi'+\pi-\Phi} \quad \text{etc.}$$

Circa

Circa has expressiones primum notandum est, aggregata $\alpha+b$; $\mathfrak{b}+c$; $\gamma+d$; $\delta+e$; $\varepsilon+f$ etc. esse oportere positiua quippe quibus lentium interualla exprimuntur. Erit nempe

Interuallum
Lentium

$$\text{I et II} = \frac{A\mathfrak{B}\alpha\pi}{\mathfrak{B}\pi-\Phi} > 0$$

$$\text{II et III} = \frac{AB\alpha\Phi(\mathfrak{C}\pi'-(1-\mathfrak{B})\pi)}{(\mathfrak{B}\pi-\Phi)(\mathfrak{C}\pi'-\pi+\Phi)} > 0$$

$$\text{III et IV} = \frac{ABC\alpha\Phi(\mathfrak{D}\pi''-(1-\mathfrak{C})\pi')}{(\mathfrak{C}\pi'-\pi+\Phi)(\mathfrak{D}\pi''-\pi'+\pi-\Phi)} > 0$$

$$\text{IV et V} = \frac{ABCD\alpha\Phi(\mathfrak{E}\pi'''-(1-\mathfrak{D})\pi'')}{(\mathfrak{D}\pi''-\pi'+\pi-\Phi)(\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi)} > 0$$

$$\text{V et VI} = \frac{ABCDE\alpha\Phi(\mathfrak{F}\pi''''-(\ -\mathfrak{E})\pi''')}{(\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi)(\mathfrak{F}\pi''''-\pi'''+\pi''-\pi'+\pi-\Phi)} > 0$$

etc.

Praeterea vt lentes sequentes omnes radios a prima exceptos transmittant, debet esse

Semid: aperturae

Lentis secundae $> \frac{\Phi}{\mathfrak{B}\pi-\Phi}x$

Lentis tertiae $> \frac{\Phi}{\mathfrak{C}\pi'-\pi+\Phi}x$

Lentis quartae $> \frac{\Phi}{\mathfrak{D}\pi''-\pi'+\pi-\Phi}x$

Lentis quintae $> \frac{\Phi}{\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi}x$

Lentis sextae $> \frac{\Phi}{\mathfrak{F}\pi''''-\pi'''+\pi''-\pi'+\pi-\Phi}x$

etc.

Denique

Denique ex §. 214 posito $\mu = 0,938191$ et $\nu = 0,232692$ erunt litterarum maiuscularum P, Q, R, etc. valores

$$P = \frac{\mu}{A^3 a^3}(A+1)(\lambda(A+1)^2+\nu A)$$
$$Q = \frac{\mu}{B^3 b^3}(B+1)(\lambda'(B+1)^2+\nu B)$$
$$R = \frac{\mu}{C^3 c^3}(C+1)(\lambda''(C+1)^2+\nu C)$$
$$S = \frac{\mu}{D^3 d^3}(D+1)(\lambda'''(D+1)^2+\nu D)$$
$$T = \frac{\mu}{E^3 e^3}(E+1)(\lambda''''(E+1)^2+\nu E)$$
$$V = \frac{\mu}{F^3 f^3}(F+1)(\lambda'''''(F+1)^2+\nu F)$$

etc.

His ita constitutis denominationibus, sit O distantia oculi post lentem vltimam, cuius distantia iusta ponatur $= l$. Deinde statuatur exponens multiplicationis $= m$, relatus ad distantiam b, ita vt diameter obiecti per instrumentum visi m vicibus maior cernatur, quam si idem obiectum a nudo oculo in distantia $= b$ aspiceretur: multiplicationi autem adiungi conuenit situm indicando, vtrum obiectum situ erecto an inuerso sit appariturum. Porro gradus claritatis y denotet semidiametrum coni luminosi, qui a quouis obiecti puncto in oculum transmittitur posito semidiametro pupillae $= \omega$. Semidiametrum denique confusionis voco semidiametrum apparentem circulorum qui in nudo oculo aeque magni depinguntur, atque singula obiecti puncta per instrumentum in oculo.

Coroll. 1.

340. Quia pro apertura ſingularum lentium geminos limites inuenimus, ſemidiameter aperturae cuiusque conuenientiſſime aggregato amborum limitum aequalis aſſumitur, vel ſaltem non minor. Vtrque autem limes etſi forte alter prodeat negatiuus, affirmatiue accipi debet.

Coroll. 2.

341. Hinc ergo ſequentes conſequimur formulas pro ſingularum lentium aperturis.

Semidiam: apert:

Lentis ſecundae $= \frac{A\mathfrak{B}a\pi \pm x}{\mathfrak{B}\pi - \Phi} \cdot \Phi$

Lentis tertiae $= \frac{AB\mathfrak{C}a\pi' \pm x}{\mathfrak{C}\pi' - \pi + \Phi} \cdot \Phi$

Lentis quartae $= \frac{ABC\mathfrak{D}a\pi'' \pm x}{\mathfrak{D}\pi'' - \pi' + \pi - \Phi} \cdot \Phi$

Lentis quintae $= \frac{ABCD\mathfrak{E}a\pi''' \pm x}{\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi} \cdot \Phi$

Lentis ſextae $= \frac{ABCDE\mathfrak{F}a\pi'''' \pm x}{\mathfrak{F}\pi'''' - \pi''' + \pi'' - \pi' + \pi - \Phi} \cdot \Phi$

etc.

Coroll. 3.

342. Ex diſtantiis binis determinatricibus cum numero arbitrario quaelibet lens facile conſtruitur; id quod pro lente prima ex ſuperioribus repetamus. Nempe ſi ſit $\lambda > 1$ lens ſimplex ſatisfaciet cuius conſtructio poſito breuitatis gratia:

$\varrho = 0,190781$; $\sigma = 1,627401$ et $\tau = 0,905130$

ita

ita ſe habet:

	anterioris	poſterioris
radius faciei . .	$\frac{a\alpha}{\rho\alpha+\sigma a\pm\tau(a+\alpha)\surd(\lambda-1)}$;	$\frac{a\alpha}{\rho a+\sigma\alpha\mp\tau(a+\alpha)\surd(\lambda-1)}$

ſeu numeris ſubſtitutis ſi ſit $\lambda=1+v$

radius faciei $\begin{cases}\text{anterioris} = \frac{a\alpha}{+0,150781\alpha+1,627401a\pm0,905133(a+\alpha)\surd v}\\ \text{poſterioris} = \frac{a\alpha}{+1,627401\alpha+0,190781a\mp0,905133(a+\alpha)\surd v}\end{cases}$.

Coroll. 4.

343. Sin autem ſit $\lambda<1$, ſed tamen $>0,191827$ lens eſt duplicanda ſeu ex duabus ſimplicibus componenda, quarum conſtructio, ſi ponatur $\lambda=0,191827+v$ ita ſe habet

Pro Lente radius faciei

priori $\begin{cases}\text{anterioris} = \frac{a\alpha}{-0,622915\alpha+0,113700a\pm0,905133(a+\alpha)\surd v}\\ \text{poſterioris} = \frac{a\alpha}{+1,532010\alpha+0,055390a\mp0,905133(a+\alpha)\surd v}\end{cases}$

poſteriori $\begin{cases}\text{anterioris} = \frac{a\alpha}{+0,055390\alpha+1,532010a\pm0,905133(a+\alpha)\surd v}\\ \text{poſterioris} = \frac{a\alpha}{+0,113700\alpha-0,622515a\mp0,905133(a+\alpha)\surd v}\end{cases}$

Coroll. 5.

344. At ſi ſit $\lambda=0,042165+v$, lente vtendum eſt triplicata ita conſtruenda:

Pro lente radius

Priori $\begin{cases} \text{anterioris} = \dfrac{a\alpha}{-0,854153\alpha+0,542467a \pm 0,905133(a+\alpha)\sqrt{v}} \\ \text{posterioris} = \dfrac{a\alpha}{+1,500214\alpha+0,063594a \mp 0,905133(a+\alpha)\sqrt{v}} \end{cases}$

Media $\begin{cases} \text{anterioris} = \dfrac{a\alpha}{-0,415280\alpha+1,021340a \pm 0,905133(a+\alpha)\sqrt{v}} \\ \text{posterioris} = \dfrac{a\alpha}{+1,021340\alpha-0,415280a \mp 0,905133(a+\alpha)\sqrt{v}} \end{cases}$

Posteriori $\begin{cases} \text{anterioris} = \dfrac{a\alpha}{+0,063594\alpha+1,500214a \pm 0,905133(a+\alpha)\sqrt{v}} \\ \text{posterioris} = \dfrac{a\alpha}{+0,542467\alpha-0,854153a \mp 0,905133(a+\alpha)\sqrt{v}} \end{cases}$

Coroll. 6.

345. Si denique sit $\lambda = -0,010216+v$ lens facienda est quadruplicata ita construenda:

Pro lente radius faciei

Prima $\begin{cases} \text{anterioris} = \dfrac{a\alpha}{-1,085770\alpha+0,406450a \pm 0,905133(a+\alpha)\sqrt{v}} \\ \text{posterioris} = \dfrac{a\alpha}{+1,484315\alpha+0,047655a \mp 0,905133(a+\alpha)\sqrt{v}} \end{cases}$

Secunda $\begin{cases} \text{anterioris} = \dfrac{a\alpha}{-0,670615\alpha+0,766005a \pm 0,905133(a+\alpha)\sqrt{v}} \\ \text{posterioris} = \dfrac{a\alpha}{+1,125160\alpha-0,314600a \mp 0,905133(a+\alpha)\sqrt{v}} \end{cases}$

Tertia $\begin{cases} \text{anterioris} = \dfrac{a\alpha}{-0,314600\alpha+1,125160a \pm 0,905133(a+\alpha)\sqrt{v}} \\ \text{posterioris} = \dfrac{a\alpha}{+0,766005\alpha-0,670615a \mp 0,905133(a+\alpha)\sqrt{v}} \end{cases}$

Quarta $\begin{cases} \text{anterioris} = \dfrac{a\alpha}{+0,047655\alpha+1,484315a \pm 0,905133(a+\alpha)\sqrt{v}} \\ \text{posterioris} = \dfrac{a\alpha}{+0,406450\alpha-1,085770a \mp 0,905133(a+\alpha)\sqrt{v}} \end{cases}$

Scholion

Scholion.

346. Quae in his corollariis de constructione lentis primae sunt allata, mutatis litteris a et α ad reliquas lentes facile accommodari manifestum est quarum loco, si vsus requirat, vt numeri λ', λ'' etc. sint vnitate minores, etiam lentes siue duplicatae siue triplicatae siue adeo quadruplicatae adhiberi debebunt. Ceterum hae formulae ad lentes quotcunque patent ita vt proposito lentium numero quocunque tantum literae, quae ad lentes sequentes pertinerent, sint omittendae. In his autem denominationibus statim introduximus campum apparentem numero Φ contentum, cum sit semidiameter spatii in obiecto conspicui $z = a\,\Phi$. Interim tamen campum apparentem non ad lubitum augere licet, siquidem is multiplicationis ratione et numeris π, π', π'' etc. determinatur. Hi autem numeri semper infra $\frac{1}{2}$ imo $\frac{1}{3}$ subsistunt, ac plerumque $\frac{1}{4}$ vel adeo $\frac{1}{5}$ superare nequeunt; quandoque etiam minores accipi debent, id quod formulae coroll. 2. quouis casu declarabunt, quibus semidiameter aperturae cuiusque lentis exprimitur. Quare retentis his denominationibus momenta constructionis pro quouis lentium numero determinato expendamus.

Problema 4.

347. Si instrumentum dioptricum duabus constet lentibus, quarum crassitiem negligere liceat, definire momenta quibus constructio continetur.

Solutio.

Cum hic lens fecunda fit vltima, momenta conftructionis ita fe habebunt.

1. Vt oculus imaginem in diftantia iufta confpiciat, debet effe $O = \mathfrak{E} + l$ ideoque

$O = l + \frac{ABa\Phi}{\mathfrak{B}\pi - \Phi}$. et ex conditione campi apparentis

$O = \frac{\mathfrak{B}b\pi}{\pi - \Phi}$.

2. Exponens multiplicationis m ad diftantiam b relatus praebet hanc aequationem

$m = AB\frac{b}{l}$ pro fitu erecto

fin autem fitus inuerfus defideretur, numerus m negatiue eft capiendus.

3. Pro gradu claritatis femper eft $y = \frac{bx}{ma}$.

4. Pro campo apparente habebitur haec aequatio: $\pi - \Phi = \frac{-ma\Phi}{b}$ vnde fit $\Phi = \frac{-\pi b}{ma - b}$.

5. Si diftantia O hinc prodeat negatiua, capi debet $O = 0$ feu $\frac{ABa\Phi}{\mathfrak{B}\pi - \Phi} = -l$ tum vero π eiusmodi valorem induit, vt apertura lentis ocularis non fuperet aperturam pupillae; fiet nempe $\pi\mathfrak{B}b = \omega$, hincque

$\Phi = \frac{\mathfrak{B}\pi\omega}{A\mathfrak{B}.\pi + \omega}$. vel $(B+1)\omega = -\pi l$.

6. Semidiameter confufionis fequenti formula exprimetur.

$\frac{Bbb}{4AAl}x^3\left(\frac{AAaa}{bb}P + \frac{bb}{AAaa}Q\right)$

quae

quae factis substitutionibus abit in hanc:

$$\frac{\mu m x^3}{4 a \alpha b}\left(\frac{(A+1)(\lambda(A+1)^2+\nu A)}{A^3}+\frac{\Phi(B+1)(\lambda'(B+1)^2+\nu B)}{A^2 a(\mathfrak{B}\pi-\Phi)}\right)$$

7. Vt obiectum sine margine colorato appareat, si distantia O fuerit positiua, necesse est sit:

$$\frac{\pi}{\mathfrak{B}\pi-\Phi}=0$$

8. Sin autem capiatur $O=0$, fieri debet $\frac{(A+1)\mathfrak{B}\pi}{\Phi}=0$ seu $\frac{\pi}{\mathfrak{A}\Phi}=0$.

9. Omnis autem confusio a diuersa radiorum refrangibilitate oriunda penitus tolletur si insuper fuerit

$$\frac{1}{\mathfrak{A}}+\frac{\Phi}{A\mathfrak{B}(\mathfrak{B}\pi-\Phi)}=0$$

10. Praeterea autem erit distantia lentium

$\frac{A\mathfrak{B}a\pi}{\mathfrak{B}\pi-\Phi}$ quae debet esse positiua.

11. Denique Semidiameter aperturae lentis ocularis est;

$$\frac{A\mathfrak{B}a\pi\pm\pi}{\mathfrak{B}\pi-\Phi}.\Phi$$

vbi signorum $\pm$ id capi debet, quod valorem praebet maximum siue positiuum siue negatiuum.

Coroll. I.

348. Cum sit (n°. 4) $\Phi=\frac{-\pi b}{ma-b}$, et ob multiplicationem $B=\frac{ml}{Ab}$, hincque $\mathfrak{B}=\frac{ml}{ml+Ab}$; habebitur pro loco oculi $O=\frac{-Abl(ma-b)}{mmal+Abb}$.

Tum

Tum vero eſt $\mathfrak{B}\pi - \Phi = \frac{ml\pi}{ml + Ab} + \frac{\pi b}{ma - b} = \frac{\pi(mmal + Abb)}{(ma - b).(ml + Ab)}$, vnde fit lentium diſtantia $\frac{A\mathfrak{B}a\pi}{\mathfrak{B}\pi - \Phi} = \frac{mAa(ma - b)}{mmal + Abb} l$.

Coroll. 2.

349. Cum ſit $\frac{A}{A+1} = \mathfrak{A}$ et $A = \frac{\mathfrak{A}}{1 - \mathfrak{A}}$ ſemidiameter confuſionis etiam ita exprimi poteſt vt ſit

$$\frac{\mu m x^3}{4aab}\left(\frac{\lambda + \nu\mathfrak{A}(1 - \mathfrak{A})}{\mathfrak{A}^3} + \frac{\Phi^4\lambda' + \nu\mathfrak{B}(1 - \mathfrak{B}))}{A^3\mathfrak{B}^3(\mathfrak{B}\pi - \Phi)}\right).$$

Coroll. 3.

350. Cum ſit lentium diſtantia $= \frac{A\mathfrak{B}a\pi}{\mathfrak{B}\pi - \Phi}$, obiectum ſine coloribus cerni nequit, niſi lentium diſtantia euaneſcat; ſiquidem oculus in eo loco, quem campus apparens poſtulat, teneatur. Et quoniam huic conditioni ſatisfieri nequit, multo minus tota confuſio ex n°. 9 tolli poterit.

Scholion.

351. Hic ſtatim ab inſtrumentis duabus lentibus inſtructis incepi, quoniam lens vnica facillime expeditur. Primum enim neceſſe eſt vt ſit $O = l + \alpha = l + Aa$. Tum vero multiplicatio praebet $m = \frac{b}{a}$ pro ſitu erecto, et campus apparens non terminatur ſumendo $Aa + l = 0$, ſeu $O = 0$, ita vt oculus lenti immediate debeat applicari. Semidiameter vero confuſionis erit

$= \frac{a}{4l}x^3 . P = \frac{-Aax^3}{4l} . P = -\frac{1}{4}x^3 P = \frac{-max^3}{4b} . P$ ob $l = -Aa$ et $ma = b$

Quare ſemidiameter confuſionis erit

$$\frac{\mu m x^3}{4aab}\left(\frac{(A+1)(\lambda(A+1)^2 + \nu A)}{A^3}\right) = \frac{\mu m x^3}{4aab} . \frac{\lambda + \nu\mathfrak{A}(1 - \mathfrak{A})}{\mathfrak{A}^3}.$$

Pro

Pro gradu claritatis autem habetur vt semper $y = \frac{bx}{m\alpha}$.

Obiectum porro hoc casu sine margine colorato cernetur, tota autem confusio tolli nequit, nisi sit $\dot{\alpha} = 0$ seu $A = -1$, id quod scopo lentium repugnat. Quare ad considerationem plurium lentium progrediamur.

Problema 5.

352. Si instrumentum dioptricum tribus instructum sit lentibus, quarum crassities tam sit parva, vt negligi queat, definire cuncta momenta ad constructionem dirigendam necessaria.

Solutio.

Hic igitur lens tertia erit vltima seu ocularis, ideoque momenta sequenti modo se habebunt:

1. Vt oculo imago in distantia iusta spectanda offeratur debet esse $O = \gamma + l$, ac loco γ valore substituto

$O = l + \frac{ABC\alpha\Phi}{\mathfrak{C}\pi' - \pi + \Phi}$ et ex conditione campi apparentis

$O = \frac{\mathfrak{C} c \pi'}{\pi' - \pi + \Phi}$.

2. Exponens multiplicationis m ad distantiam b relatus praebet hanc aequationem:

$m = -\frac{ABCb}{l}$ pro situ erecto.

vnde si situs inuersus desideretur, numerum m negatiue accipi conuenit.

3. Pro gradu claritatis habemus, vt semper, $y = \frac{bx}{ma}$.

4. Pro campo autem apparente definiendo habemus hanc aequationem

$$\Phi = \frac{-\pi + \pi'}{ma - b} . b.$$

5. At si distantia O hinc prodeat negatiua, vt capi oporteat $O = 0$, ideoque $\frac{ABCa\Phi}{\mathfrak{C}\pi' - \pi + \Phi} = -l$ campus apparens definiri debet ex hac aequatione $\pi'\mathfrak{C}c = \omega$ seu $\frac{AB\mathfrak{C}a\Phi\pi'}{\mathfrak{C}\pi' - \pi + \Phi} = \omega$ vel $(C+1)\omega = -\pi' l$.

6. Semidiameter confusionis vero ita exprimetur:

$$\frac{Ccc'}{4ABal} x^3 \left(\frac{AABBaa}{cc} P + \frac{BBb^4}{AAaacc} Q + \frac{cc}{AABBaa} R \right)$$

quae ob $\frac{1}{l} = \frac{-m}{ABCb}$ abit signo mutato in hanc:

$$\frac{\mu m x^3}{4aab} \left\{ \begin{array}{l} + \frac{(A+1)(\lambda(A+1)^2 + \nu A)}{A^3} \\ + \frac{\Phi(B+1)(\lambda'(B+1)^2 + \nu B)}{A^3 B^3 (\mathfrak{B}\pi - \Phi)} \\ + \frac{\Phi(C+1)(\lambda''(C+1)^2 + \nu C)}{A^3 B^3 C^3 (\mathfrak{C}\pi' - \pi + \Phi)} \end{array} \right. .$$

7. Vt obiectum sine margine colorato appareat, si quidem distantia O prodierit positiua, esse oportet.

$$\frac{\pi}{\mathfrak{B}\pi - \Phi} + \frac{\pi'}{\mathfrak{C}\pi' - \pi + \Phi} = 0$$

8. Sin autem ob istam distantiam prodeuntem negatiuam capiatur $O = 0$, margo coloratus euanescet faciendo:

$$\frac{\pi'}{\mathfrak{A}\Phi} + \frac{\pi'}{A\mathfrak{B}(\mathfrak{B}\pi - \Phi)} = \frac{\pi}{AB\mathfrak{C}(\mathfrak{B}\pi - \Phi)}$$

9. Quod

9. Quod ſi marginem coloratum tollere liceat, praeterea viſio ab omni confuſione liberabitur, ſi huic aequationi ſatisfiat.

$$\frac{1}{a}+\frac{\Phi}{A\mathfrak{B}(\mathfrak{B}\pi-\Phi)}+\frac{\Phi}{AB\mathfrak{C}(\mathfrak{C}\pi'-\pi+\Phi)}=0.$$

10. Denique effici debet, vt diſtantia lentium fiat poſitiua vnde habebitur:

Interuallum
Lentium

I et II $=\frac{A\mathfrak{B}a\Phi}{\mathfrak{B}\pi-\Phi}>0$

II et III $=\frac{ABa\Phi(\mathfrak{C}\pi'-(1-\mathfrak{B})\pi)}{(\mathfrak{B}\pi-\Phi)(\mathfrak{C}\pi'-\pi+\Phi)}>0$

11. Tandem numeros π et π' ita accipi oportet, vt aperturae lentium non fiant nimis magnae: eſt vero

Semidiameter
aperturae

Lentis ſecundae $=\frac{A\mathfrak{B}a\pi+x}{\mathfrak{B}\pi-\Phi}.\Phi$

Lentis tertiae $=\frac{AB\mathfrak{C}a\pi'+x}{\mathfrak{C}\pi'-\pi+\Phi}.\Phi.$

Coroll. I.

353. Cum ſit $C=\frac{-ml}{ABb}$, ideoque $\mathfrak{C}=\frac{-ml}{ABb-ml}$; et $\Phi=\frac{-\pi+\pi'}{ma-b}.b$, ſi hi valores ſubſtituantur erit pro loco oculi.

$$O=\frac{ABbl(ma-b)\pi'}{(mmal-ABbb)\pi'+ma(ABb-ml)\pi}.$$

Coroll. 2.

354. Semidiameter confusionis etiam ita exprimi potest vt sit

$$\frac{\mu m x^3}{4 a a b}\left(\frac{\lambda + \nu \mathfrak{A}(1-\mathfrak{A})}{\mathfrak{A}^3} + \frac{\Phi(\lambda' + \nu \mathfrak{B}(1-\mathfrak{B}))}{A^3 \mathfrak{B}^3 (\mathfrak{B}\pi - \Phi)} + \frac{\Phi(\lambda'' + \nu \mathfrak{C}(1-\mathfrak{C}))}{A^3 B^3 \mathfrak{C}^3 (\mathfrak{C}\pi' - \pi + \Phi)}\right)$$

cuius formae cohaerentia cum praecedentibus iam ita est manifesta, vt ad plures lentes facile extendi queat.

Problema 6.

355. Si instrumentum dioptricum quatuor lentibus sit instructum, quarum crassities in computum duci non mereatur, definire omnia momenta ad constructionem dirigendam necessaria.

Solutio.

Quia hic lens quarta est vltima.

1. Vt oculo imago postrema in distantia iusta offeratur debet esse $O = \delta + l$, ideoque

$O = l + \frac{ABCDa\Phi}{\mathfrak{D}\pi'' - \pi' + \pi - \Phi}$ et ex conditione campi apparentis

$O = \frac{\mathfrak{D} d \pi''}{\pi'' - \pi' + \pi - \Phi}$.

2. Exponens multiplicationis m ad distantiam b relatus, si obiectum vt hactenus situ erecto exhiberi assumatur, erit

$m = + ABCD. \frac{b}{r}$ pro situ erecto.

sin autem situs desideretur inuersus numerus m negatiue est accipiendus.

3. Pro

3. Pro gradu claritatis perpetuo habetur $y = \frac{bx}{ma}$.

4. Campus autem apparens definiri debet ex hac aequatione

$$\Phi = \frac{-\pi + \pi' - \pi''}{m\,a - b}\, b$$

hinc enim $a\Phi$ praebebit femidiametrum fpatii in obiecto confpicui, fiquidem diftantia O fuerit pofitiua.

5. Sin autem diftantia O prodeat negatiua quo cafu capi oportet $O = o$, feu $\frac{ABCDa\Phi}{\mathfrak{D}\pi'' - \pi' + \pi - \Phi} = -l$ quantitas Φ determinari debet ex hac aequatione $\pi''\mathfrak{D}d = \omega$ feu hac

$$\frac{ABC\mathfrak{D}a\pi''\Phi}{\mathfrak{D}\pi'' - \pi' + \pi - \Phi} = \omega \text{ fiue } (D+1)\omega = -\pi'' l.$$

6. Semidiameter confufionis ita reperietur expreffus:

$$\frac{\mu\, m\, x^3}{4\, a\, a\, b}\left\{\begin{array}{l} + \frac{(A+1)(\lambda(A+1)^2 + \nu A)}{A^2} \\ + \frac{\Phi(B+1)(\lambda'(B+1)^2 + \nu B)}{A^3 B^2(\mathfrak{B}\pi - \Phi)} \\ + \frac{\Phi(C+1)(\lambda''(C+1)^2 + \nu C)}{A^3 B^3 C^2(\mathfrak{C}\pi' - \pi + \Phi)} \\ + \frac{\Phi(D+1)(\lambda'''(D+1)^2 + \nu D)}{A^3 B^3 C^3 D^2(\mathfrak{D}\pi'' - \pi' + \pi - \Phi)} \end{array}\right.$$

7. Vt obiectum fine margine colorato appareat, fiquidem diftantia O fit pofitiua effe oportet:

$$\frac{\pi}{\mathfrak{B}\pi - \Phi} + \frac{\pi'}{\mathfrak{C}\pi' - \pi + \Phi} + \frac{\pi''}{\mathfrak{D}\pi'' - \pi' + \pi - \Phi} = o.$$

8. Verum fi acceperimus $O = o$, margo coloratus euanefcet, huic aequationi fatisfaciendo.

$$\frac{\pi''}{\mathfrak{A}\Phi} + \frac{\pi''}{A\mathfrak{B}(\mathfrak{B}\pi - \Phi)} + \frac{\pi''}{AB\mathfrak{C}(\mathfrak{C}\pi' - \pi + \Phi)} = \frac{\pi}{ABC\mathfrak{D}(\mathfrak{B}\pi - \Phi)} + \frac{\pi'}{ABC\mathfrak{D}(\mathfrak{C}\pi' - \pi + \Phi)}.$$

 9. Omnis

9. Omnis autem confusio a diuersa radiorum natura oriunda penitus tolletur, si satisfieri liceat huic aequationi:

$$\frac{1}{a}+\frac{\Phi}{A\mathfrak{B}(\mathfrak{B}\pi-\Phi)}+\frac{\Phi}{AB\mathfrak{C}(\mathfrak{C}\pi'-\pi+\Phi)}+\frac{\Phi}{ABC\mathfrak{D}(\mathfrak{D}\pi''-\pi'+\pi-\Phi)}=0.$$

10. Denique efficiendum est vt distantiae lentium sint positiuae vnde oriuntur formulae sequentes:

Interuallum
Lentium

I et II $=\frac{A\mathfrak{B}a\pi}{\mathfrak{B}\pi-\Phi}>0$

II et III $=\frac{ABa\Phi(\mathfrak{C}\pi'-(1-\mathfrak{B})\pi)}{(\mathfrak{B}\pi-\Phi)(\mathfrak{C}\pi'-\pi+\Phi)}>0$

III et IV $=\frac{ABCa\Phi(\mathfrak{D}\pi''-(1-\mathfrak{C})\pi')}{(\mathfrak{C}\pi'-\pi+\Phi)(\mathfrak{D}\pi''-\pi'+\pi-\Phi)}>0.$

11. Aperturae autem singularum lentium ita sunt capiendae vt sit

Semid: apert:

Lentis secundae $=\frac{A\mathfrak{B}a\pi+x}{\mathfrak{B}\pi-\Phi}.\ \Phi$

Lentis tertiae $=\frac{AB\mathfrak{C}a\pi'+x}{\mathfrak{C}\pi'-\pi+\Phi}.\ \Phi$

Lentis quartae $=\frac{ABC\mathfrak{D}a\pi''+x}{\mathfrak{D}\pi''-\pi+\pi-\Phi}.\ \Phi.$

Coroll. I.

356. Cum sit ob multiplicationem $D=\frac{ml}{ABCb}$ ideoque $\mathfrak{D}=\frac{ml}{ABCb+ml}$ et $\Phi=\frac{-\pi+\pi'-\pi''}{ma-b}.\ b$; pro loco oculi habebitur haec expressio:

$$O=\frac{-ABCbl(ma-b)\pi''}{(mmal+ABCbb)\pi''-ma(ABCb+ml)(\pi'-\pi)}.$$

Coroll.

Coroll. 2.

357. Quia hic tot occurrunt litterae determinandae, dubium nullum esse videtur, quin formulis sub n°. 7 vel 8 et 9 contentis satisfieri queat, et certum saltem est, id fieri posse, si plura adhibeantur mediorum refringentium genera.

Problema 7.

358. Si instrumentum dioptricum quinque lentibus sit instructum, quarum crassities ob paruitatem negligi queat, definire omnia momenta constructionem dirigentia.

Solutio.

Cum hic lens quinta vltimum locum teneat, habebimus.

1. Pro loco oculi, vt imago visa ipsi in iusta distantia offeratur debet esse $O = \varepsilon + l$ ideoque

$O = l + \frac{ABCDE\alpha\Phi}{\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi}$ et ex conditione campi apparentis

$$O = \frac{\mathfrak{E}\, e\, \pi'''}{\pi''' - \pi'' + \pi' - \pi + \Phi}.$$

2. Exponens multiplicationis m ad distantiam $= b$ relatus, si quidem obiectum situ erecto sit spectandum

$$m = -ABCDE. \frac{b}{r}.$$

pro situ autem inuerso numerus m negatiue capi debet.

3. Pro

3. Pro gradu claritatis eſt vt hactenus $y = \frac{bx}{ma}$.

4. Campus apparens definiri debet ex hac aequatione

$$\Phi = \frac{-\pi + \pi' - \pi'' + \pi'''}{ma - b}\, b$$

ſiquidem diſtantia illa O prodierit poſitiua.

5. Sin autem haec diſtantia prodeat negatiua vt capiatur $O = 0$, ſeu $\frac{ABCDEa\Phi}{\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi} = -l$, campus apparens non ex formula n°. 4 ſed ex hac $\pi''' \mathfrak{E} e = \omega$ definiri debet, vnde fit

$$\frac{ABCD\mathfrak{E}a\pi'''\Phi}{\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi} = \omega \text{ ſeu } (E+1)\omega = -\pi''' l$$

6. Semidiameter confuſionis ab apertura lentium oriundae ſequenti modo exprimitur,

$$\frac{\mu m x^3}{4aab}\left\{\begin{array}{l} +\frac{(A+1)(\lambda(A+1)^2+\nu A)}{A^3} \\ +\frac{\Phi(B+1)(\lambda'(B+1)^2+\nu B)}{A^3 B^3(\mathfrak{B}\pi - \Phi)} \\ +\frac{\Phi(C+1)(\lambda''(C+1)^2+\nu C)}{A^3 B^3 C^3(\mathfrak{C}\pi' - \pi + \Phi)} \\ +\frac{\Phi(D+1)(\lambda'''(D+1)^2+\nu D)}{A^3 B^3 C^3 D^3(\mathfrak{D}\pi'' - \pi' + \pi - \Phi)} \\ +\frac{\Phi(E+1)(\lambda''''(E+1)^2+\nu E)}{A^3 B^3 C^3 D^3 E^3(\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi)}. \end{array}\right.$$

7. Vt obiectum ſine margine colorato appareat, ſiquidem diſtantia O fuerit poſitiua, huic aequationi erit ſatisfaciendum:

$$\frac{\pi}{\mathfrak{B}\pi - \Phi} + \frac{\pi'}{\mathfrak{C}\pi' - \pi + \Phi} + \frac{\pi''}{\mathfrak{D}\pi'' - \pi' + \pi - \Phi} + \frac{\pi'''}{\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi} = 0.$$

8. At

8. At si distantia O sit negatiua, capiaturque $O = 0$ pro hoc scopo obtinendo oportet sit

$$\frac{\pi'''}{\mathfrak{A}\Phi} + \frac{\pi'''}{A\mathfrak{B}(\mathfrak{B}\pi-\Phi)} + \frac{\pi'''}{AB\mathfrak{C}(\mathfrak{C}\pi'-\pi+\Phi)} + \frac{\pi'''}{ABC\mathfrak{D}(\mathfrak{D}\pi''-\pi'+\pi-\Phi)}$$
$$= \frac{1}{ABCD\mathfrak{E}}\left(\frac{\pi}{\mathfrak{B}\pi-\Phi} + \frac{\pi'}{\mathfrak{C}\pi'-\pi+\Phi} + \frac{\pi''}{\mathfrak{D}\pi''+\pi'+\pi-\Phi}\right).$$

9. Omnis autem confusio a diuersa radiorum natura oriunda penitus tolletur, si fieri possit

$$\frac{1}{\mathfrak{A}} + \frac{\Phi}{A\mathfrak{B}(\mathfrak{B}\pi-\Phi)} + \frac{\Phi}{AB\mathfrak{C}(\mathfrak{C}\pi'-\pi+\Phi)} + \frac{\Phi}{ABC\mathfrak{D}(\mathfrak{D}\pi''-\pi'+\pi-\Phi)} + \frac{\Phi}{ABCD\mathfrak{E}(\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi)} = 0$$

10. Tum vero efficiendum est, vt interualla lentium omnia sint positiua, vnde oriuntur formulae sequentes.

Interuallum
Lentium

I. et II. $= \frac{A\mathfrak{B}a\pi}{\mathfrak{B}\pi-\Phi} > 0$

II. et III. $= \frac{ABa\Phi(\mathfrak{C}\pi'-(1-\mathfrak{B})\pi)}{(\mathfrak{B}\pi-\Phi)(\mathfrak{C}\pi'-\pi+\Phi)} > 0$

III. et IV. $= \frac{ABCa\Phi(\mathfrak{D}\pi''-(1-\mathfrak{C})\pi')}{(\mathfrak{C}\pi'-\pi+\Phi)(\mathfrak{D}\pi''-\pi'+\pi-\Phi)} > 0$

IV. et V. $= \frac{ABCDa\Phi(\mathfrak{E}\pi'''-(1-\mathfrak{D})\pi'')}{(\mathfrak{D}\pi''-\pi'+\pi-\Phi)(\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi)} > 0$

11. Aperturae denique singularum lentium ita sunt capiendae, vt sit:

Semid. apert:

Lentis secundae $= \frac{A\mathfrak{B}a\pi + x}{\mathfrak{B}\pi-\Phi}\Phi$

Lentis tertiae $= \frac{AB\mathfrak{C}a\pi' + x}{\mathfrak{C}\pi'-\pi+\Phi}\Phi$

Lentis quartae $= \frac{ABC\mathfrak{D}a\pi'' + x}{\mathfrak{D}\pi''-\pi'+\pi-\Phi}\Phi$

Lentis quintae $= \frac{ABCD\mathfrak{E}a\pi''' + x}{\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi}\Phi.$

Corollarium.

359. Ob multiplicationem ergo erit:

$$E = \frac{-ml}{ABCDb} \text{ et } \mathfrak{E} = \frac{-ml}{ABCDb - ml}$$

vnde oritur:

$$O = \frac{ABCDbl(ma-b)\pi'''}{(mmal - ABCDbb)\pi''' + ma(ABCDb - ml)(\pi'' - \pi' + \pi)}.$$

Scholion.

360. Hactenus ſuppoſui oculum vel in eo loco teneri, quem viſio campi apparentis poſtulat, vel lenti poſtremae immediate applicari; quorum vtroque caſu efficiendum eſt, vt imago vltima oculo ad diſtantiam iuſtam offeratur. Hic quidem locum oculi, quem campus praebet ſtatim coniunxi cum iuſta imaginis ſpectandae diſtantia; ſed etiam ſine reſpectu ſiue ad hanc diſtantiam ſiue ad multiplicationem habito definiri poteſt. Ex ſuperioribus enim §. 271 habemus

Pro caſu

vnius lentis $O = 0$

duarum lentium $O = \frac{A\mathfrak{B}a\pi\Phi}{(\pi-\Phi)(\mathfrak{B}\pi-\Phi)}$

trium lentium $O = \frac{AB\mathfrak{C}a\pi'\Phi}{(\pi'-\pi+\Phi)(\mathfrak{C}\pi'-\pi+\Phi)}$

quatuor lentium $O = \frac{ABC\mathfrak{D}a\pi''\Phi}{(\pi''-\pi'+\pi-\Phi)(\mathfrak{D}\pi''-\pi'+\pi-\Phi)}$

quinque lentium $O = \frac{ABCD\mathfrak{E}a\pi'''\Phi}{(\pi'''-\pi''+\pi'-\pi+\Phi)(\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi)}$

etc.

Verum

Verum si nolimus ad campum apparentem respicere semper eiusmodi locum oculo assignare licet, vnde obiectum sine margine colorato cernatur. Haud abs re autem fore arbitror hunc locum determinasse, quia ex eius distantia a loco oculi ob rationem campi assumto facilius iudicare licebit, quanto margine cinctum apparere debeat obiectum si oculus in loco isto posteriori teneatur. Si enim aequationi n°. 7 appositae satisfieri nequeat, nil aliud intelligimus, nisi obiectum non sine margine colorato esse appariturum at si insuper constet ille locus, vnde obiectum sine huiusmodi margine spectari posset, facilius quantitatem istius marginis colligere poterimus.

Problema 8.

361. Si instrumentum dioptricum ex lentibus quotcunque constet, quarum crassitiem negligere liceat, eum pro oculo locum assignare, vnde obiectum sine margine colorato conspiciatur.

Solutio.

Ponatur haec oculi post lentem vltimam distantia $= \Omega$, atque ex aequationibus §. 325 datis, si pro O scribatur Ω haec ipsa distantia Ω, quam quaerimus, elici potest. Scilicet pro quouis lentium numero erit vt sequitur.

 1. Pro

I. Pro vnica Lente.

Habetur haec aequatio $\frac{\Omega\, d\alpha}{\alpha(\alpha-\Omega)}=0$ vnde fit $\frac{\Omega}{\alpha(\alpha-\Omega)}=0$, seu $\frac{\Omega}{\alpha-\Omega}=0$. Verum etiam campus apparens exigit $O=0$, vnde hoc casu ambo loca oculi O et Ω conveniunt, neque margo coloratus est pertimescendus.

II. Pro duabus Lentibus

Hoc casu reperta est ista aequatio

$$\frac{\Omega\, d\beta}{\beta(\beta-\Omega)}=d\alpha\left(\frac{1}{\alpha}+\frac{1}{b}\right)$$

vnde substitutis valoribus supra assignatis oritur

$$\frac{\Omega}{\beta(\beta-\Omega)}=\frac{\pi}{\mathfrak{A}\Phi}:(B+1)AB\alpha\left(\frac{1}{\mathfrak{a}}+\frac{\Phi}{A\mathfrak{B}(\mathfrak{B}\pi-\Phi)}\right)$$

seu si ponamus

$$Y=\frac{\pi}{\mathfrak{A}\Phi}\ \text{et}\ Z=\frac{1}{\mathfrak{a}}+\frac{\Phi}{A\mathfrak{B}(\mathfrak{B}\pi-\Phi)}$$

erit

$$\frac{\Omega}{\beta-\Omega}=\frac{Y}{AB(B+1)\alpha Z}.\ \beta=\frac{Y}{(B+1)Z}\cdot\frac{\Phi}{\mathfrak{B}\pi-\Phi}.$$

III. Pro tribus Lentibus.

Eodem modo si ponamus

$$Y=\frac{\pi'}{\mathfrak{A}\Phi}+\frac{\pi'}{A\mathfrak{B}(\mathfrak{B}\pi-\Phi)}-\frac{\pi}{AB\mathfrak{C}(\mathfrak{B}\pi-\Phi)}\quad \text{et}$$

$$Z=\frac{1}{\mathfrak{a}}+\frac{\Phi}{A\mathfrak{B}(\mathfrak{B}\pi-\Phi)}+\frac{\Phi}{AB\mathfrak{C}(\mathfrak{C}\pi'-\pi+\Phi)}$$

erit pro casu trium lentium

$$\frac{\Omega}{\gamma-\Omega}=\frac{Y}{ABC(C+1)\alpha Z}.\ \gamma=\frac{Y}{(C+1)Z}\cdot\frac{\Phi}{\mathfrak{C}\pi'-\pi+\Phi}$$

vbi

vbi notandum eſt $Y=0$ et $Z=0$ eſſe eas ipſas aequationes, quae in ſuperioribus problematibus ſub numeris 8 et 9 retulimus.

IV. Pro quatuor lentibus

Si iam ponatur:

$$Y=\frac{\pi''}{\mathfrak{A}\Phi}+\frac{\pi''}{A\mathfrak{B}(\mathfrak{B}\pi-\Phi)}+\frac{\pi''}{AB\mathfrak{C}(\mathfrak{C}\pi'-\pi+\Phi)}-\frac{1}{ABC\mathfrak{D}}\left(\frac{\pi}{(\mathfrak{B}\pi-\Phi)}+\frac{\pi'}{\mathfrak{C}\pi'-\pi+\Phi}\right)$$

$$Z=\frac{1}{\mathfrak{A}}+\frac{\Phi}{A\mathfrak{B}(\mathfrak{B}\pi-\Phi)}+\frac{\Phi}{AB\mathfrak{C}(\mathfrak{C}\pi'-\pi+\Phi)}+\frac{\Phi}{ABC\mathfrak{D}(\mathfrak{D}\pi''-\pi'+\pi-\Phi)}$$

erit:

$$\frac{\Omega}{\delta-\Omega}=\frac{Y}{ABCD(D+1)\alpha Z}\,\delta$$

hincque $\frac{\Omega}{\delta-\Omega}=\frac{Y}{(D+1)Z}\cdot\frac{\Phi}{\mathfrak{D}\pi''-\pi'+\pi-\Phi}$.

V. Pro quinque Lentibus.

Poſito ſecundum aequationes n°. 8 et 9 exhibitas

$$Y=\frac{\pi'''}{\mathfrak{A}\Phi}+\frac{\pi'''}{A\mathfrak{B}(\mathfrak{B}\pi-\Phi)}+\frac{\pi'''}{AB\mathfrak{C}(\mathfrak{C}\pi'-\pi+\Phi)}+\frac{\pi'''}{ABC\mathfrak{D}(\mathfrak{D}\pi''-\pi'+\pi-\Phi)}$$
$$-\frac{1}{ABCD\mathfrak{E}}\left(\frac{\pi}{\mathfrak{B}\pi-\Phi}+\frac{\pi'}{\mathfrak{C}\pi'-\pi+\Phi}+\frac{\pi''}{\mathfrak{D}\pi''-\pi'+\pi-\Phi}\right)$$

$$Z=\frac{1}{\mathfrak{A}}+\frac{\Phi}{A\mathfrak{B}(\mathfrak{B}\pi-\Phi)}+\frac{\Phi}{AB\mathfrak{C}(\mathfrak{C}\pi'-\pi+\Phi)}+\frac{\Phi}{ABC\mathfrak{D}(\mathfrak{D}\pi''-\pi'+\pi-\Phi)}$$
$$+\frac{\Phi}{ABCD\mathfrak{E}(\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi)}$$

habebimus

$$\frac{\Omega}{\varepsilon-\Omega}=\frac{Y}{ABCDE(E+1)\alpha Z}\cdot\varepsilon \text{ vel}$$
$$\frac{\Omega}{\varepsilon-\Omega}=\frac{Y}{(E+1)Z}\cdot\frac{\Phi}{\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi}.$$

Lex harum formularum pro quouis maiori lentium numero hinc ſatis eſt perſpicua.

Coroll. 1.

362. Cum igitur supra pro quouis lentium numero exhibuerimus aequationes n°. 8. et n°. 9. formula n°. 8. dabit valorem Y et formula n°. 9. valorem ipsius Z, quibus cognitis locus oculi, vbi margo coloratus euanescit, facile innotescit.

Coroll. 2.

363. Si praeterea ex aequatione n°. 7. exposita ponatur

$$X = \frac{\pi}{\mathfrak{B}\pi - \Phi} + \frac{\pi'}{\mathfrak{C}\pi' - \pi + \Phi} + \frac{\pi''}{\mathfrak{D}\pi'' - \pi' + \pi - \Phi} + \frac{\pi'''}{\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi}$$

erit quidem pro casu quinque lentium

$$Y = \frac{\pi'''}{\Phi} Z - \frac{1}{ABCD\mathfrak{E}} X$$

Pro quatuor lentibus $Y = \frac{\pi''}{\Phi} Z - \frac{1}{ABC\mathfrak{D}} X$

Pro tribus lentibus $Y = \frac{\pi'}{\Phi} Z - \frac{1}{AB\mathfrak{C}} X$

et pro duabus lentibus $Y = \frac{\pi}{\Phi} Z - \frac{1}{A\mathfrak{B}} X$.

Scholion.

364. Quo facilius haec oculi loca littera Ω designata cum praecedentibus littera O designatis comparare liceat, notandum est esse.

Pro casu

Vnius Lentis $\frac{O}{\alpha - O} = 0$

Duarum Lentium $\frac{O}{\beta - O} = \frac{\pi}{(B+1)(\mathfrak{B}\pi - \Phi)}$

Trium Lentium $\frac{O}{\gamma - O} = \frac{\pi'}{(C+1)(\mathfrak{C}\pi' - \pi + \Phi)}$

Quatuor Lentium $\frac{O}{\delta - O} = \frac{\pi''}{(D+1)(\mathfrak{D}\pi'' - \pi' + \pi - \Phi)}$

Quinque Lentium $\frac{O}{\varepsilon - O} = \frac{\pi'''}{(E+1)(\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi)}$ etc.

Quare

Quare vt ambo loca congruant retentis pro quouis lentium numero litteris Y et Z modo adhibitis, oportet ſit

Pro vna Lente $\quad 0 = 0$

Pro duabus Lentibus $\frac{Y}{Z} = \frac{\pi}{\Phi}$

Pro tribus Lentibus $\frac{Y}{Z} = \frac{\pi'}{\Phi}$

Pro quatuor Lentibus $\frac{Y}{Z} = \frac{\pi''}{\Phi}$

Pro quinque Lentibus $\frac{Y}{Z} = \frac{\pi'''}{\Phi}$ etc.

vnde generatim fit $X = 0$, quae eſt ipſa aequatio n°. 7. allata. Haec igitur ſunt principia generalia, ex quibus conſtructio inſtrumentorum dioptricorum erit perficienda, quae quidem inſtrumenta hic potiſſimum ad viſionem accommodaui. Nihilo vero minus ad repraeſentationem obiectorum in camera obſcura ſuper Tabula alba adhiberi poſſunt, vbi praeter iam data praecepta tenendum eſt, has effigies etiam ſine margine colorato eſſe apparituras, ſi aequationibus n°. 7. ſatisfiat, omnem vero confuſionem a diuerſa radiorum natura oriundam penitus tolli, ſi inſuper aequationibus n°. 9 ſatisfiat.

Supplementum VII.

Si ratio refractionis in ſingulis lentibus fuerit diverſa et ſecundum lentium ordinem litteris n, n', n'', n''' etc. exhibeatur, inde pro lentium conſtructione

I°. Litterae reſpondentes ϱ, σ, τ et pro confuſione

fufione litterae μ et ν conuenienter definiri debent fecundum formulas fupra datas §. 55

$$\varrho = \frac{1}{2(n-1)} + \frac{1}{n+2} - 1 \,;\; \sigma = \frac{1}{2(n-1)} - \frac{1}{n+2} + 1$$

$$\tau = \tfrac{1}{3}\left(\frac{1}{2(n-1)} + \frac{1}{n+2}\right) \sqrt{(4n-1)}$$

ita, vt fit

$$\varrho + \sigma = \frac{1}{n-1} \text{ et } \sigma - \varrho = \frac{-2}{n+2} + 2 = \frac{2(n+1)}{n+2}.$$

Porro autem

$$\mu = \frac{1}{4(n+2)} + \frac{1}{4(n-1)} + \frac{1}{8(n-1)^2}$$

$$\nu = \frac{4(n-1)^2}{4n-1};$$

vnde intelligitur, quemadmodum ex ratione refractionis n' litterae refpondentes ϱ', σ', τ', μ', ν' definiri debeant.

II°. Quod iam ad elementa occafione campi apparentis introducta π, π', π'' etc. attinet; diftantiae determinatrices lentium inde eodem modo manent determinatae, vt fupra, ita, vt non opus fit, eas formulas hic transfcribere; interim tamen meminiffe iuuabit; formulas primitiuas, vnde illae funt natae, quae funt

1°. $\frac{\mathfrak{A}\pi - \Phi}{\Phi} = \frac{Aa}{b}$

2°. $\frac{\mathfrak{C}\pi' - \pi + \Phi}{\Phi} = \frac{ABa}{c}$

3°. $\frac{\mathfrak{D}\pi'' - \pi' + \pi - \Phi}{\Phi} = \frac{ABCa}{d}$

4°. $\frac{\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi}{\Phi} = \frac{ABCDa}{e}$

etc.

III.

III. Verum valores litterarum P, Q, R etc. ob diuerſas refractiones mutantur, vt ſequitur.

$$P = \frac{\mu}{A^3 a^3}(A+1)(\lambda(A+1)^2+\nu A)$$

$$Q = \frac{\mu'}{B^3 b^3}(B+1)(\lambda'(B+1)^2+\nu' B);$$

$$R = \frac{\mu''}{C^3 c^3}(C+1)(\lambda''(C+1)^2+\nu'' C);$$

$$S = \frac{\mu'''}{D^3 d^3}(D+1)(\lambda'''(D+1)^2+\nu''' D);$$

etc.

IV. Diſtantiae lentium etiam manent, vt ante, vna cum ſemidiametris aperturarum ſingularum lentium; at radii binarum facierum vtriusque lentis ita ſunt ad praeſentem caſum accommodandae, vt pro lente cuius refractio eſt n', ei reſpondentes litterae ϱ', σ', et τ' vſurpari debeant, ſimilique modo etiam pro ſequentibus lentibus, quarum refractio litteris n'', n''' etc. indicatur.

His praemiſſis ſingula momenta, quae in conſtructione inſtrumentorum dioptricorum ſunt obſeruanda, ſequenti modo repraeſentabimus.

I. Pro loco oculi ſeu eius diſtantia poſt vltimam lentem $= O$

habebimus pro ſingulis lentium numeris ſequentes determinationes:

Num. lentium.	
I.	$O = o = \alpha + l$; ideoque $\alpha = -l = Aa$
II.	$O = \frac{\mathfrak{B}b\pi}{\pi - \Phi} = \mathfrak{b} + l$; ideoque $\mathfrak{b} = \frac{\mathfrak{B}b\pi}{\pi - \Phi} - l = Bb$; $l = \frac{-Bb(\mathfrak{B}\pi - \Phi)}{\pi - \Phi}$
III.	$O = \frac{\mathfrak{C}c\pi'}{\pi' - \pi + \Phi} = \gamma + l$; hinc $\gamma = \frac{\mathfrak{C}c\pi'}{\pi' - \pi + \Phi} - l = Cc$;
	hinc $l = -\frac{Cc(\mathfrak{C}\pi' - \pi + \Phi)}{\pi' - \pi + \Phi}$
IV.	$O = \frac{\mathfrak{D}d\pi''}{\pi'' - \pi' + \pi - \Phi} = \delta + l$; hinc $\delta = \frac{\mathfrak{D}d\pi''}{\pi'' - \pi' + \pi - \Phi} - l$;
	adeoque $l = \frac{-Dd(\mathfrak{D}\pi'' - \pi' + \pi - \Phi)}{\pi'' - \pi' + \pi - \Phi}$
V.	$O = \frac{\mathfrak{E}e\pi'''}{\pi''' - \pi'' + \pi' - \pi + \Phi} = \varepsilon + l$; hinc $\varepsilon = \frac{\mathfrak{E}e\pi'''}{\pi''' - \pi'' + \pi' - \pi + \Phi} - l$;
	adeoque $l = -\frac{Ee(\mathfrak{E}\pi''' - \pi'' + \pi' - \pi + \Phi)}{\pi''' - \pi'' + \pi' - \pi + \Phi}$
	etc.

II. Pro ratione multiplicationis ad diſtantiam *b* relata

habebimus pro quolibet lentium numero ſequentes valores litterae *m*.

I.	$m = -A\frac{b}{l}$	Pro ſitu erecto, ſi *m* hinc prodierit poſitiuum; pro ſitu autem inuerſo, ſi *m* obtinuerit valorem negatiuum.
II.	$m = +AB\frac{b}{l}$	
III.	$m = -ABC\frac{b}{l}$	
IV.	$m = +ABCD\frac{b}{l}$	
V.	$m = -ABCDE\frac{b}{l}$	
	etc.	

III. Pro gradu claritatis y

ex ſuperioribus liquet, eum ſemper pari modo exprimi; quantuſcunque fuerit lentium numerus; perpetuo enim erit $y = \frac{bx}{ma}$

IV. Pro Campo apparente ſi quidem O habeat valorem poſitiuum,

eius ſemidiameter Φ pro quouis lentium numero ſequenti modo definietur:

Num. lentium.	
I.	$\Phi = \infty$ ſeu indefinitum
II.	$\Phi = \frac{-\pi b}{ma-b}$
III.	$\Phi = \frac{-\pi+\pi'}{ma-b} \cdot b$
IV.	$\Phi = \frac{-\pi+\pi'-\pi''}{ma-b} \cdot b$
V.	$\Phi = \frac{-\pi+\pi'-\pi''+\pi'''}{ma-b} \cdot b$ etc.

(IV.) Pro Campo apparente,

ſi diſtantia O fuerit negatiua, quo caſu oculus lenti vltimae immediate adplicari debet, Φ definietur ex aequationibus ſequentibus.

Num. lentium.	
I.	$\Phi = \infty$ indefinitum
II.	$\frac{A\mathfrak{B}a\Phi}{\mathfrak{B}\pi-\Phi} = \omega$
III.	$\frac{AB\mathfrak{C}a\Phi}{\mathfrak{C}\pi'-\pi+\Phi} = \omega$
IV.	$\frac{ABC\mathfrak{D}a\Phi}{\mathfrak{D}\pi''-\pi'+\pi\ \Phi} = \omega$
V.	$\frac{ABCD\mathfrak{E}a\Phi}{\pi'''-\pi''+\pi'-\pi+\Phi} = \omega$. etc.

V. Pro femidiametro confufionis habebuntur fequentes expreffiones

I. $\frac{m x^3}{4aab} \cdot \frac{\mu(A+1)(\lambda(A+1)^2+\nu A)}{A^3}$

II. $\frac{m x^3}{4aab}\left(\frac{\mu(A+1)(\lambda(A+1)^2\nu A)}{A^3}\right)+\frac{\mu'(B+1)(\lambda'(B+1)^2+\nu' B)}{A^3 B (\mathfrak{B}\pi-\Phi)}\Phi)$

III. $\frac{m x^3}{4aab}\left\{\frac{\mu(A+1)(\lambda(A+1)^2+\nu A)}{A^3}+\frac{\mu'(B+1)(\lambda'(B+1)^2+\nu' B)}{A^3 B (\mathfrak{B}\pi-\Phi)}\Phi\right.$
$\left. +\frac{\mu''(C+1)(\lambda''(C+1)^2+\nu'' C)}{A^3 B^3 C^3 (\mathfrak{C}\pi'-\pi+\Phi)}\Phi\right]$

IV. $\frac{m x^3}{4aab}\left\{\frac{\mu(A+1)(\lambda(A+1)^2+\nu A)}{A^3}+\frac{\mu'(B+1)(\lambda'(B+1)^2+\nu' B)}{A^3 B^3 (\mathfrak{B}\pi-\Phi)}\Phi\right.$
$\left.\frac{\mu''(C+1)(\lambda''(C+1)^2\nu'' C)}{A^3 B^3 C^3 (\mathfrak{C}\pi'-\pi+\Phi)}\Phi+\frac{\mu'''(D+1)(\lambda'''(D+1)^2+\nu''' D)}{A^3 B^3 C^3 D^3 (\mathfrak{D}\pi''-\pi'+\pi-\Phi)}\Phi\right.$

V. $\frac{m x^3}{4aab}\left\{\frac{\mu(A+1)(\lambda(A+1)^2+\nu A)}{A^3}+\frac{\mu'(B+1)(\lambda'(B+1)^2+\nu' B)}{A^3 B^3 (\mathfrak{B}\pi-\Phi)}\Phi\right.$
$+\frac{\mu''(C+1)(\lambda''(C+1)^2+\nu'' C)}{A^3 B^3 C^3 (\mathfrak{C}\pi'-\pi+\Phi)}\Phi+\frac{\mu'''(D+1)(\lambda'''(D+1)^2+\nu''' D)}{A^3 B^3 C^3 D^3 (\mathfrak{D}\pi''-\pi'+\pi+\Phi)}\Phi$
$\left.+\frac{\mu''''(E+1)(\lambda''''(E+1)^2+\nu'''' E)}{A^3 B^3 C^3 D^3 E^3 (\mathfrak{E}\pi'''-\pi''+\pi'-\pi+\Phi)}\Phi\right.$
etc.

VI. Pro tollendo margine colorato,

fi diftantia O prodierit pofitiua, obiectum fine margine colorato apparebit, fatisfaciendo fequentibus aequationibus:

Num. lentium.	
I.	$o = o$
II.	$o = \frac{bdn'}{n'-1} \cdot \frac{\pi}{Aa\Phi}$
III.	$o = \frac{bdn'}{n'-1} \cdot \frac{\pi}{Aa\Phi} + \frac{cdn''}{n''-1} \cdot \frac{\pi'}{ABa\Phi}$
VI.	$o = \frac{bdn'}{n'-1} \cdot \frac{\pi}{Aa\Phi} + \frac{cdn''}{n''-1} \cdot \frac{\pi'}{ABa\Phi} + \frac{ddn'''}{n'''-1} \cdot \frac{\pi''}{ABCa\Phi}$ etc.

(VI.)

(VI). Pro tollendo margine colorato.

Sin autem distantia O prodierit negatiua, margo coloratus euanescet, si sequentibus aequationibus satisfiat

I. $0 = 0$

II. $0 = \frac{adn}{n-1}(A+1)B\pi$

III. $0 = \frac{adn}{n-1}(A+1)BC\pi' + \frac{bdn'}{n'-1}\frac{(B+1)C\pi' - (C+1)\pi}{A}$

IV. $0 = \frac{adn}{n-1}(A+1)BCD\pi'' + \frac{bdn'}{n'-1}\left(\frac{(B+1)CD\pi'' - (D+1)\pi}{A}\right)$
$+ \frac{cdn''}{n''-1}\left(\frac{(C+1)D\pi'' - (D+1)\pi'}{AB}\right)$

V. $0 = \frac{adn}{n-1}(A+1)BCDE\pi''' + \frac{bdn'}{n'-1}\left(\frac{(B+1)CDE\pi''' - (E+1)\pi}{A}\right)$
$+ \frac{cdn''}{n''-1}\left(\frac{(C+1)DE\pi''' - (E+1)\pi'}{AB}\right)$
$+ \frac{ddn'''}{n'''-1}\left(\frac{(D+1)E\pi''' - (E+1)\pi''}{ABC}\right)$

etc.

VII. Pro tollenda confusione omni

insuper sequentibus aequationibus est satisfaciendum

I. $0 = \frac{adn}{n-1} \cdot \frac{A+1}{A}$

II. $0 = \frac{adn}{n-1} \cdot \frac{A+1}{A} + \frac{bdn'}{n'-1} \cdot \frac{B+1}{A^2B}$

III. $0 = \frac{adn}{n-1} \cdot \frac{A+1}{A} + \frac{bdn'}{n'-1} \cdot \frac{B+1}{A^2B} + \frac{cdn''}{n''-1} \cdot \frac{C+1}{A^2B^2C}$

IV. $0 = \frac{adn}{n-1} \cdot \frac{A+1}{A} + \frac{bdn'}{n'-1} \cdot \frac{B+1}{A^2B} + \frac{cdn''}{n''-1} \cdot \frac{C+1}{A^2B^2C}$
$+ \frac{ddn'''}{n'''-1} \cdot \frac{D+1}{A^2B^2C^2D}$

etc.

 Distan-

Diſtantiae autem lentium et ſemidiametri aperturarum perinde definiuntur, ac ſupra; tantum notetur inſuper, formulas pro ſemidiametro confuſionis exhibitas, niſi penitus ad nihilum redigi queant, aequales poni debere formulae $\frac{1}{4k^3}$, exiſtente circiter $k = 40$ (§. 193) vel adhuc minore, prout circumſtantiae poſtulauerint.

Supplementum VIII.

De Lentibus obiectiuis perfectis.

In Cap. III. nullas alias lentes obiectiuas commemoranimus, niſi [illegible] vitri ſpecie et ex principio minimi ſunt paratae, neque tales lentes ex diuerſis vitri ſpeciebus conficere conuenit, quoniam eae ad confuſionem ex diuerſa radiorum refrangibilitate oriundam, tollendam ſunt ineptae. Cum igitur iſta confuſio aliter tolli nequeat, niſi diuerſae materiae refringentes adhibeantur; hic adcuratius perpendamus, quemadmodum ex diuerſis mediis diaphanis eiusmodi lentes conſtrui queant, in quibus non ſolum prior confuſio ex apertura lentium, ſed etiam poſterior ex diuerſa radiorum refrangibilitate oriunda ad nihilum redigi queat, cuiusmodi lentes compoſitas merito *perfectas* appellare licebit.

I. Primo igitur examinemus, quomodo poſteriori confuſioni ſit occurrendum, quando quotcunque lentes inuicem iungantur, ita, vt earum diſtantiae quaſi euaneſcant. Quem in finem conſiderentur formulae ſupra

ſupra datae tam pro margine colorato vitando, quam pro confuſione penitus tollenda N°. VI, VII. At ex aequatione N°. VI. patet, marginem coloratum euaneſcere, ſi litterae π, π', π'' etc. ſint $=0$, quod quidem ſponte euenit, ſi lentes immediate coniungantur; quare in genere nullus margo coloratus eſt metuendus, ſtatim atque interualla omnia lentium euaneſcunt.

II. Vt autem aequationibus N°. VII. ſatisfiat, pro caſu duarum lentium immediate iunctarum habemus.

$$0 = \frac{a\,dn}{n-1} \cdot \frac{A+1}{A} + \frac{b\,dn'}{n'-1} \cdot \frac{B+1}{A^2 B}$$

$$0 = \frac{a\,dn}{n-1} \cdot \frac{a+\alpha}{\alpha} + \frac{b\,dn'}{n'-1} \cdot \frac{(b+\beta)a^2}{\alpha^2 \cdot \beta}$$

$$0 = \frac{dn}{n-1} \cdot \frac{a+\alpha}{a\alpha} + \frac{b\,dn'}{n'-1} \cdot \frac{b+\beta}{\alpha^2 \beta}; \text{ ſeu ob } \alpha = -b$$

$$0 = \frac{dn}{n-1} \cdot \frac{a+\alpha}{a\alpha} + \frac{dn'}{n'-1} \cdot \frac{b+\beta}{b\beta},$$

$$0 = \frac{dn}{n-1} \cdot \left(\frac{1}{a} + \frac{1}{\alpha}\right) + \frac{dn'}{n'-1} \cdot \left(\frac{1}{b} + \frac{1}{\beta}\right)$$

vbi duo hi coëfficientes denotant reciproca diſtantiarum focalium primae et ſecundae lentis.

III. Nunc inſuper effici debet, vt prior confuſio ab apertura lentium oriunda ad nihilum redigatur; vbi obſeruare debemus, hoc caſu eſſe debere $\alpha + b = 0$; $\beta + c = 0$; $\gamma + d = 0$ etc. pro numero lentium, quas coniungere velimus. Vnde ſemidiameter

confu-

confusionis ex lentibus iunctis ortae euanescet, si reddatur

$$0 = \frac{\mu(A+1)(\lambda(A+1)^2+\nu A)}{A^3} - \frac{\mu'(B+1)(\lambda'(B+1)^2+\nu' B)}{A^3\,B^3}$$
$$+ \frac{\mu''(C+1)(\lambda''(C+1)^2+\nu'' C)}{A^3\,B^3\,C^3} \text{ etc.}$$

quoniam supra iam vidimus, fore $\pi=0$, $\pi'=0$ etc. atque hinc pro quouis numero lentium inuicem iungendarum constructionem lentium perfectarum petere debemus; quas inuestigationes hic suscipiamus.

De lentibus perfectis ex duabus lentibus compositis.

Quoniam hoc casu distantiae determinatrices ipsius lentis compositae sunt a et $\mathfrak{b}$; statuamus

$$\frac{1}{a}+\frac{1}{\alpha} = f\left(\frac{1}{a}+\frac{1}{\mathfrak{b}}\right) \text{ et } \frac{1}{b}+\frac{1}{\mathfrak{b}} = g\left(\frac{1}{a}+\frac{1}{\mathfrak{b}}\right) \text{ siue}$$
$$\frac{a\alpha}{a+\alpha} = \frac{1}{f}\left(\frac{a\mathfrak{b}}{a+\mathfrak{b}}\right);\ \frac{b\mathfrak{b}}{b+\mathfrak{b}} = \frac{1}{g}\cdot\frac{a\mathfrak{b}}{a+\mathfrak{b}};$$

vnde pro prima conditione implenda sequitur:

$0 = \frac{dn}{n-1}\cdot f + \frac{dn'}{n'-1}\cdot g$. Porro ob $\frac{1}{\alpha} = \frac{f-1}{a} + \frac{f}{\mathfrak{b}}$ et $\frac{1}{b} = \frac{g}{a} + \frac{g-1}{\mathfrak{b}}$ et $\alpha + b = 0$, erit $\frac{f+g-1}{a} + \frac{f+g-1}{\mathfrak{b}} = 0$, seu $f+g=1$.

Sit breuitatis gratia $\frac{dn}{n-1} = \zeta$ et $\frac{dn'}{n'-1} = \eta$, vt sit $\zeta f + \eta g = 0$;

ex qua aequatione cum altera $f+g=1$ coniuncta vtraque littera f et g determinabitur, ita, vt sit

$$f = \frac{\eta}{\eta-\zeta} \text{ et } g = -\frac{\zeta}{\eta-\zeta}$$

deinde

deinde cum fit

$$A=\frac{\alpha}{a}=\frac{\mathfrak{b}}{\mathfrak{b}(f-1)+af}=\frac{\mathfrak{b}}{af-\mathfrak{b}g}=\frac{(\eta-\zeta)\mathfrak{b}}{a\eta+\mathfrak{b}\zeta}\text{ et}$$

$$B=\frac{\mathfrak{b}}{b}=\frac{g\mathfrak{b}+(g-1)a}{a}=\frac{g\mathfrak{b}-fa}{a}=\frac{-\zeta\mathfrak{b}-\eta a}{(\eta-\zeta)a};$$

$$\text{ergo } A+1=\frac{\eta(a+\mathfrak{b})}{a\eta+\mathfrak{b}\zeta}\text{ et } B+1=\frac{-\zeta(a+\mathfrak{b})}{a(\eta-\zeta)};$$

vnde pro confusione priori tollenda habebimus hanc aequationem

$$0=\frac{\mu\lambda.\eta^3(a+\mathfrak{b})^3}{(\eta a+\zeta\mathfrak{b})^3}+\frac{\mu\nu\eta(\eta-\zeta)(a+\mathfrak{b})\mathfrak{b}}{(\eta a+\zeta\mathfrak{b})^2}$$
$$\frac{-\mu'\lambda'.\zeta^3(a+\mathfrak{b})^3}{(\eta a+\zeta\mathfrak{b})^3}+\frac{\mu'\nu'\zeta(\eta-\zeta)(a+\mathfrak{b})a}{(\eta a+\zeta b)^2}$$

quae reducitur ad hanc

$$0=\frac{(a+\mathfrak{b})^2}{\eta a+\zeta\mathfrak{b}}(\mu\lambda\eta^3-\mu'\lambda'\zeta^3)+(\eta-\zeta)(\mu\nu\eta\mathfrak{b}+\mu'\nu'\zeta a)$$

Breuius etiam ex §. 214 deducitur haec aequatio, cui fatisfieri debet $P+Q=0$ ob $\alpha=-b$, exiftente pro hoc cafu

$$P=\mu f\left(\frac{1}{a}+\frac{1}{\mathfrak{b}}\right)\left(\lambda\left(\frac{1}{a}+\frac{1}{\mathfrak{b}}\right)^2+\frac{\nu}{a}\left(\frac{f-1}{a}+\frac{f}{\mathfrak{b}}\right)\right)\text{ et}$$

$$Q=\mu'.g\left(\frac{1}{a}+\frac{1}{\mathfrak{b}}\right)\left(\lambda'g^2\left(\frac{1}{a}+\frac{1}{\mathfrak{b}}\right)^2+\frac{\nu'}{\mathfrak{b}}\left(\frac{g}{a}+\frac{g-1}{\mathfrak{b}}\right)\right).$$

ex qua aequatione fiue λ, fiue λ' inueniri poteft, dummodo hae litterae non minores prodeant vnitate; quamobrem quosdam eiusmodi cafus euoluamus et ad praxin accommodemus.

IV. Tria autem mediorum diaphanorum genera hic potiffimum contemplabimur, quorum primum fit aqua pluuia, pro qua eft $n=\frac{4}{3}=1,3333$; fecundum vitrum ordinarium, pro quo eft $n=\frac{31}{20}=1,5500$ et

tertium vitrum Cryſtallinum, anglice Flint-Glaſs, pro quo eſt $n = \frac{8}{5} = 1,6000$; factoque calculo inuenimus, pro litteris graecis inde pendentibus ſequentes valores:

	Aqua	Vitrum commune	Vitrum Cryſtall.
n	$=1,3333$	$=1,5500$	$=1,6000$
ϱ	$=0,8000$	$=0,1907$	$=0,1111$
σ	$=2,2000$	$=1,6274$	$=1,5555$
τ	$=1,2490$	$=0,9051$	$=0,8606$
μ	$=1,9500$	$=0,9381$	$=0,8333$
ν	$=0,1025$	$=0,2326$	$=0,2666$
$\mu\nu$	$=0,1999$	$=0,2184$	$=0,2221.$

V. Quod autem ad valores differentialium dn attinet, optandum eſſet, vt ii per experimenta accuratiſſime definirentur; interim tamen duae ſequentes hypotheſes ex theoria deductae perpendi mererentur, quandoquidem *Newtoniana*, ex qua dn ipſi $n-1$ foret proportionale, ad confuſionem tollendam eſſet inepta. Prima autem hypotheſis, quam dudum propoſueram, facit dn ipſi $n.$ log. n proportionale; altera autem ex theoria attractionis deducta dat dn ipſi $\frac{nn-1}{n}$ proportionale; atque hinc valores formulae $\frac{dn}{n-1}$ pro vtraque hypotheſi euoluamus:

I^{ma} Hypotheſis $dn = n$ log. n

	Aqua	Vitrum commune	Vitrum Cryſtall.
dn	0,1665	0,2950	0,3265
$\frac{dn}{n-1}$	0,4995	0,5364	0,5442

II^{da}

II^{de} Hypothesis $dn = \frac{nn-1}{n}$

	Aqua	Vitrum commune	Vitrum Cryftall.
$\frac{dn}{n-1}$	1, 7500	1, 6452	1, 6250

VI. Verum neutra harum hypothefium cum iis experimentis, quae Celeb. *Dollondus* circa prismata inftituit, fubfiftere poteft. Vfus nempe eft duobus prismatibus, quorum alterum ex vitro communi, erat paratum, angulo refringente exiftente triginta graduum; alterum autem ex vitro chryftallino confectum continebat angulum 19°; hisque duobus prismatibus inuerfe iunctis obferuauit, in fpectro inde ad parietem proiecto iridis colores nullos adparere; vnde conclufit, (fi n pro ratione refractionis vitri communis, n' vero pro refractione vitri chryftallini affumatur) fore $dn : dn' = 2 : 3$, quam rationem vocat rationem difperfionis; quo experimento admiffo fequitur, $\frac{dn}{n-1} : \frac{dn'}{n'-1} = 8 : 11$; ita, vt pro noftro calculo foret $\zeta = 8$ et $\eta = 11$, quae ratio ab allatis hypothefibus enormiter aberrat. Prima enim dat $\zeta : \eta = 0,5364 : 0,5442$ hoc eft proxime, vt $68 : 69$. Altera autem praebet $\zeta : \eta = 1,6452 : 1,6250$ vel proxime, vti $81 : 80$, vnde patet, hypothefin pofteriorem non poffe fubfiftere, quia ex ea fequeretur $\zeta > \eta$; prior vero tantum adhuc difcrepat, vt cum experimento neutiquam conciliari poffit; ex quo merito ancipites haeremus, quomodo nos in calculo hoc gerere debe-

amus. Interim experimentum *Dollondi* nonnullis adhuc difficultatibus premi videtur, quas hic ob oculos ponere visum est.

Digressio de refractione vitri chrystallini secundum experimenta *Dollondi*.

Tab. III. Fig. 17.

I. Quo clarius theoriam, cui haec experimenta innituntur, ob oculos ponamus, rem aliquanto generalius, quam *Dollondus* fecit, consideremus. Repraesentet ergo triangulum ABC prisma triangulare ex vitro communi paratum, cuius angulus ad C, quem *Dollondus* 30° assumit, vocetur $=f$; alterum vero prisma ex vitro chrystallino factum DBE ita inuerse priori sit adplicatum, vt sit angulus $CBD=g$, ipseque angulus huius prismatis $DBE=h$ vbi in experimento examinando erat $g=0$ et $h=19^\circ$ tum vero refractio radiorum mediorum in prismate priori sit vt $m:1$, in posteriori vero vt $n:1$, ita vt facultates radios dispergendi nobis per differentialia dm et dn repraesententur. Quibus positis *Dollondus* se obseruasse affirmat, si radius solis per haec duo prismata transmittatur, spectrum inde in parietem proiectum nullis coloribus inquinari. Vnde conclusit, fore $dm:dn=2:3$, quam conclusionem adcuratius examinemus.

II. Hic primum notari conuenit, angulum, sub quo radius solis Op per faciem prioris lentis AC immittitur, non definiri, quasi perinde sit, sub quocunque

cunque angulo haec immerfio fiat, id quod pleniorem expofitionem poftulat. Ponamus igitur angulum $OPA = a$, qui eft complementum anguli incidentiae et anguli fequentes quauis refractione orti fint p, q, r, t, u, v, e et radius iterum emergens VZ; ex quo ftatim perfpicitur, fore $q = p + f$, $r = g + t$ et $u = v + b$; tum vero ex ratione refractionis intelligimus I) Cof. $a = m$. Cof. p. II) Cof. $r = m$. Cof. q. III) Cof. $t = n$. Cof. u. IV) Cof. $e = n$. Cof. v.

III. Quia iam hoc tranfitu nulla colorum difperfio obferuata effe perhibetur, neceffe eft, vt in emerfione V omnes radii colorati redditi fint inter fe paralleli, ideoque angulus e conftans perinde ac primus angulus a, etiamfi rationes refractionis m et n fuis differentialibus dm et dn augeantur; quippe qua mutatione tantum anguli p, q, r, t, u, v, fiunt variabiles.

IV. Nunc igitur primo hanc angulorum mutationem in primo prismate euoluamus, differentiando fcilicet aequationes fupra notatas; vnde erit

$dq = dp$ deinde $o = dm.\ \text{cof.}\ p - m\,dp.\ \text{fin.}\ p$ feu

$dp = \frac{dm.\,\text{cof.}\,p}{m.\,\text{fin.}\,p} = dq$;

tum vero $-dr.\ \text{fin.}\ r = dm.\ \text{cof.}\ q - m\,dq.\ \text{fin.}\ q$ feu

$-dr.\ \text{fin.}\ r = dm.\ \text{cof.}\ q - \frac{dm.\,\text{cof.}\,p.\,\text{fin.}\,q}{\text{fin.}\,p}$

$= \frac{dm}{\text{fin.}\,p}(\text{cof.}\,q\ \text{fin.}\,p - \text{cof.}\,p.\ \text{fin.}\,q) = \frac{dm.\,\text{fin.}\,(p-q)}{\text{fin.}\,p}$

hincque tandem $dr = \frac{dm.\,\text{fin.}\,f}{\text{fin.}\,p.\,\text{fin.}\,r}$ ob $p - q = -f$

Simili

Simili modo euoluantur refractiones per alterum prisma, incipiendo ab angulo constante e vbi aequatio

$$\text{cos. } e = n.\ \text{cos. } v \text{ dat } 0 = dn.\ \text{cos. } v - ndv.\ \text{sin. } v \text{ siue}$$

$$dv = \frac{dn.\ \text{cos. } v}{n.\ \text{sin. } v} = du \text{ ob } u = v + b;$$

deinde vero aequatio cos. $t = n$. cos. u dabit

$$-dt.\ \text{sin. } t = dn.\ \text{cos. } u - ndu.\ \text{sin. } u = dn.\ \text{cos. } u - \frac{dn.\ \text{cos. } v.\ \text{sin. } u}{\text{sin. } v} = \frac{dn}{\text{sin. } v}$$

$$(\text{cos. } u.\ \text{sin. } v - \text{cos. } v.\ \text{sin. } u) = \frac{dn}{\text{sin. } v}.\ \text{sin. } (v - u)$$

ergo ob $u - v = b$ habebitur $dt = + \frac{dn.\ \text{sin. } b}{\text{sin. } v.\ \text{sin. } t}$; Cum autem sit

$$r = g + t, \text{ erit } dr = dt; \text{ ergo } \frac{dm.\ \text{sin. } f}{\text{sin. } s.\ \text{sin. } r} = \frac{dn.\ \text{sin. } b}{\text{sin. } v.\ \text{sin. } t}$$

ex qua aequatione haec relatio inter dm et dn concluditur

$$dm : dn = \frac{\text{sin. } b}{\text{sin. } v.\ \text{sin. } t} : \frac{\text{sin. } f}{\text{sin. } s.\ \text{sin. } r}$$

$$dm : dn = \frac{\text{sin. } s.\ \text{sin. } r}{\text{sin. } f} : \frac{\text{sin. } v.\ \text{sin. } t}{\text{sin. } b}$$

quae proportio generaliter locum habet, quoties angulus ad e prodit constans

V. Iam hanc conclusionem ad *Dollondi* experimentum adplicemus et quia ibi est $g = 0$, hincque

$r = t$ erit ratio inter vtramque dispersionem seu

$$dm : dn = \frac{\text{sin. } p}{\text{sin. } s} : \frac{\text{sin. } v}{\text{sin. } b} = \frac{CQ}{PQ} : \frac{BU}{VU};$$

ex qua aequatione iam satis clare elucet, rationem $\frac{dm}{dn}$ ab angulo incidentiae a neutiquam esse independentem, vti *Dollondus* supposuisse videtur; ita, vt etsi difsu-

diffusio colorum pro certo quodam angulo a euanescens sit deprehensa, hanc conclusionem neutiquam ad omnes angulos incidentiae extendere liceat. Quoniam igitur hic angulus a *Dollondo* non est assignatus, ex hoc experimento nihil certi determinari poterit.

VI. Vt autem hic nullum dubium relinquatur, aliquot casus praecipuos pro angulo a calculo euoluamus, vt adpareat, quanta diuersitas inde in rationem $\frac{dm}{dn}$ ingrediatur, dum tamen plus vna vera esse nequit. Quare cum *Dollondus* inuenisset pro vtraque vitri specie a se adhibita I. $m = 1,53$ et II) $n = 1,58$, tum vero angulos $f = 30^\circ$ et $b = 19^\circ$ sequentia exempla hinc expediamus:

Exemplum I.

Angulum a ita constitutum concipiamus, vt fiat q angulus rectus, ideoque etiam r, t et u recti; erit sin. $p =$ cos. f et sin. $v =$ cos. b, ex quo coligitur $dm : dn =$ cotang. f : cotang. b; siue $\frac{dm}{dn} = 0,59639$, quae iam minor est, quam $\frac{2}{3}$

Exemplum II.

Sit $a = 90^\circ$, qui forte est ipse casus *Dollondi*; atque hinc inuenientur sequentes anguli $p = 90^\circ$; $q = 120^\circ$ $r = 139^\circ, 53' = t$. $u = 118^\circ\ 57'$. $v = 99^\circ\ 57'$. $e = 105^\circ\ 50'$. Vnde sequitur $\frac{dm}{dn} = \frac{\text{sin. } p . \text{sin. } b.}{\text{sin } s . \text{sin. } t.} = 0,6615$. ideoque proxime $dm : dn = 2 : 3$ vt habet *Dollondus*.

Cete-

Ceterum hinc iam euidens eſt, hanc rationem pendere etiam ab angulo incidentiae, quae tamen in hoc experimento non eſt commemorata.

Si ergo hanc rationem cum *Dollondo* aſſumamus, fiet porro $\frac{dm}{m-1} : \frac{dn}{n-1} = 116 : 159 = 7 : 10$, ita, vt futurum ſit pro lentibus obiectiuis duplicatis, quibus *I'ollondus* vtitur, $\zeta = 7$, $\eta = 10$, quarum conſtructionem hic ex noſtris principiis inueſtigemus.

Lens Obiectiua Clariſſ. *Dollondi* duplicata prior, anteriore lente ex vitro communi, poſteriore vero ex vitro chryſtallino parata.

Cum igitur ſit $n = 1,53$ et $n' = 1,58$, $\zeta = 7$, $\eta = 10$, erit ſtatim $f = \frac{\eta}{\eta-\zeta} = \frac{10}{3}$ et $g = \frac{-\zeta}{\eta-\zeta} = -\frac{7}{3}$;

vnde diſtantiae determinatrices vtriusque lentis erunt

Pro prima a, et $\alpha = \frac{(\eta-\zeta)a\mathfrak{b}}{\eta a+\zeta\mathfrak{b}}$

Pro poſteriori b et $\beta = \frac{-(\eta-\zeta)a\mathfrak{b}}{\eta a+\zeta\mathfrak{b}} = -\alpha$.

Dum ipſius lentis duplicatae diſtantiae determinatrices ſunt a et $\mathfrak{b}$.

Tantum igitur reſtat, vt pro his binis lentibus numeri arbitrarii λ et λ' definiantur; et quoniam quaeſtio circa lentes obiectiuas verſatur, ſtatuamus ſtatim $a = \infty$, vt adhuc reſolui debeat haec aequatio

$0 = \mu\lambda\eta^3 - \mu'\lambda'\zeta^3 + \eta(\eta-\zeta)\zeta\mu'\nu'$.

ſeu $\mu'\lambda'\zeta^3 = \mu\lambda\eta^3 + \eta\zeta(\eta-\zeta)\mu'\nu'$. quae ob

$\zeta = 7$ et $\eta = 10$ abit in hanc

$343.\ \mu'\lambda' = 1000.\ \mu\lambda + 210.\ \mu'\nu'$. II.

II. Vt igitur *Dollondum* ſequamur, pro his valoribus $n = 1,53$ et $n' = 1,58$ litteras inde deriuatas quaerere debemus.

Eruitur autem ex formulis ſuperioribus

$\mu = 0,9875$	$\mu' = 0,8724$
$l\mu = 9.9945449.$	$l\mu' = 9.9407397$
$\nu = 0,2194$	$\nu' = 0,2529$
$l.\nu = 9.3413418$	$l.\nu' = 9.4030044$
$l.\mu\nu = 9.3358867.$	$l.\mu'\nu' = 9.3437441.$

Hinc coëfficientes trium terminorum noſtrae aequationis computentur.

$l.\ 343 = 2,5352941$	$l.\ 1000 = 3,0000000$	$l.\ 210 = 2,3222193$
$l.\ \mu' = 9.9407397$	$l.\ \mu = 9.9945449$	$l.\ \mu'\nu' = 9.3437441$
2.4760338	2.9945449	1.6659634
Subtrah. - - - -	2.4760338	2.4760338
	0.5185111	9.1899296
	3,3000	0,1548

ita, vt ſit

$$\lambda' = 3,3000\,\lambda + 0,1548$$

quare ſumto

$\lambda = 1$, ſit $\lambda' = 3,4548$; ergo

$\lambda' - 1 = 2,4548$, et log. $\sqrt{\lambda' - 1} = 0,1950080.$

III. Iam ad radios facierum harum lentium definiendos, cum ſit

$$a = \infty \text{ et } \alpha = \frac{(n-\zeta)e}{\eta} \text{ et } b = -\frac{(\eta-\zeta)e}{\eta}$$

 erit

erit pro lente priore radius faciei

anterioris $= \frac{\alpha}{\sigma} = \frac{3\mathfrak{E}}{10\sigma}$; posterioris $= \frac{\alpha}{\varrho} = \frac{3\mathfrak{E}}{10\varrho}$.

Pro lente posteriore radius faciei

anter. $= \frac{-3\mathfrak{E}}{10\varrho' - 3\sigma' \pm 7\tau'\sqrt{\lambda' - 1}}$

poster. $= \frac{-3\mathfrak{E}}{10\sigma' - 3\varrho' \mp 7\tau'\sqrt{(\lambda' - 1)}}$

quare pro duplici refractione $n = 1,53$ et $n' = 1,58$ valores litterarum ϱ, σ, τ quaeri oportet, qui ita se habebunt

$n = 1,53$	$n' = 1,58$
$\varrho = 0,2266$	$\varrho' = 0,1413$
$\sigma = 1,6602$	$\sigma' = 1,5827$
$\tau = 0,9251$	$\tau' = 0,8775.$

IV. Cum igitur $\mathfrak{E}$ denotet distantiam focalem ipsius lentis duplicatae, quam deinceps littera P indicabimus, pro lente priori ex vitro communi subviridi, cuius refractio est $n = 1,53$, reperiemus vtramque faciem; erit radius faciei.

Pro prima lente

anterioris $= 0,1807.\,P$; posterioris $= 1,3239.\,P$.

Pro altera ex vitro chrystallino, cuius refractio $n = 1,58$ facies ita erunt comparatae.

Pro secunda lente

anterioris $= \frac{-3P}{-3,3351 \pm 5.6240}$

posterioris $= \frac{-3P}{15,4031 \mp 5.6240}$

quia

quia nunc radios curuaturae minores euitari conuenit, fignorum ambiguorum ea funt fumenda, quae denominatores minores producunt; id quod fit, fi figna fuperiora valeant; hinc ergo obtinebimus fequentes determinationes:

radius faciei anterioris $= \frac{-3P}{6,2889} = 0.4770.P$

posterioris $= \frac{-3P}{5,7791} = -0,5191\,P$

quae lens obiectiua capax eft aperturae, cuius femidiameter eft $= 0.0452\,P$.

Lens obiectiua duplicata altera, anteriore lente ex vitro chryftallino, pofteriori ex vitro communi parata.

I. Cum igitur hic fit

$n = 1,58$ et $n' = 1,53$ erit $\zeta = 10$ et $\eta = 7$; hinc

$f = \frac{\eta}{\eta - \zeta} = -\frac{7}{3}$. et

$g = -\frac{\zeta}{\eta - \zeta} = \frac{10}{3}$; Hincque

$a = \frac{-3\alpha\mathfrak{C}}{7\alpha + 10\mathfrak{C}} = -\frac{3\mathfrak{C}}{7}$ ob

$\alpha = \infty$, et $b = \frac{3\mathfrak{C}}{7}$.

Confufio autem primi generis vt euanefcat, fieri debet

$$0 = 343\,\mu\lambda - 1000\,\mu'\lambda' - 210\,\mu'\nu' \text{ feu}$$

$$343\,\mu\lambda = 1000\,\mu'\lambda' + 210\,\mu'\nu'.$$

II. Huius aequationis terni coëfficientes iam quaerantur:

$l.\ 343 = 2,5352941.$	$l.\ 1000 = 3,0000000$	$l.\ 210 = 2,3222193$
$l.\ \mu = 9,9407397$	$l.\ \mu' = 9,9945449$	$l.\ \mu'\nu' = 9\ 3358867$
2.4760338	2.9945449	1.6581060
	2.4760338	2.4760338
	0.5185111	9.1820722
	$3,3000$	$0,1520$

ita, vt fit

$$\lambda = 3,3000\ \lambda' + 0,1520;$$ quare fumto

$\lambda' = 1$ fiet $\lambda = 3.4520$ ac propterea

$$\lambda - 1 = 2,4520 \text{ et log. } \sqrt{(\lambda - 1)} = 0,3895205.$$

III. Cum nunc fit

$$a = \infty \text{ et } \alpha = \frac{-3\mathfrak{B}}{7} \text{ et } b = \tfrac{3}{7}\mathfrak{B}$$

erunt radii facierum vtriusque lentis

Pro lente priore radius faciei

anterioris $= \frac{-3\mathfrak{B}}{7(\sigma \pm \tau\sqrt{(\lambda-1)})}$; pofter. $= \frac{-3\mathfrak{B}}{7\varrho \pm 7\tau\sqrt{(\lambda-1)}}$

Pro lente pofteriore radius faciei

anterioris $= \frac{3\mathfrak{B}}{7\varrho' + 3\sigma'}$; pofterioris $= \frac{3\mathfrak{B}}{7\sigma' + 3\varrho'}$

eft autem

$$\varrho = 0,1413;\ \sigma = 1,5827;\ \tau = 0,8775.$$
$$\varrho' = 0,2266;\ \sigma' = 1,6602:$$

quibus

quibus ſubſtitutis erit pro priore lente radius faciei

$$\text{anter. } \frac{-3\beta}{11,3785 \pm 5,6187}; \text{ poſter.} = \frac{-3\beta}{0,3851 \mp 5,6187}$$

valeant autem ob rationes ſupra allegatas ſigna inferiora, eritque radius faciei:

$$\text{anterioris} = \frac{-3\beta}{1,4602} = -2,0545\ P$$

$$\text{poſterioris} = \frac{-3\beta}{10,5076} = -0,2828\ P.$$

Pro lente autem poſteriore erit radius faciei

$$\text{anterioris} = \frac{3\beta}{6,5661} = 0,4568\ P$$

$$\text{poſterioris} = \frac{3\beta}{12,3012} = 0,2438\ P.$$

Inter quos quatuor radios minimus eſt 0, 2438 P vnde haec lens duplicata aperturam admittere poteſt, cuius ſemidiameter $= 0, 0609$. P ita, vt haec lens maiorem admittat aperturam, quam praecedens.

De Lentibus obiectiuis ex tribus lentibus compoſitis.

I. Sit pro prima lente ratio refractionis $= n$, pro ſecunda $= n'$, pro tertia $= n''$, deinde diſtantiae determinatrices primae a et α, ſecundae b et β tertiae c et γ cum numeris arbitrariis λ, λ', λ'' eritque $\alpha + b = 0$, et $\beta + c = 0$, eruntque a et γ di-

 ſtantiae

ſtantiae determinatrices ipſius lentis triplicatae. Iam vero ſtatuatur

$$\frac{1}{a}+\frac{1}{\alpha}=f\left(\frac{1}{a}+\frac{1}{\gamma}\right);$$
$$\frac{1}{b}+\frac{1}{\beta}=g\left(\frac{1}{a}+\frac{1}{\gamma}\right);$$
$$\frac{1}{c}+\frac{1}{\gamma}=h\left(\frac{1}{a}+\frac{1}{\gamma}\right);$$

fietque

$$\frac{1}{\alpha}=\frac{f-1}{a}+\frac{f}{\gamma};\ \alpha=\frac{a\gamma}{\gamma(f-1)+af}=-b$$
$$\frac{1}{\beta}=\frac{g}{a}+\frac{g}{\gamma}+\frac{f-1}{a}+\frac{f}{\gamma}$$
$$\frac{1}{\beta}=\frac{g+f-1}{a}+\frac{g+f}{\gamma}=-\frac{1}{c}$$

ac proinde

$$\frac{-f-g+1}{a}-\frac{f-g+1}{\gamma}=\frac{h}{a}+\frac{h}{\gamma}$$

vnde ſequitur fore

$$\frac{-f-g-h+1}{a}-\frac{(f+g+h-1)}{\gamma}=0$$

hinc $f+g+h=1$.

II. Conditio vero confuſionem colorum penitus tollens poſtulat, vt ſupra vidimus, hanc aequationem

$$0=\frac{dn}{n-1}\left(\frac{1}{a}+\frac{1}{\alpha}\right)+\frac{dn'}{n'-1}\left(\frac{1}{b}+\frac{1}{\beta}\right)+\frac{dn''}{n''-1}\left(\frac{1}{c}+\frac{1}{\gamma}\right);$$

quae ponendo

$$\frac{dn}{n-1}=\zeta;\ \frac{dn'}{n'-1}=\eta;\ \frac{dn''}{n''-1}=\vartheta$$

abit in hanc formam $0=\zeta f+\eta g+\vartheta h$.

III. Vt autem confuſio ab apertura pendens deſtruatur, huic aequationi ſatisfieri debet:

$$0=\frac{\mu(A+1)(\lambda(A+1)^2+\nu A)}{A^3}-\frac{\mu'(B+1)(\lambda'(B+1)^2+\nu' B)}{A^3 B^3}$$
$$+\frac{\mu''(C+1)(\lambda''(C+1)^2+\nu'' C)}{A^3 B^3 C^3}.$$

vbi

vbi notetur esse

$$\frac{A+1}{A}=af\left(\frac{1}{a}+\frac{1}{\gamma}\right) \text{ et}$$

$$\frac{B+1}{B}=bg\left(\frac{1}{a}+\frac{1}{\gamma}\right) \text{ et}$$

$$\frac{C+1}{C}=cb\left(\frac{1}{a}+\frac{1}{\gamma}\right);$$

vnde termini priores continentes λ erunt

$$a^2\left(\frac{1}{a}+\frac{1}{\gamma}\right)^2(\mu\lambda f^2+\mu'\lambda' g^2+\mu''\lambda'' b^2)$$

termini vero tres posteriores fiunt

$$a^2\left(\frac{1}{a}+\frac{1}{\gamma}\right)\left(\frac{\mu\nu f}{a\alpha}+\frac{\mu'\nu' g}{b\beta}+\frac{\mu''\nu'' b}{c\gamma}\right)$$

adeoque prodit haec aequatio

$$0=\mu\lambda f^2+\mu'\lambda' g^2+\mu''\lambda'' b^2$$

$$+\frac{1}{\left(\frac{1}{a}+\frac{1}{\gamma}\right)^2}\left(\frac{\mu\nu f}{a\alpha}+\frac{\mu'\nu' g}{b\beta}+\frac{\mu''\nu'' b}{c\gamma}\right)$$

vbi notandum est, ex praecedentibus esse

$$\frac{1}{a\alpha}=\frac{1}{a}\left(\frac{f-1}{a}+\frac{f}{\gamma}\right);$$

$$\frac{1}{b\beta}=-\left(\frac{f-1}{a}+\frac{f}{\gamma}\right)\left(\frac{g+f-1}{a}+\frac{f+g}{\gamma}\right)$$

$$\frac{1}{c\gamma}=-\frac{1}{\gamma}\left(\frac{f+g-1}{a}+\frac{f+g}{\gamma}\right)$$

IV. Quodsi nunc ponamus $a=\infty$, sequentes aequationes habebuntur resoluendae:

$$a=\frac{\gamma}{f} \text{ et } b=\frac{-\gamma}{f};\ \frac{1}{\beta}=\frac{g+f}{\gamma} \text{ seu}$$

$$\beta=\frac{\gamma}{f+g} \text{ et } c=\frac{-\gamma}{f+g}.$$

Quibus

Quibus adiungi debent hae aequationes:

$$f+g+h=1\text{ , et }\zeta f+\eta g+\vartheta h=0\text{ et}$$

pro omni confusione tollenda

$$\text{ob }\frac{1}{a\alpha}=0\text{ ; }\frac{1}{b\beta}=\frac{-(f+g)}{\gamma^2}\text{ ; }\frac{1}{c\gamma}=\frac{-f+g}{\gamma^2}$$

habebitur haec aequatio

$$0=\mu\lambda f^3+\mu'\lambda' g^3+\mu''\lambda'' h^3$$
$$-(f+g)(\mu'\nu' fg+\mu''\nu''.h).$$

V. Hic quidem assumsimus, omnes tres materias, ex quibus hae lentes sunt confectae, esse diuersas; in praxi autem nulla ratio suadet, vt tres diuersas vitri species adhibeamus; sed potius primam et tertiam lentem ex eadem materia parari conueniet; ex quo consequimur statim

$$n''=n\text{ ; }\mu''=\mu\text{ ; }\nu''=\nu\text{ et }\vartheta=\zeta,$$

ita, vt sit

$$f+h+\frac{\eta}{\zeta}.g=0\text{ ;}$$

quae ab altera subtracta dat

$$g-\frac{\eta}{\zeta}.g=1\text{ ; hincque }g=\frac{\zeta}{\zeta-\eta}$$

vnde porro fit

$$f+h=-\frac{\eta}{\zeta-\eta}.$$

Ponamus hunc in finem

$$f-h=\frac{x}{\zeta-\eta}\text{ eritque}$$
$$f=\frac{x-\eta}{2(\zeta-\eta)}\text{ et }h=\frac{-x-\eta}{2(\zeta-\eta)}$$

ex

ex quibus valoribus prodit

$$a = \frac{2\gamma(\zeta-\eta)}{x-\eta},\ b = \frac{-2\gamma(\zeta-\eta)}{x-\eta};$$

$$\mathfrak{b} = \frac{2\gamma(\zeta-\eta)}{x+2\zeta-\eta};\ c = \frac{-2\gamma(\zeta-\eta)}{x+2\zeta-\eta}.$$

VI. Aequatio ergo, cui adhuc ſatisfieri oportet, erit

$$0 = \tfrac{1}{4}\mu\lambda(x-\eta)^3 + 2\mu'\lambda'\zeta^3 - \tfrac{1}{4}\mu\lambda''(x+\eta)^3$$
$$-\tfrac{1}{2}(x+2\zeta-\eta)(\mu'\nu'(x-\eta)\zeta-\mu\nu(x+\eta)(\zeta-\eta))$$

ex qua aequatione vel vna litterarum λ vel etiam numerus x definiri poteſt; quo poſteriori caſu hoc commodi conſequeremur, vt ſingulae litterae $\lambda = 1$ ſtatui poſſent ſicque ſingulae lentes vtpote ex principio minimi deductae facillime conſtrui poſſent; hanc autem inueſtigationem non aliter, niſi in exemplis particularibus ſuſcipere licebit.

Ponamus igitur

$$\lambda = 1,\ \lambda' = 1 : \lambda'' = 1 \text{ fietque}$$

$$0 = -\tfrac{1}{2}\mu(3\eta x^2+\eta^3)+2\mu'\zeta^3-\tfrac{1}{2}\mu'\nu'\zeta(x^2+2(\zeta-\eta)x-\eta(2\zeta-\eta))$$
$$+\tfrac{1}{2}\mu\nu(\zeta-\eta)(x^2+2\zeta x+\eta(2\zeta-\eta))$$

quae aequatio reducitur ad hanc

$$x^2(-3\mu\eta-\mu'\nu'\zeta+\mu\nu(\zeta-\eta))$$
$$+2x.(\zeta-\eta)\zeta(\mu\nu-\mu'\nu')-\mu\eta^3+4\mu'\zeta^3+\mu'\nu'\eta\zeta(2\zeta-\eta)+\mu\nu.\eta(\zeta-\eta)(2\zeta-\eta)=0.$$

VII. Adplicemus haec ad illas duas ſpecies vitri, quibus *Dollondus* vſus eſt, ac duo caſus euoluendi occurrunt. Primus igitur caſus eſto, quo tam prima, quam tertia lens ex vitro chryſtallino con-

ficitur; media autem ex vitro communi subuiridi; ita, vt sit $n = n'' = 1,58$ et $n' = 1,53$, ideoque $\zeta = 10$ et $\eta = 7$ tum vero $\mu = 0,8724$, $\nu = 0,2529$ et $l.\ \mu\nu = 9.3437441$ deinde $\mu' = 0,9875$; $\nu' = 0,2194$ et $l.\ \mu'\nu' = 9.3358867$ vnde superior aequatio reducetur ad hanc formam

$$+19,8255\, x^2 = 0,2340\, x + 4177,6081, \text{ siue}$$

$$x^2 = 0.0118\, x + 210,7190$$

cuius resolutio praebet:

$$x = 0,0059 \pm \sqrt{210,7190}$$

$$= 0,0059 \pm 14,5161$$

vnde ambo valores ipsius x sunt

$$x = 14,5220$$

$$x = -14,5102$$

Lens obiectiua triplicata perfecta prior cuius lens prima et tertia ex vitro chrystallino, pro quo $n = 1,58$, media vero ex vitro communi subuiridi, $n = 1,53$, est confecta.

I. Ob duplicem valorem ipsius x duas etiam eiusmodi lentes exhibere poterimus; sit igitur primo $x = 14,5220$ et quia est $\zeta = 10$ et $\eta = 7$, erit $f = 1,2537$; $g = 3,3333$, $h = -3.5870$

$$\frac{1}{a} = \frac{1,2537}{\gamma};\ \frac{1}{b} = \frac{-1,2537}{\gamma};\ \frac{1}{c} = \frac{4,5870}{\gamma} \text{ et } \frac{1}{e} = \frac{-4,5870}{\gamma};$$

quodsi

quodſi iam harum trium lentium radios facierum anteriorum ponamus F, F', F'', poſteriorum vero G, G', G'' ob $\lambda = \lambda' = \lambda'' = 1$,
habebimus

$$\frac{1}{F} = \frac{\varrho}{a} + \frac{\sigma}{\alpha} = \frac{\sigma}{\alpha} = \frac{1,2537 \cdot \sigma}{\gamma}; \quad \frac{1}{G} = \frac{\varrho}{\alpha} + \frac{\sigma}{a} = \frac{\varrho}{\alpha} = \frac{1,2537\,\varrho}{\gamma} \text{ et}$$

$$\frac{1}{F'} = \frac{\varrho'}{b} + \frac{\sigma'}{\beta}; \quad \frac{1}{G'} = \frac{\varrho'}{\beta} + \frac{\sigma'}{b}$$

et ſubſtitutis valoribus

$$\frac{1}{F'} = \frac{-1,2537 \cdot \varrho'}{\gamma} + \frac{4,5870 \cdot \sigma'}{\gamma}$$

$$\frac{1}{G'} = \frac{4,5870 \cdot \varrho'}{\gamma} - \frac{1,2537 \cdot \sigma'}{\gamma}$$

ſimili modo pro lente tertia

$$\frac{1}{F''} = \frac{\varrho}{c} + \frac{\sigma}{\gamma}; \quad \frac{1}{G''} = \frac{\varrho}{\gamma} + \frac{\sigma}{c}$$

$$\frac{1}{F''} = \frac{-4,5870 \cdot \varrho + \sigma}{\gamma}; \quad \frac{1}{G''} = \frac{\varrho - 4,5870 \cdot \sigma}{\gamma}$$

exiſtente

$$\varrho = 0,\ 1413;\ \sigma = 1,\ 5827 \text{ et}$$
$$\varrho' = 0,\ 2266;\ \sigma' = 1,\ 6602$$

ac ſi ipſius lentis triplicatae diſtantia focalis ponatur $= P$, vt hic ſit $\gamma = P$, reperiuntur valores ſequentes

$\frac{1}{F} = \frac{1,9843}{P}$, ſiue $F = +0,\ 5039.\ P.$

$\frac{1}{G} = \frac{0,1771}{P}$, hincque $G = +5,\ 6450.\ P.$

$\frac{1}{F'} = \frac{7,3312}{P}$, ſiue $F' = +0,\ 1364\ P.$

$\frac{1}{G'} = \frac{-1,0420}{P}$, hincque $G' = -0,\ 9597.\ P.$

$\frac{1}{F''} = \frac{0,9346}{P}$, hincque $F'' = +1,\ 0699.\ P.$

$\frac{1}{G''} = \frac{-7,1186}{P}$, hincque $G'' = -0,\ 1404.\ P.$

II. Inter hos ſex radios cum minimus ſit o, 1364. P, eius pars quarta $= 0,0341$. P dat ſemidiametrum maximae aperturae, cuius haec lens triplicata eſt capax. Contra vero haec lens hac praerogatiua gaudet, quod eius conſtructio ob $\lambda = 1$; $\lambda' = 1$; $\lambda'' = 1$ in praxi minimae difficultati ſit obnoxia.

Altera lens triplicata, pro qua pariter, vt ante, lens prima et tertia ex vitro chryſtallino, media vero ex vitro communi ſubviridi paratur.

I. Sumatur iam

$$x = -14,5102, \text{ ob } \zeta = 10, \text{ et } \eta = 7 \text{ erit}$$

$$f = -3,5850, g = 3,3333; h = +1,2517.$$

$$\frac{1}{a} = \frac{-3,5850}{\gamma}; \frac{1}{b} = \frac{3,5850}{\gamma}$$

$$\frac{1}{\mathfrak{c}} = \frac{-0,2517}{\gamma}; \frac{1}{c} = +\frac{0,2517}{\gamma}$$

vnde pro radiis ſingularum facierum prodibunt ſequentes formulae:

$$\frac{1}{F} = \frac{\sigma}{a} = \frac{-3,5850.\sigma}{\gamma}; \quad \frac{1}{G} = \frac{\rho}{a} = \frac{-3,5850.\rho}{\gamma}$$

$$\frac{1}{F'} = \frac{\rho'}{b} + \frac{\sigma'}{\mathfrak{c}} = \frac{3,5850.\rho' - 0,2517.\sigma'}{\gamma}$$

$$\frac{1}{G'} = \frac{\rho'}{\mathfrak{c}} + \frac{\sigma'}{b} = \frac{-0,2517\rho' + 3,5850\sigma'}{\gamma}$$

$$\frac{1}{F''} = \frac{\rho}{c} + \frac{\sigma}{\gamma} = \frac{0,2517\rho + \sigma}{\gamma}$$

$$\frac{1}{G''} = \frac{\rho}{\gamma} + \frac{\sigma}{c} = \frac{\rho + 0,2517.\sigma}{\gamma}$$

vnde

vnde sequentes valores determinati pro his radiis inveniuntur:

$\frac{1}{F} = -\frac{5,6729}{P}$ hincque $F = -0,1762.\ P.$

$\frac{1}{G} = -\frac{0,5065}{P}$ hincque $G = -1,9741.\ P.$

$\frac{1}{F'} = \frac{0,3945}{P}$ hincque $F' = +2,5349\ P.$

$\frac{1}{G'} = \frac{5,8962}{P}$, hincque $G' = +0,1696.\ P.$

$\frac{1}{F''} = \frac{1,6145}{P}$, hincque $F'' = +0,6194.\ P.$

$\frac{1}{G''} = \frac{0,5396}{P}$ hincque $G'' = +1,8532.\ P.$

quorum sex radiorum minimus est 0,1696 P, cuius pars quarta 0,0424. P dat semidiametrum aperturae maximae, cuius haec lens triplicata est capax. Ideoque haec lens obiectiua praecedenti est anteferenda.

VIII. Secundum §. VII. adhuc alius casus euolui debet, quo lens prima et tertia ex vitro communi subuiridi $n = n'' = 1,53$; media autem ex Chrystallino $n = 1,58$ parari potest; sicque erit $\zeta = 7$, $\eta = 10$. Vnde cum fiat

$$\mu = 0.9875,\ l.\ \mu\nu = 9.3358867$$
$$\mu' = 0.8724.\ l.\ \mu'\nu' = 9.3437441.$$

aequatio nostra pro confusione tollenda sequentem induet formam:

$$31,8198\ x^2 = +0,1638\ x + 245,2121 \text{ siue}$$
$$x^2 = 0,0051.\ x + 7,7063$$

 cuius

cuius resolutio dat

$$x = 0,0025 \pm 2,7760$$

vnde bini valores ipsius x erunt

$$\text{I. } x = 2,7785$$

$$\text{II. } x = -2.7735.$$

Lens triplicata perfecta prior, cuius lens prima et tertia ex vitro communi, $n=1,53$. media ex chrystallino, $n=1,58$ est confecta.

Cum igitur sit

$x = 2,7785$ et $\zeta = 7, \eta = 10$ erit

$f = +1,2036$; $g = -2,3333$. $h = 2,1297$;

hincque

$$\frac{1}{\alpha} = \frac{1,2036}{\gamma}, \quad \frac{1}{b} = \frac{-1,2036}{\gamma}, \quad \frac{1}{\mathfrak{b}} = \frac{-1,1297}{\gamma}, \quad \frac{1}{c} = \frac{1,1297}{\gamma}$$

ex quibus colligitur

$$\frac{1}{F} = \frac{\sigma}{\alpha} = \frac{1,2036 \, \sigma}{\gamma}; \quad \frac{1}{G} = \frac{\rho}{\alpha} = \frac{1,2036 \, \rho}{\gamma}$$

$$\frac{1}{F'} = \frac{\rho'}{b} + \frac{\sigma'}{\mathfrak{b}} = \frac{-1,2036 \, \rho' - 1,1297 \, \sigma'}{\gamma}$$

$$\frac{1}{G'} = \frac{\rho'}{\mathfrak{b}} + \frac{\sigma'}{b} = \frac{-1,1297 \, \rho' - 1,2036 \, \sigma'}{\gamma}$$

$$\frac{1}{F''} = \frac{\rho}{c} + \frac{\sigma}{\gamma} = \frac{1,1297 \, \rho + \sigma}{\gamma}$$

$$\frac{1}{G''} = \frac{\sigma}{c} + \frac{\rho}{\gamma} = \frac{1,1297 \, \sigma + \rho}{\gamma}$$

vbi

vbi $\varrho = 0,2266$; $\sigma = 1,6602$

$\varrho' = 0,1413$; $\sigma' = 1,5827$

Facto igitur calculo obtinebimus:

$\frac{1}{F} = \frac{1,9982}{P}$ hincque $F = 0,5004.\ P.$

$\frac{1}{G} = \frac{0,2727}{P}$ hincque $G = 3,6665.\ P.$

$\frac{1}{F'} = \frac{-1,9580}{P}$ hincque $F' = -0,5107.\ P.$

$\frac{1}{G'} = \frac{-2,0645}{P}$ hincque $G' = -0,4843.\ P.$

$\frac{1}{F''} = \frac{1,0161}{P}$ hincque $F'' = 0,5219.\ P.$

$\frac{1}{G''} = \frac{2,1021}{P}$ hincque $G'' = 0,4757.\ P.$

inter quos radios cum minimus sit 0, 4757 P eius pars quarta 0, 1189. P dat semidiametrum aperturae, quam haec lens admittit; ideoque haec lens duabus praecedentibus est anteferenda.

Lens triplicata perfecta altera, cuius lens prima et tertia ex vitro communi $n = 1,53$ media ex chrystallino, $n = 1,58$ est parata.

Sumatur iam ex binis ipsius x valoribus alter negatiuus $x = -2.7735$; et cum sit $\zeta = 7$, $\eta = 10$, inuenitur

$f = +2,1289$; $g = -2,3333$ $h = +1,2044$;

hincque

$\frac{1}{\alpha} = \frac{2,1280}{\gamma}$; $\frac{1}{b} = \frac{-2,1286}{\gamma}$

$\frac{1}{\beta} = \frac{-0,1044}{\gamma}$; $\frac{1}{c} = \frac{0,1044}{\gamma}$

ex

ex quibus consequimur

$$\frac{1}{F}=\frac{\sigma}{\alpha}=\frac{2,1289.\sigma}{\gamma};\quad \frac{1}{G}=\frac{\rho}{\alpha}=\frac{[illegible]\,\rho}{\gamma}$$

$$\frac{1}{F'}=\frac{\rho'}{b}+\frac{\sigma'}{6}=\frac{-2,1285.\rho'-0,1044\,\sigma'}{\gamma}$$

$$\frac{1}{G'}=\frac{\rho'}{6}+\frac{\sigma'}{b}=\frac{-0,1044.\rho'-2,1285.\sigma'}{\gamma}$$

$$\frac{1}{F''}=\frac{\rho}{c}+\frac{\sigma}{\gamma}=\frac{0,1044\,\rho+\sigma}{\gamma}$$

$$\frac{1}{G''}=\frac{\sigma}{c}+\frac{\rho}{\gamma}=\frac{0,1044\,\sigma+\rho}{\gamma}.$$

Vnde facto calculo pro radiis facierum singularum lentium sequentes inuenientur radii.

$\frac{1}{F}=\frac{3,5344}{p}$; seu $F=0,2829.\ P.$

$\frac{1}{G}=\frac{0,4824}{p}$; seu $G=2,0729.\ P.$

$\frac{1}{F'}=\frac{-0,4660}{p}$; seu $F'=-2,1459.\ P.$

$\frac{1}{G'}=\frac{-3,3841}{p}$; seu $G'=-0,2955.\ P.$

$\frac{1}{F''}=\frac{1,6828}{p}$; seu $F''=0,5938.\ P.$

$\frac{1}{G''}=\frac{0,3999}{p}$; seu $G''=2,5006.\ P.$

Inter quos radios minimus est 0, 2829. P, cuius pars quarta 0, 0707. P, dat semidiametrum aperturae maximae, cuius haec lens triplicata est capax.

IX. Hae quatuor lentes triplicatae ideo prae ceteris, quas proponere liceret, sunt commendandae, quod in iis assumsimus $\lambda=1$; $\lambda'=1$; $\lambda''=1$. Quam ob caussam in praxi facillime construi possunt, ex iisdem vero principiis etiam constructio eiusmodi lentium com-

compoſitarum deduci poſſet qua inter binas lentes vitreas aqua aliudue fluidum includeretur, ita, vt lens media ex fluido conſtaret: tum autem praeter conditiones ante tractatas duae nouae eſſent implendae; ſcilicet vt radii facierum mediae lentis aequales eſſent ſtatuendi et contrarii radiis facierum internarum primae ac tertiae lentis. Ob has igitur nouas conditiones non amplius liceret numeros λ, λ' et λ'' vnitati aequales aſſumere; qua poſitione conſtructioni practicae quam maxime conſulitur; deinde vero etiam radii facierum tam parui prodirent, vt tales lentes nimis exiguam aperturam eſſent admiſſurae; quam ob cauſſam operae pretium haud videtur, in earum conſtructionem adcuratius inquirere.

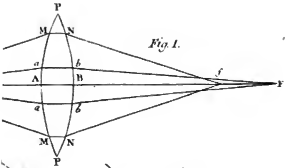


Fig. 1.

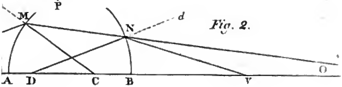


Fig. 2.

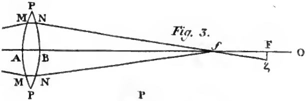


Fig. 3.

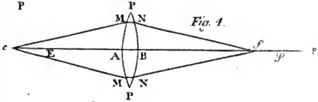


Fig. 4.

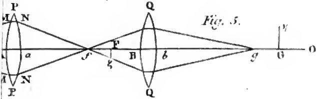


Fig. 5.

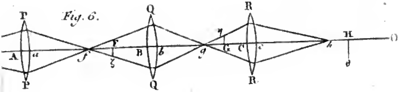


Fig. 6.

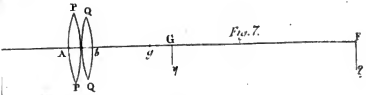


Fig. 7.

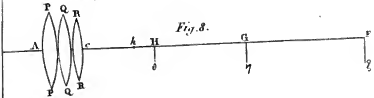


Fig. 8.

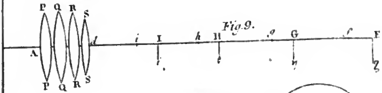


Fig. 9.

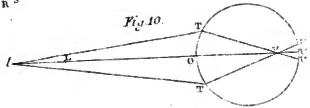


Fig. 10.

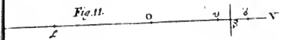


Fig. 11.

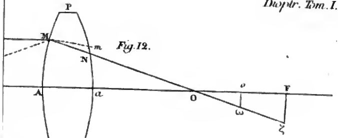


Fig. 12.

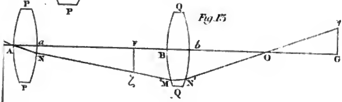


Fig. 13.

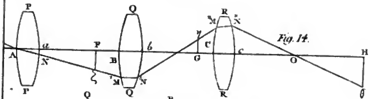


Fig. 14.

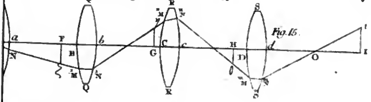


Fig. 15.



Fig. 16.

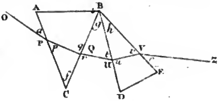


Fig. 17.